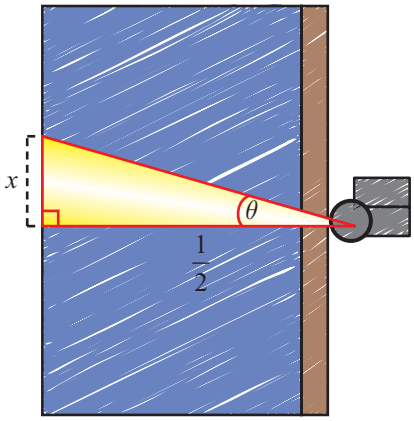


FIGURE 9

Example 4: Using Inverse Trigonometric Functions

A lighthouse is to be constructed half a mile from a long, straight reef, as shown in Figure 9. In order to ensure the light illuminates certain portions of the reef within specified lengths of time, the engineer needs a formula for θ in terms of x. Find such a formula.

Solution

From Figure 9, we see that $\tan\theta = \dfrac{x}{\frac{1}{2}} = 2x$, so the formula for θ is

$$\theta = \tan^{-1}(2x).$$

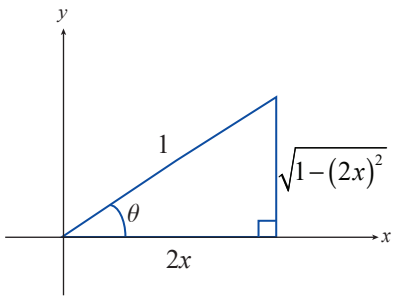


FIGURE 10

Example 5: Using Inverse Trigonometric Functions

Express $\sin\left(\cos^{-1}(2x)\right)$ as an algebraic function of x, assuming $-\dfrac{1}{2} \le x \le \dfrac{1}{2}$.

Solution

Let $\theta = \cos^{-1}(2x)$. Then $\cos\theta = 2x$, and we are led to consider a sketch like the one in Figure 10.

In the sketch, we have chosen the simplest lengths for the adjacent side and the hypotenuse that make $\cos\theta = 2x$, though any positive multiple of these lengths would also work. And as always, once the lengths of two sides of the right triangle have been determined, the Pythagorean Theorem provides the length of the third side. Now we can refer to the sketch to see that

$$\sin\left(\cos^{-1}(2x)\right) = \sin\theta = \frac{\sqrt{1-4x^2}}{1} = \sqrt{1-4x^2}.$$

8.6 EXERCISES

PRACTICE

Evaluate each of the following expressions without the use of a calculator. See Example 1.

1. $\sin^{-1}(-1)$ **2.** $\cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$ **3.** $\tan^{-1}1$ **4.** $\cot^{-1}\left(-\dfrac{\sqrt{3}}{3}\right)$

5. $\sec^{-1}\left(\dfrac{2\sqrt{3}}{3}\right)$ **6.** $\csc^{-1}(-2)$ **7.** $\arcsin 0$ **8.** $\arccos(-1)$

9. $\arctan\left(-\sqrt{3}\right)$ **10.** $\operatorname{arccot}\left(-\sqrt{3}\right)$ **11.** $\operatorname{arcsec} 2$ **12.** $\operatorname{arccsc}\left(\sqrt{2}\right)$

13. $\operatorname{arccot}(-1)$ **14.** $\tan^{-1}\left(\dfrac{\sqrt{3}}{3}\right)$ **15.** $\cos^{-1}\left(-\dfrac{1}{2}\right)$ **16.** $\csc^{-1}2$

17. $\arcsin\left(-\frac{1}{2}\right)$ **18.** $\sec^{-1}(-1)$ **19.** $\operatorname{arccsc} 1$ **20.** $\arctan 0$

21. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ **22.** $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ **23.** $\operatorname{arcsec}(-2)$ **24.** $\cot^{-1}\left(-\sqrt{3}\right)$

Evaluate each of the following expressions, if possible, using a calculator and rounding your answers to four decimal places if necessary.

25. $\sin^{-1}(-0.2)$ **26.** $\cos^{-1} 4$ **27.** $\sin^{-1}(-0.9)$

28. $\tan^{-1} 5$ **29.** $\cos^{-1}(-0.4)$ **30.** $\tan^{-1}(0.8)$

Some calculators are not equipped with arccosecant, arcsecant, and arccotangent buttons, but expressions involving these functions can still be evaluated. To evaluate, for example, $\csc^{-1} x$, let $\theta = \csc^{-1} x$. Then

$$\csc\theta = x$$
$$\frac{1}{\sin\theta} = x$$
$$\sin\theta = \frac{1}{x}$$
$$\theta = \sin^{-1}\left(\frac{1}{x}\right).$$

This means that arccosecant, arcsecant, and arccotangent can all be evaluated using the following formulas.

$$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$$
$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$$
$$\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right), \text{ with } \cot^{-1} 0 = \frac{\pi}{2}$$

Use these formulas to evaluate each of the following expressions, if possible, using a calculator and rounding your answers to four decimal places if necessary.

