



FIGURE 7: $f(x) = 2 \sin\left(x - \frac{\pi}{3}\right)$

9.2 EXERCISES

💡 PRACTICE

Use the sum and difference identities to determine the exact value of each of the following expressions. See Examples 1, 2, and 3.

1. $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$
2. $\sin\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)$
3. $\tan\left(\frac{4\pi}{3} + \frac{5\pi}{4}\right)$
4. $\sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$
5. $\tan\left(\frac{\pi}{3} - \frac{3\pi}{4}\right)$
6. $\tan\left(\frac{4\pi}{3} - \frac{5\pi}{4}\right)$
7. $\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$
8. $\sin\left(\frac{5\pi}{4} - \frac{\pi}{3}\right)$
9. $\sin\left(\frac{7\pi}{4} + \frac{2\pi}{3}\right)$
10. $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$
11. $\tan 75^\circ$
12. $\tan 15^\circ$
13. $\sin 165^\circ$
14. $\cos(-15^\circ)$
15. $\tan 255^\circ$
16. $\cos 195^\circ$
17. $\cos 165^\circ$
18. $\sin\left(\frac{\pi}{12}\right)$
19. $\tan\left(\frac{5\pi}{12}\right)$
20. $\cos\left(\frac{7\pi}{12}\right)$
21. $\cos\left(\frac{25\pi}{12}\right)$
22. $\sin\left(\frac{13\pi}{12}\right)$
23. $\sin\left(\frac{11\pi}{12}\right)$
24. $\tan\left(\frac{7\pi}{12}\right)$
25. $\sin\left(\frac{5\pi}{12}\right)$
26. $\tan\left(\frac{\pi}{12}\right)$

27. Suppose that $\sin \alpha = \frac{4}{5}$ and $\sin \beta = \frac{5}{13}$ and both α and β are in quadrant I. Find $\cos(\alpha - \beta)$.
28. Suppose that $\sin \alpha = -\frac{15}{17}$ and $\cos \beta = -\frac{3}{5}$ and the terminal sides of both α and β are in quadrant III. Find $\sin(\alpha - \beta)$.
29. Suppose that $\cos \alpha = -\frac{15}{17}$ and $\cos \beta = -\frac{3}{5}$, the terminal side of α is in quadrant II, and the terminal side of β is in quadrant III. Find $\sin(\alpha + \beta)$.
30. Suppose that $\cos \alpha = -\frac{24}{25}$ and $\sin \beta = \frac{5}{13}$, the terminal side of α is in quadrant III, and β is in quadrant I. Find $\cos(\alpha + \beta)$.
31. Suppose that $\cos \alpha = \frac{2}{5}$ and $\cos \beta = \frac{1}{5}$ and both α and β are in quadrant I. Find $\sin(\beta - \alpha)$.
32. Suppose that $\cos \alpha = -\frac{2}{3}$ and $\sin \beta = -\frac{2\sqrt{2}}{3}$, the terminal side of α is in quadrant III, and the terminal side of β is in quadrant IV. Find $\tan(\alpha + \beta)$.

Use the sum and difference identities to rewrite each of the following expressions as a trigonometric function of one angle, and then evaluate the result. See Example 4.

33. $\sin 15^\circ \cos 30^\circ + \cos 15^\circ \sin 30^\circ$
34. $\cos\left(\frac{5\pi}{12}\right) \cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{5\pi}{12}\right) \sin\left(\frac{2\pi}{3}\right)$
35. $\frac{\tan 100^\circ + \tan 35^\circ}{1 - \tan 100^\circ \tan 35^\circ}$
36. $\sin 125^\circ \cos 35^\circ - \cos 125^\circ \sin 35^\circ$
37. $\frac{\tan\left(\frac{5\pi}{16}\right) - \tan\left(\frac{\pi}{16}\right)}{1 + \tan\left(\frac{5\pi}{16}\right) \tan\left(\frac{\pi}{16}\right)}$
38. $\cos 15^\circ \cos 15^\circ - \sin 15^\circ \sin 15^\circ$
39. $\sin 70^\circ \cos 80^\circ + \cos 70^\circ \sin 80^\circ$
40. $\cos\left(\frac{\pi}{5}\right) \cos\left(\frac{3\pi}{10}\right) - \sin\left(\frac{\pi}{5}\right) \sin\left(\frac{3\pi}{10}\right)$
41. $\cos 182^\circ \cos 47^\circ + \sin 182^\circ \sin 47^\circ$
42. $\frac{\tan\left(\frac{5\pi}{12}\right) + \tan\left(\frac{3\pi}{4}\right)}{1 - \tan\left(\frac{5\pi}{12}\right) \tan\left(\frac{3\pi}{4}\right)}$
43. $\frac{\tan 70^\circ - \tan 10^\circ}{1 + \tan 70^\circ \tan 10^\circ}$

$$44. \sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$$

Evaluate each of the following expressions. See Example 6.

