We've now gained quite a lot of experience with the process of graphing transformed trigonometric functions. The periodic nature of trigonometric functions means, however, that the reverse process of identifying a function from its graph is not so clear-cut, as our last example illustrates.

Example 5: Identifying a Trigonometric Graph

Find a function corresponding to the graph shown in Figure 6.

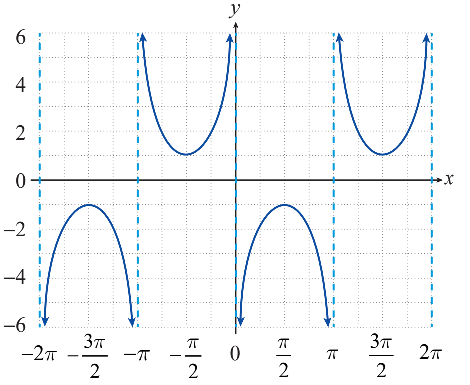

FIGURE 6

Solution

The graph shown in Figure 6 is certainly similar to the graph of cosecant in many ways, but after a transformation of some sort. One such transformation that may come to mind is that the graph appears to be the reflection of cosecant with respect to the y-axis, which corresponds to replacing x with $-x$ in the definition of the function. So the graph in Figure 6 could be of the function $f(x)=\csc(-x)$. However, someone else may look at the graph and see it as a horizontal shift to the left by π units of the graph of cosecant, which would be the function $g(x)=\csc(x+\pi)$. A third person may recognize it as the graph of cosecant reflected with respect to the x-axis, which would be the function $h(x)=-\csc x$. Once we recall that cosecant is an odd function, we remember that $\csc(-x)=-\csc x$, so we shouldn't be surprised that f and h are two possible functions corresponding to the given graph. In fact, f, g, and h are just three of an infinite number of functions all having the graph in Figure 6.

8.5 EXERCISES

PRACTICE

Sketch the graph of each of the following functions. See Examples 1 through 4.

1. $f(x)=\csc\left(\frac{3\pi}{4}x\right)$

2. $g(x)=\tan\left(3\pi x-\frac{\pi}{2}\right)$

3. $g(x)=\frac{1}{3}\sec(2x)$

4. $f(x)=-5\cot(\pi x)$

5. $g(x) = \csc\left(\frac{3\pi}{2}x - \frac{1}{2}\right)$

6. $g(x) = \cot\left(\frac{\pi x}{4}\right)$

7. $f(x) = 5\tan\left(3\pi - \frac{\pi}{2}x\right)$

8. $f(x) = 4 + \csc\left(1 - \frac{5\pi}{4}x\right)$

9. $f(x) = 1 - \cot\left(x - \frac{\pi}{2}\right)$

10. $g(x) = 1 + \tan\left(\pi x - \frac{\pi}{4}\right)$

11. $f(x) = 4 + \tan\left(x + \frac{3\pi}{2}\right)$

12. $f(x) = 1 - 2\sec(2\pi x)$

13. $g(x) = 2 + \frac{5}{6}\sec\left(\frac{1}{2}x - \pi x\right)$

14. $f(x) = \frac{1}{2}\tan\left(\frac{3}{4}x - 2\pi\right) + 3$

WRITING & THINKING

15. Sketch the graph of the cotangent function, using each of the following approaches and noting how they produce the same result.

a. Use the identity $\cot x = \frac{\cos x}{\sin x}$.

b. Use the identity $\cot x = \frac{1}{\tan x}$.

c. Use the identity $\cot x = \tan\left(\frac{\pi}{2} - x\right)$ and the fact that tangent is an odd function.

16. Sketch the graph of the secant function, using each of the following approaches and noting how they produce the same result.

a. Use the identity $\sec x = \frac{1}{\cos x}$.

b. Use the identity $\sec x = \csc\left(\frac{\pi}{2} - x\right)$ and the fact that cosecant is an odd function.