

Example 9: Solving Equations Using Inverse Trigonometric Functions

Solve the equation $6\sin x - 2 = \sin x$ on the interval $[0, 2\pi)$.

Solution

$$\begin{aligned} 6\sin x - 2 &= \sin x \\ 5\sin x &= 2 \\ \sin x &= 0.4 \end{aligned}$$

The trigonometric function can be easily isolated.

$$x = \sin^{-1}(0.4) \quad \text{or} \quad x = \pi - \sin^{-1}(0.4)$$

Note that since $\sin x$ is positive, x lies in the first or second quadrant. The solution in the first quadrant is $\sin^{-1}(0.4)$, and the solution in the second quadrant has $\sin^{-1}(0.4)$ as its reference angle.

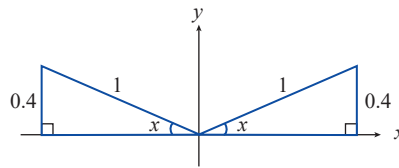


FIGURE 5

9.4 EXERCISES**PRACTICE**

Use trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equations. See Examples 1 through 7.

- $2\sin x + 1 = 0$
- $4\sin^2 x + 2 = 3$
- $\sqrt{2} - 2\cos x = 0$
- $4\cos x = 2$
- $2\cos x - \sqrt{3} = 0$
- $\sin(2x) = \sqrt{3}\cos(2x)$
- $-\frac{1}{\sqrt{48}\sin x} = \frac{1}{8}$
- $\sin^2 x - \sin x = 2\sin x - 2$
- $\sqrt{3}\tan x + 1 = -2$
- $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$
- $\sec^2 x - 1 = 0$
- $\sin^2 x = \sin^2 x + \cos^2 x$
- $\sec x + \tan x = 1$
- $\cos x + \sin x \tan x = 2$
- $\sin^2 x + \cos^2 x + \tan^2 x = 0$
- $\cos^2 x = 3\sin^2 x$
- $2\cos^2 x - 3 = 5\cos x$
- $\sin^3 x = \sin x$
- $\frac{\cot(2x)}{\sqrt{3}} = -1$
- $\cos x - 1 = \sin x$

Use trigonometric identities, algebraic methods, and inverse trigonometric functions, as necessary, to solve the following trigonometric equations on the interval $[0, 2\pi)$. See Examples 8 and 9.

21. $2 \sin^2 x + 7 \sin x = 4$ 22. $\tan^2 x = \tan x + 6$
23. $2 \cos^2 x - 1 = 0$ 24. $\sec^2 x - 3 = -\tan x$
25. $0.05 \sin^3 x = 0.1 \sin x$ 26. $12 \sin x - 1 = 8 \sin x$
27. $2 \cos^2 x + 11 \cos x = -5$ 28. $10 \sin x - 16 = -11 + 18 \sin x$
29. $\sec^2 x - 2 + 8 \tan x = 42$ 30. $\sin(-x) = 5 \sin x - 3$

Verify that the x -values given are solutions to the given equation.

31. $2 \cos x + 1 = 0$ 32. $3 \sec^2 x - 4 = 0$
- a. $x = \frac{2\pi}{3}$ b. $x = \frac{4\pi}{3}$ a. $x = \frac{\pi}{6}$ b. $x = \frac{5\pi}{6}$
33. $2 \sin^2 x - \sin x - 1 = 0$ 34. $\tan^2(3x) = 3$
- a. $x = \frac{\pi}{2}$ b. $x = \frac{7\pi}{6}$ a. $x = \frac{\pi}{9}$ b. $x = \frac{2\pi}{9}$
35. $\csc^4 x - 4 \csc^2 x = 0$ 36. $3 \cot^2 x - 1 = 0$
- a. $x = \frac{\pi}{6}$ b. $x = \frac{5\pi}{6}$ a. $x = \frac{\pi}{3}$ b. $x = \frac{2\pi}{3}$
37. $2 \cot x + 1 = -1$ 38. $\csc^2 x = 2 \cot x$
- a. $x = \frac{3\pi}{4}$ b. $x = \frac{7\pi}{4}$ a. $x = \frac{\pi}{4}$ b. $x = \frac{5\pi}{4}$
39. $2 \sec x + 1 = \sec x + 3$ 40. $2 \sin^2(2x) = 1$
- a. $x = \frac{\pi}{3}$ b. $x = \frac{5\pi}{3}$ a. $x = \frac{\pi}{8}$ b. $x = \frac{3\pi}{8}$

Determine if the value given is a solution to the trigonometric equation. If the value of x is not a solution, give all solutions to the equation.

