Example 9: Modeling Damped Harmonic Motion

Sketch the graph of $f(t) = -4e^{-t}\cos(6\pi t)$.

Solution

We've already graphed the function $f(t) = -4\cos(6\pi t)$ (this function was our simple model for the motion of the grocer's basket in Example 8). The factor of e^{-t} provides the desired damping effect. In Figure 17, the graphs of $4e^{-t}$ and $-4e^{-t}$ are included to show how they describe the "envelope" of amplitude modulation. The result is that the magnitude of the displacement of the grocer's basket decreases over time. Notice, however, the period is unaffected.

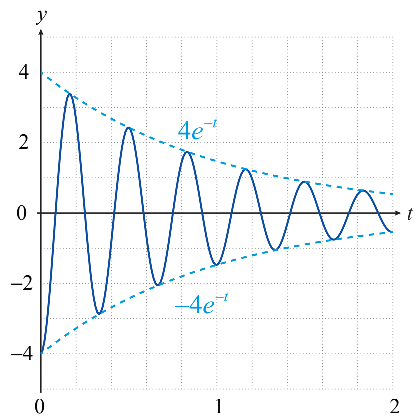

FIGURE 17

In the exercises to follow, you'll be asked to sketch graphs of similar products of damping factors and trigonometric functions.

8.4 EXERCISES

PRACTICE

Determine the amplitude and frequency of each of the following functions. Then sketch one complete cycle of each function starting at $x = 0$. See Example 4.

1. $f(x) = 2\cos(2x)$

2. $g(x) = -5\sin(\pi x)$

3. $g(x) = \dfrac{\sin(3x)}{2}$

4. $f(x) = \dfrac{\cos\left(\frac{\pi x}{2}\right)}{3}$

Determine the amplitude, period, and phase shift of each of the following functions. See Examples 6 and 7.

5. $f(x)=2\cos x$

6. $g(x)=\frac{3}{2}\sin x$

7. $f(x)=5+4\cos x$

8. $f(x)=\sin(x-5)$

9. $h(x)=-\sin x$

10. $h(x)=\frac{\cos x}{2}$

11. $g(x)=-3\cos(x+7)$

12. $f(x)=\frac{2}{3}\sin x$

13. $f(x)=2\sin(2x)$

14. $h(x)=-3\cos\left(\frac{1}{2}x\right)$

15. $g(x)=\frac{3\sin(\pi\theta)}{2}$

16. $g(x)=\cos(3\pi\theta-2)$

17. $f(x)=0.5\sin(8x+1)$

18. $h(x)=7\cos\left(x\cdot\frac{\pi}{2}+\frac{3}{2}\right)$

19. $g(x)=\frac{8\cos(2\pi x+4)}{5}$

20. $g(x)=2-\frac{3}{4}\sin(-3+x)$

Sketch the graph of each of the following functions. See Examples 6 and 7.

21. $f(x)=\cos(\pi x)$

22. $g(x)=-2\sin(5x)$

23. $g(x)=3\sin(x-2\pi)$

24. $g(x)=\sin\left(x-\frac{\pi}{4}\right)$

25. $f(x)=4\cos\left(\frac{3x}{2}+\frac{\pi}{2}\right)$

26. $g(x)=2\cos(4x-2)$

27. $f(x)=\cos(x-\pi)$

28. $g(x)=3\sin(4x)$

29. $f(x)=-\sin(2\pi x)$

30. $g(x)=1+\sin(x-2\pi)$

31. $f(x)=2-\cos(2\pi x)$

32. $g(x)=5-2\sin\left(x-\frac{\pi}{2}\right)$

33. $f(x)=-3+5\cos x$

34. $g(x)=2-\sin\left(2x-\frac{\pi}{4}\right)$

35. $f(x)=\frac{1}{2}-5\sin\left(\frac{1}{2}x-\frac{\pi}{2}\right)$

36. $g(x)=1-\frac{1}{4}\cos\left(\frac{1}{4}x-\frac{\pi}{2}\right)$

Sketch each of the following functions modeling damped harmonic motion. See Example 9.

37. $g(t) = -2e^{-t}\cos(5\pi t)$

38. $f(t) = e^{-t}\sin\left(\frac{3\pi}{4}t\right)$

39. $g(t) = e^{t}\sin\left(3t - \frac{\pi}{2}\right)$

40. $g(t) = 3e^{-t}\cos\left(5t - \frac{\pi}{2}\right)$

41. $f(t) = -3 + 5e^{-t}\cos t$

42. $f(t) = -5e^{t}\cos\left(\frac{3\pi}{2}t\right)$

43. $f(t) = \frac{1}{2}e^{-t}\sin\left(\frac{5}{6}t - 4\pi\right) + 2$

44. $g(t) = 2 + e^{-t}\sin\left(t - \frac{\pi}{4}\right)$

APPLICATIONS

In Exercises 45–46, use the relationship between frequency and period to answer the question. See Example 5.

45. Many grandfather clocks have a pendulum that swings with a period of two seconds. What is the frequency of such a pendulum?

46. A heart rate of 1200 beats per minute (bpm) is typical for a hummingbird. What is the length of the period, in seconds, of such a heart rate?

47. A baby is playing with a toy attached above his head on a coiled spring. The baby pulls the toy down a distance of 3 inches from its equilibrium position, and then releases it. The time for one oscillation is 2 seconds. Find the amplitude and period, then give the function for its displacement.

48. A pull cord for a lighted ceiling fan is swinging back and forth. The end of the cord swings a total distance of 4 inches from end to end at an average speed of 9 inches per second. Find the period of oscillation.

49. Marcel is bouncing a basketball at an average speed of 10 ft/s with the ball coming up to his waist on each bounce. The distance from the ground to his waist is approximately 3 feet. Find the amplitude and period, then give the function for its displacement.

50. A leaf floating on the water of a perfectly calm pond is suddenly disturbed by a series of waves caused by a landing duck. At time $t = 0$ seconds, the leaf initially bobs upward 5 centimeters, and then continues to oscillate up and down with a period of 2 seconds.

a. Find a function that models the simple harmonic motion described.
b. Assuming the amplitude of the waves diminishes over time by a factor of $e^{\frac{-t}{5}}$, modify your SHM model accordingly and graph the result.

WRITING & THINKING

51. **a.** Find a transformation of cosine that shifts its graph to the left so as to coincide with the graph of sine.

b. Using n to represent an arbitrary integer, find an expression that describes the infinite number of transformations of cosine that are equal to sine.

52. Prove the even and odd identities for the secant and cotangent functions.