

What we were not given initially is the length labeled  $y$  and the actual angle  $\theta$ . But from Figure 16, we can now recognize that the triangle is a familiar one and that  $\theta' = \frac{\pi}{6}$ , so  $\theta = \frac{5\pi}{6}$ . Finally, it must be the case that  $y = 1$ , and therefore

$$\tan \theta = -\frac{1}{\sqrt{3}}.$$

### Example 8: Using the Relationships between Trigonometric Functions

Given that  $\cot \theta = 0.4$  and  $\theta$  lies in the first quadrant, determine  $\sin \theta$ .

#### Solution

All trigonometric functions are ratios, so it will probably be useful to express cotangent as a fraction. The result will help us construct a right triangle that relates to the given information. To that end, note that  $\cot \theta = 0.4 = \frac{4}{10} = \frac{2}{5}$ . If we take the numerator and denominator as the lengths of the adjacent and opposite sides of a right triangle, we are led to the diagram in Figure 17.

Now we can use the Pythagorean Theorem to determine that  $r = \sqrt{4 + 25} = \sqrt{29}$ , and so  $\sin \theta = \frac{5}{\sqrt{29}}$ .

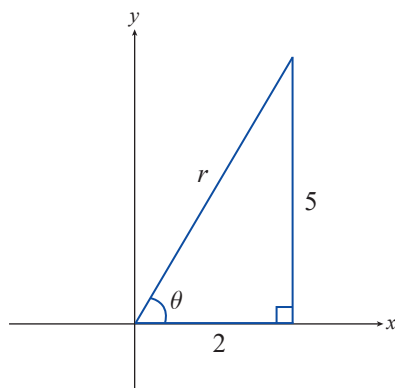


FIGURE 17

## 8.3 EXERCISES

### PRACTICE

Determine the point  $(x, y)$  on the unit circle associated with each real number  $s$ . See Example 1.

- |                         |                    |                          |
|-------------------------|--------------------|--------------------------|
| 1. $s = \frac{\pi}{6}$  | 2. $s = 225^\circ$ | 3. $s = -120^\circ$      |
| 4. $s = -\frac{\pi}{4}$ | 5. $s = \pi$       | 6. $s = -\frac{8\pi}{3}$ |
| 7. $s = -750^\circ$     | 8. $s = 855^\circ$ | 9. $s = \frac{31\pi}{6}$ |

Determine all real numbers  $s$  associated with each point  $(x, y)$  on the unit circle. See Example 2.

- |  |   |
|--|---|
| 10. $(x, y) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ | 11. $(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  |
| 12. $(x, y) = (0, -1)$   | 13. $(x, y) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ |

Determine the values of the six trigonometric functions of each angle  $\theta$ , using a calculator and rounding your answers to four decimal places if necessary. See Examples 3, 4, and 5.

- |                               |                               |                           |
|-------------------------------|-------------------------------|---------------------------|
| 14. $\theta = 45^\circ$       | 15. $\theta = \frac{\pi}{2}$  | 16. $\theta = 60^\circ$   |
| 17. $\theta = \frac{3\pi}{4}$ | 18. $\theta = \frac{5\pi}{2}$ | 19. $\theta = -520^\circ$ |
| 20. $\theta = 305^\circ$      | 21. $\theta = -1105^\circ$    | 22. $\theta = 6\pi$       |
| 23. $\theta = 670^\circ$      | 24. $\theta = \frac{3\pi}{2}$ | 25. $\theta = -215^\circ$ |
| 26. $\theta = \frac{5\pi}{4}$ | 27. $\theta = 780^\circ$      | 28. $\theta = -445^\circ$ |

Determine the reference angle associated with the given angle. See Example 4.

- |                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| 29. $\theta = 98^\circ$       | 30. $\theta = \frac{9\pi}{2}$ | 31. $\theta = -60^\circ$      |
| 32. $\theta = \frac{5\pi}{4}$ | 33. $\theta = \frac{5\pi}{2}$ | 34. $\theta = 313^\circ$      |
| 35. $\theta = \frac{7\pi}{6}$ | 36. $\theta = -168^\circ$     | 37. $\theta = \frac{6\pi}{5}$ |
| 38. $\theta = 216^\circ$      | 39. $\theta = \frac{3\pi}{2}$ | 40. $\theta = -330^\circ$     |
| 41. $\theta = \frac{7\pi}{4}$ | 42. $\theta = 718^\circ$      | 43. $\theta = 105^\circ$      |

Determine the quadrant in which the terminal side of the angle  $\theta$  is located.