31. $\csc^{-1} 5$ **32.** $\sec^{-1}(-0.5)$ **33.** $\cot^{-1} 150$

34. $\cot^{-1}(0.2)$ **35.** $\csc^{-1}(-8.9)$ **36.** $\sec^{-1} 2$

Evaluate each of the following expressions, if possible. See Example 2.

37. $\cos^{-1}\left(\cos\left(\frac{2\pi}{4}\right)\right)$ **38.** $\sin^{-1}\left(\sin\left(\frac{3\pi}{2}\right)\right)$ **39.** $\tan\left(\tan^{-1}(0.5)\right)$

40. $\sin^{-1}\left(\sin\left(\frac{7\pi}{6}\right)\right)$ **41.** $\cos\left(\cos^{-1}(-0.8)\right)$ **42.** $\tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right)$

Evaluate each of the following expressions, if possible, using a calculator and rounding your answers to four decimal places if necessary. See Example 3.

43. $\sin\left(\arctan\left(0.4\right)\right)$

44. $\sin^{-1}\left(\cos\left(\frac{3\pi}{2}\right)\right)$

45. $\cos\left(\tan^{-1}\left(0.5\right)\right)$

46. $\arcsin\left(\tan 1\right)$

47. $\tan\left(\cos^{-1}\left(-0.8\right)\right)$

48. $\tan^{-1}\left(\cos 5\right)$

Find the value of each of the following expressions without using a calculator.

49. $\sin\left(\arctan\left(\sqrt{3}\right)\right)$

50. $\cos\left(\sec^{-1}\left(-2\right)\right)$

51. $\tan\left(\operatorname{arccot} 1\right)$

52. $\csc\left(\arccos\left(-\frac{\sqrt{3}}{2}\right)\right)$

53. $\tan\left(\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

54. $\sec\left(\csc^{-1}\left(\frac{2\sqrt{3}}{3}\right)\right)$

55. $\cos\left(\cot^{-1}\left(-1\right)\right)$

56. $\sec\left(\arcsin\left(-\frac{1}{2}\right)\right)$

57. $\cot\left(\operatorname{arcsec}\left(\sqrt{2}\right)\right)$

58. $\cot\left(\operatorname{arccsc}\left(-2\right)\right)$

59. $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

60. $\sec\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)$

61. $\sec\left(\arccos\left(-\frac{\sqrt{2}}{2}\right)\right)$

62. $\tan\left(\csc^{-1}\left(-2\right)\right)$

63. $\sin\left(\operatorname{arcsec}\left(\sqrt{2}\right)\right)$

64. $\csc\left(\cot^{-1}\left(\sqrt{3}\right)\right)$

65. $\cot\left(\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

66. $\cos\left(\arctan\left(-\frac{\sqrt{3}}{3}\right)\right)$

Express each of the following functions as a purely algebraic function. See Example 5.

67. $\tan\left(\cos^{-1} x\right)$

68. $\cot\left(\sin^{-1}\left(\frac{2}{x}\right)\right)$

69. $\sec\left(\tan^{-1}\left(3x\right)\right)$

70. $\tan\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+3}}\right)\right)$

71. $\sin\left(\sec^{-1} x\right)$

72. $\cos\left(\tan^{-1}\left(\frac{x}{4}\right)\right)$

Using a calculator, find the value of θ in degrees. Remember to make sure your calculator is in the correct mode.

73. $\theta = \sin^{-1}\left(0.74184113\right)$

74. $\theta = \arctan\left(-0.258416\right)$

75. $\theta = \operatorname{arccsc}\left(1.847526\right)$

76. $\theta = \sec^{-1}\left(-1.1224539\right)$

77. $\theta = \cot^{-1}\left(0.57496998\right)$

Using a calculator, find the value of θ in radians. Remember to make sure your calculator is in the correct mode.

78. $\theta = \arccos(-0.1115598)$

79. $\theta = \operatorname{arccot}(1.547773)$

80. $\theta = \tan^{-1}(5.999999)$

81. $\theta = \csc^{-1}(-1.333333)$

82. $\theta = \arcsin(0.65937229)$

Sketch the graph of each of the following functions. Then graph the function using a graphing utility to check your answer.

83. $f(x) = \sin^{-1}(x-3)$

84. $f(x) = \sec^{-1}(2x)$

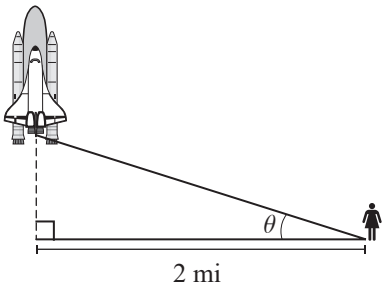
85. $f(x) = \arctan\left(\frac{x}{2}\right)$

86. $f(x) = 2\arccos x$

APPLICATIONS

87. Kim is watching a space shuttle launch from an observation spot two miles away from the launch pad. Find the angle of elevation to the shuttle for each of the following heights. Round your answers to four decimal places.

a. 0.5 miles **b.** 2 miles **c.** 2.8 miles



88. Jesse is rowing in the men's singles race. The length of the oar from the side of the shell to the water is 7 feet. At what angle is the oar from the side of the boat when the blade is at the following distance from the boat? Round your answers to four decimal places.

a. 2 feet **b.** 3 feet **c.** 5 feet