41. $2 \cos x = -1$; $x = \frac{4\pi}{3} + 2n\pi$ 42. $\tan(3x)(\tan x - 1) = 0$; $x = \frac{\pi}{4} + n\pi$
43. $3 \sec^2 x = 4$; $x = \frac{\pi}{6} + n\pi$ 44. $\sin^2 x - 3 \cos^2 x = 0$; $x = \frac{\pi}{3} + 2n\pi$
45. $\sqrt{3} \csc x = 2$; $x = \frac{2\pi}{3} + 2n\pi$ 46. $2 \sin^2 x - 1 = 0$; $x = \frac{\pi}{4} + n\pi$
47. $\tan x = -\sqrt{3}$; $x = \frac{\pi}{6} + n\pi$ 48. $\tan^2(3x) - 3 = 0$; $x = \frac{2\pi}{9} + \frac{n\pi}{3}$
49. $3 \cot^2 x = 1$; $x = \frac{\pi}{3} + n\pi$
50. $\cos(2x)(2 \cos x + 1) = 0$; $x = \frac{5\pi}{6} + n\pi$

Use trigonometric identities and algebra, as necessary, to rewrite the following equations to be quadratic-like, and then solve each on the interval $[0, 2\pi)$.

51. $2\sin^2 x - \sin x - 1 = 0$

52. $2\sin^2 x + 3\cos x - 3 = 0$

53. $\sin x - \cos x - 1 = 0$

54. $\tan x + \sqrt{3} = \sec x$

55. $\cos(2x) - \cos x = 0$

56. $2\cos^2 x - \sqrt{3}\cos x = 0$

57. $\csc^2 x - 2\cot x = 0$

Solve the following equations on the interval $[0^\circ, 360^\circ)$. Give the exact answers when appropriate; otherwise, round your answers to one decimal place.

58. $\sin^2 x \cos x - \cos x = 0$

59. $\cos^2 x = \sin^2 x$

60. $\tan x = \cot x$

61. $2\sin x = \csc x + 1$

62. $\sec^2 x - 2\tan x = 4$

63. $\sin^2 x = 2\sin x - 3$

64. $2\cos^2 x - 1 = -2\cos x$

65. $2\sin x \cot x + \sqrt{3}\cot x - 2\sqrt{3}\sin x - 3 = 0$

Solve the algebraic and trigonometric equations given. Restrict the solutions of the trigonometric equations to the interval $[0, 2\pi)$. Give the exact answers for s ; round your answers for t to four decimal places.

66. $6s^2 - 13s + 6 = 0$; $6\cos^2 t - 13\cos t + 6 = 0$

67. $s^2 + s - 12 = 0$; $\sin^2 t + \sin t - 12 = 0$

68. $2s^2 + 7s - 15 = 0$; $2\tan^2 t + 7\tan t - 15 = 0$

69. $4s^2 - 4s - 1 = 0$; $4\cos^2 t - 4\cos t - 1 = 0$

APPLICATIONS

70. If an arrow is shot by an archer with an initial velocity v_0 at an angle of θ in reference to the horizontal, then its range, the horizontal distance it travels, is given by $r = \frac{1}{32}v_0^2 \sin(2\theta)$. If the initial velocity is $v_0 = 100$ feet per second and the arrow hits a target 300 feet from where the archer is standing, what is the value, in degrees, of the angle θ ? Round your answer to one decimal place.

71. A baseball leaves a bat at an angle of θ in reference to the horizontal. The initial velocity is $v_0 = 95$ feet per second. The ball is caught 160 feet from where it is hit. What is the value, in degrees, of the angle θ if the range, the horizontal distance traveled by the ball, is given by $r = \frac{1}{32}v_0^2 \sin(2\theta)$? Round your answer to one decimal place.

 TECHNOLOGY

Use a graphing utility to approximate the solutions of the given equation on the interval $[0, 2\pi)$. Round your answers to four decimal places.

72. $x \tan x - 3 = 0$

73. $2 \sin x + \cos x = 0$

74. $2 \cos^2 x - \sin x = 0$

75. $\cot^2 x - \sec^2 x = 0$

76. $2 \sin x - \csc^2 x = 0$

77. $2 \sin x = 1 - 2 \cos x$

78. $\log x = -\sin x$

79. $\sin\left(\frac{x}{2}\right) = 2 \cos(2x)$

Use a graphing utility to solve the following equations on the interval $[0^\circ, 360^\circ)$. Remember to change the mode to degrees.

80. $2 \sin(2x) = \sqrt{3}$

81. $\sin(3x) - \frac{1}{2} = 0$

82. $2 \sin(4x) - 1 = 0$

 WRITING & THINKING

83. While working Exercises 66–69, what did you observe as the maximum number of real solutions the algebraic equations can have?
84. While working Exercises 66–69, what did you observe as the maximum number of real solutions the trigonometric equations can have on the interval $[0, 2\pi)$?
85. While working Exercises 80–82, what did you observe about the solutions to the equations of the form $y = \sin(ax)$ on the interval $[0^\circ, 360^\circ)$?