- |   |   |
|---|---|
| 44. $\sin \theta > 0$ and $\tan \theta < 0$ | 45. $\sin \theta < 0$ and $\cot \theta > 0$ |
| 46. $\tan \theta > 0$ and $\sec \theta > 0$ | 47. $\cos \theta > 0$ and $\cot \theta < 0$ |
| 48. $\sec \theta < 0$ and $\csc \theta < 0$ | 49. $\cot \theta > 0$ and $\csc \theta > 0$ |
| 50. $\cot \theta > 0$ and $\cos \theta < 0$ | 51. $\sin \theta > 0$ and $\sec \theta < 0$ |

In Exercises 52–61, match the given angle  $\theta$  with the correct reference angle  $\theta'$  among the answer choices **a.**–**c.** Answers will be used more than once.

**a.**  $\theta' = 30^\circ$    **b.**  $\theta' = 45^\circ$    **c.**  $\theta' = 60^\circ$

- |                           |                          |                           |                          |
|---------------------------|--------------------------|---------------------------|--------------------------|
| 52. $\theta = 300^\circ$  | 53. $\theta = 150^\circ$ | 54. $\theta = -135^\circ$ | 55. $\theta = 210^\circ$ |
| 56. $\theta = -120^\circ$ | 57. $\theta = 315^\circ$ | 58. $\theta = 510^\circ$  | 59. $\theta = 600^\circ$ |
| 60. $\theta = 855^\circ$  | 61. $\theta = 480^\circ$ |                           |                          |

In Exercises 62–76, **a.** rewrite the expression in terms of the given angle's reference angle, and then **b.** evaluate the result, using a calculator and rounding your answers to four decimal places if necessary. See Example 5.

- |                                       |                                       |                       |
|---------------------------------------|---------------------------------------|-----------------------|
| 62. $\tan 98^\circ$                   | 63. $\sin\left(\frac{9\pi}{2}\right)$ | 64. $\cos(-60^\circ)$ |
| 65. $\tan\left(\frac{5\pi}{4}\right)$ | 66. $\cos\left(\frac{5\pi}{2}\right)$ | 67. $\sin 313^\circ$  |

68.  $\cos\left(\frac{7\pi}{6}\right)$       69.  $\tan(-168^\circ)$       70.  $\cos\left(\frac{6\pi}{5}\right)$   
 71.  $\sin 216^\circ$       72.  $\tan\left(\frac{3\pi}{2}\right)$       73.  $\cos(-330^\circ)$   
 74.  $\sin\left(\frac{7\pi}{4}\right)$       75.  $\tan 718^\circ$       76.  $\sin 105^\circ$

Use the appropriate identity to answer each of the following questions. Choose only one answer per question.

77. Which choice is equivalent to  $\sin 18^\circ$ ?  
 a.  $\tan 72^\circ$       b.  $\cos 72^\circ$       c.  $\csc 72^\circ$       d.  $\sec 162^\circ$       e.  $\cos 162^\circ$
78. Which choice is equivalent to  $\sec\left(\frac{\pi}{6}\right)$ ?  
 a.  $\csc\left(\frac{\pi}{3}\right)$       b.  $\cos\left(\frac{\pi}{2}\right)$       c.  $\sin\left(\frac{\pi}{6}\right)$       d.  $\cos\left(\frac{\pi}{3}\right)$       e.  $\tan\left(\frac{\pi}{6}\right)$
79. Which choice is equivalent to  $\tan\left(\frac{\pi}{12}\right)$ ?  
 a.  $\sin\left(\frac{\pi}{2}\right)$       b.  $\cos\left(\frac{\pi}{12}\right)$       c.  $\cot\left(\frac{\pi}{2}\right)$       d.  $\cot\left(\frac{\pi}{12}\right)$       e.  $\cot\left(\frac{5\pi}{12}\right)$
80. Which choice is equivalent to  $\cos 87^\circ$ ?  
 a.  $\sin 93^\circ$       b.  $\cos 93^\circ$       c.  $\sin 273^\circ$       d.  $\sec 3^\circ$       e.  $\sin 3^\circ$

Express each of the following in terms of the appropriate cofunction, and verify the equivalence of the two expressions, using a calculator and rounding your answers to four decimal places if necessary. See Example 6.