$$45. \tan\left(\cos^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{12}{13}\right)\right)$$

$$46. \cos\left(\arctan 1 + \arccos\left(\frac{1}{2}\right)\right)$$

$$47. \sin\left(\arctan \sqrt{3} + \arctan\left(\frac{4}{3}\right)\right)$$

$$48. \tan\left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \cos^{-1}\left(\frac{5}{13}\right)\right)$$

Express each of the following as an algebraic function of x . See Example 7.

$$49. \sin\left(\sin^{-1}(2x) + \cos^{-1}(2x)\right)$$

$$50. \sin\left(\arctan(2x) - \arccos(2x)\right)$$

$$51. \cos\left(\arctan(2x) - \arcsin x\right)$$

$$52. \cos\left(\cos^{-1} x - \sin^{-1} x\right)$$

$$53. \cos\left(\arccos x + \arcsin(2x)\right)$$

$$54. \sin\left(\sin^{-1} x - \cos^{-1} x\right)$$

Express each of the following functions in terms of a single sine function, and graph the result. See Example 9.

$$55. f(x) = \sin x + \cos x$$

$$56. g(x) = \sin x + \sqrt{3} \cos x$$

$$57. h(\beta) = \sin(2\beta) - \cos(2\beta)$$

$$58. f(\theta) = -\sqrt{3} \sin \theta + \cos \theta$$

$$59. g(u) = 5 \sin(5u) + 12 \cos(5u)$$

$$60. h(v) = 8 \cos\left(\frac{v}{2}\right) + 6 \sin\left(\frac{v}{2}\right)$$

WRITING & THINKING

Use the sum and difference identities to verify the following identities. See Example 5.

$$61. \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad (\text{Hint: Use sine and cosine.})$$

$$62. \cos^2 u - \sin^2 v = \cos(u+v)\cos(u-v)$$

$$63. \cos\left(\frac{3\pi}{2} - \alpha\right) = -\sin \alpha$$

$$64. \sin(\beta - \theta) + \sin(\beta + \theta) = 2 \sin \beta \cos \theta$$

$$65. \tan\left(\alpha - \frac{5\pi}{4}\right) = \frac{\tan \alpha - 1}{1 + \tan \alpha}$$

$$66. \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$67. \tan(\pi + 2\pi) = 0$$

$$68. \sin\left(\frac{5\pi}{6} + \theta\right) = \frac{1}{2}(\cos \theta - \sqrt{3} \sin \theta)$$

$$69. \sin(u+v)\sin(u-v) = \sin^2 u - \sin^2 v \quad 70. \cos\left(\frac{7\pi}{4} - \beta\right) = \frac{\sqrt{2}}{2}(\cos \beta - \sin \beta)$$

71. Use a cofunction identity to prove the sum and difference identities for sine. (Hint: Note that $\sin(u+v) = \cos\left(\frac{\pi}{2} - (u+v)\right) = \cos\left(\left(\frac{\pi}{2} - u\right) - v\right)$ and apply the difference identity for cosine.)

72. Given sum identities for sine and cosine, prove the sum identity for tangent.

73. Prove or disprove that $\sin(u+v) + \sin(u-v) = 2 \sin u \cos v$.

74. Prove or disprove that $\frac{\cos(u+v)}{\cos u \cos v} = \tan u + \tan v$.

75. Prove or disprove that $\frac{\cos(u-v)}{\cos(u+v)} = 2 \tan u \tan v$.

76. Prove or disprove that $\frac{\sin(u+v)}{\sin(u-v)} = \frac{\tan u + \tan v}{\tan u - \tan v}$.

77. Use the sine and cosine difference formulas to prove $\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$.

TECHNOLOGY

Using a graphing utility, determine whether the following identities are true or false.

(Hint: Graph both expressions on each side of the equality separately and determine if the graphs coincide.)

78. $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$

79. $\cos\left(\theta - \frac{3\pi}{2}\right) = -\sin \theta$

80. $\cot(\pi + \theta) = -\tan \theta$

81. $\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$

82. $\sin\left(\frac{\pi}{6} + \theta\right) = \frac{1}{2}(\cos \theta + \sqrt{3} \sin \theta)$

83. $\frac{1 + \tan \theta}{1 - \tan \theta} = -\tan \theta$