81.  $\cot 135^\circ$       82.  $\sec\left(\frac{\pi}{2}\right)$       83.  $\sin(-60^\circ)$   
 84.  $\cos\left(-\frac{3\pi}{4}\right)$       85.  $\csc\left(\frac{5\pi}{6}\right)$       86.  $\cot 313^\circ$   
 87.  $\cos\left(-\frac{3\pi}{6}\right)$       88.  $\csc(-168^\circ)$       89.  $\sin\left(-\frac{4\pi}{5}\right)$   
 90.  $\sec 216^\circ$       91.  $\csc\left(\frac{3\pi}{2}\right)$       92.  $\cos(-15^\circ)$   
 93.  $\cot\left(\frac{\pi}{4}\right)$       94.  $\tan(-105^\circ)$       95.  $\sec 105^\circ$

Using a calculator, determine  $\tan \theta$  and  $\cot \theta$  for each of the following exercises. Round your answers to three decimal places.

96.  $\sin \theta = 0.978$  and  $\cos \theta = 0.208$       97.  $\sin \theta = 0.588$  and  $\cos \theta = -0.809$   
 98.  $\sin \theta = -0.966$  and  $\cos \theta = -0.259$       99.  $\sin \theta = -0.866$  and  $\cos \theta = -0.5$   
 100.  $\sin \theta = -0.699$  and  $\cos \theta = 0.743$       101.  $\sin \theta = -0.995$  and  $\cos \theta = -0.105$

Use the given information about each angle to evaluate the expressions, if possible. If no angle with the stated properties exists, determine the reason. See Examples 7 and 8.

102. Given that  $\cos \theta = \frac{\sqrt{12}}{4}$  and  $\tan \theta$  is negative, determine  $\theta$  and  $\tan \theta$ .
103. Given that  $\csc \theta = 1.25$  and the terminal side of  $\theta$  lies in the second quadrant, determine  $\cot \theta$ .
104. Given that  $\tan \theta = \frac{\sqrt{3}}{3}$  and  $\sin \theta$  is positive, determine  $\theta$  and  $\sin \theta$ .
105. Given that  $\sin \theta = 2$  and  $\theta$  is positive, determine  $\tan \theta$ .
106. Given that  $\cot \theta = \frac{3}{4}$  and  $\sin \theta$  is negative, determine  $\sec \theta$ .
107. Given that  $\sin \theta = \frac{\sqrt{3}}{2}$  and  $\theta$  lies in the second quadrant, determine  $\theta$  and  $\tan \theta$ .
108. Given that  $\sec \theta = 0.3$  and the terminal side of  $\theta$  lies in the fourth quadrant, determine  $\csc \theta$ .

The three cofunction identities presented in this section have three companion identities, as follows:

$$\sin \theta = \cos \left( \frac{\pi}{2} - \theta \right), \quad \sec \theta = \csc \left( \frac{\pi}{2} - \theta \right), \quad \text{and} \quad \tan \theta = \cot \left( \frac{\pi}{2} - \theta \right).$$

Express each of the following in terms of the appropriate cofunction. Evaluate both the given expression and the cofunction expression with your calculator to verify that the expressions are equivalent.

109.  $\sin \left( \frac{7\pi}{4} \right)$       110.  $\csc \left( \frac{8\pi}{3} \right)$       111.  $\cot \left( \frac{3\pi}{4} \right)$
112.  $\cos \left( -\frac{5\pi}{3} \right)$       113.  $\tan 15^\circ$       114.  $\sec(-315^\circ)$

### WRITING & THINKING

115. Prove the three cofunction identities given in the directions for Exercises 109–114. (**Hint:** For the first identity, begin with the observation that  $\sin \theta = \sin \left( \frac{\pi}{2} - \left( \frac{\pi}{2} - \theta \right) \right)$  and then apply one of the three original cofunction identities.)