7.5 EXERCISES

PRACTICE

Solve the following exponential and logarithmic equations. Round your answer to two decimal places if necessary. See Examples 1 through 4.

1. $3e^{5x}=11$
2. $4^{3-2x}=7$
3. $11^{\frac{3}{x}}=10$
4. $8^{3x+2}=7^{2x+3}$
5. $e^{15-3x}=28$
6. $10^{\frac{5}{x}}=150$
7. $10^{2x+5}=e$
8. $e^{8x+e}=8^{ex+8}$
9. $6^{x-7}=7$
10. $2e^{3x}=145$
11. $2^{6-x}=10$
12. $e^{3x-6}=10^{x+2}$
13. $e^{-4x-2}=12$
14. $10^{x-9}=2001$
15. $e^{x-4}=4^{\frac{2x}{3}}$
16. $5^{x-2}=20$
17. $8^{x^2+1}=23$
18. $3^{\frac{4}{x}}=15$
19. $6^{3x-4}=36^{2x+4}$
20. $81^{x}=3^{2x+16}$
21. $e^{2x}=14$
22. $e^{4x}=e^{3x+14}$
23. $5^{5x-7}=10^{2x}$
24. $10^{6x}=3^{3x+4}$
25. $\log_5 x=3$
26. $\log_2 x=4$
27. $\log x+\log(4x)=2$
28. $\log_4\left(x^2\right)-\log_4 x=2$
29. $\ln(2x)-\ln 4=3$
30. $\ln(15x)-\ln 3=6$
31. $\log_4(x-3)+\log_4 2=3$
32. $\log_3 24-\log_3 4=x-5$
33. $\log_5(8x)-\log_5 3=2$
34. $e^{2\ln x}=4\log 10$
35. $9^{\log_3 x}=16$
36. $3\log_8\left(512^{x^2}\right)=36$
37. $\log_3(6x)-2\log_3(6x)=3$
38. $\ln(3e)=\log x$
39. $\ln\left(2^{4e^x}\right)=\ln\left(16^e\right)$
40. $\log(x-2)+\log(x+2)=2$
41. $\log(x-3)+\log(x+3)=4$
42. $\log_2(7x-4)=\log_2(16-3x)$
43. $\log_\pi(x-5)+\log_\pi(x+3)=\log_\pi(1-2x)$
44. $\log_3(x+3)+\log_3(x-5)=2$
45. $\log x+\log(x-3)=1$
46. $\log_7(3x+2)-\log_7 x=\log_7 4$
47. $\log_2 x+\log_2(x-7)=3$
48. $\log_{12}(x-2)+\log_{12}(x-1)=1$
49. $\log_3(x+1)-\log_3(x-4)=2$
50. $\ln(x+1)+\ln(x-2)=\ln(x+6)$
51. $\log_4(x-3)+\log_4(x-2)=\log_4(x+1)$

52. $2\ln(x+3) = \ln(12x)$

53. $\log_5(x-1) + \log_5(x+4) = \log_5(x-5)$

54. $\log_{255}(2x+3) + \log_{255}(2x+1) = 1$

55. $\log_2(x-5) + \log_2(x+2) = 3$

56. $\log_6(x+1) + \log_6(x-4) = 2$

57. $\ln(x+2) + \ln x = 0$

58. $e^{2x} - 3e^x - 10 = 0$ (**Hint:** First solve for e^x.)

59. $2^{2x} - 12(2^x) + 32 = 0$ (**Hint:** First solve for 2^x.)

60. $e^{2x} + 2e^x - 8 = 0$

61. $3^{2x} - 12(3^x) + 27 = 0$

Using the properties of logarithmic functions, simplify the following functions as much as possible. Write each function as a single term with a coefficient of 1, if possible.

62. $f(x) = 0.5\ln(x^2)$

63. $f(x) = 0.25\log(16x^8)$

64. $f(x) = 4\ln(\sqrt{5x})$

65. $f(x) = 8\ln(\sqrt[4]{3x})$

66. $f(x) = 3\ln(e^x) - 3$

67. $f(x) = 10^{2x\log 16}$

68. $f(x) = 2\ln(x^3) + \ln(x^6)$

69. $f(x) = 2\ln(x^3) - \ln(x^6)$

70. $f(x) = \ln(x^2 + x) - \ln x$

71. $f(x) = 2\ln\left(5^{x\log_{20}(2\sqrt{5})}\right)$

72. $f(x) = e^{\ln(\log(x^e)-1)}$

73. $f(x) = 2\ln(5^{\log_4 2})$

APPLICATIONS

74. Assuming that there are currently 8 billion people on Earth and a growth rate of 1.9% per year, how long will it take for Earth's population to reach 20 billion?

75. How long does it take for an investment to double in value if

a. The investment is in a monthly compounded savings account earning 4% a year?
b. The investment is in a continuously compounded account earning 7% a year?

76. Assuming a half-life of 5728 years, how long would it take for 3 grams of carbon-14 to decay to 1 gram?

77. Suppose a population of bacteria in a Petri dish has a doubling time of one and a half hours. How long will it take for an initial population of 10,000 bacteria to reach 100,000?

78. According to Newton's Law of Cooling, the temperature $T(t)$ of a hot object, at time t after being placed in an environment with a constant temperature C, is given by $T(t) = C + (T_0 - C)e^{-kt}$, where T_0 is the temperature of the object at time $t = 0$ and k is a constant that depends on the object.

If a hot cup of coffee, initially at 190 °F, cools to 125 °F in 5 minutes when placed in a room with a constant temperature of 75 °F, how long will it take for the coffee to reach 100 °F?

79. Wayne has \$12,500 in a high interest savings account at 3.66% annual interest compounded monthly. Assuming he makes no deposits or withdrawals, how long will it take for his investment to grow to \$15,000?

80. Ben and Casey both open money market accounts with 4.9% annual interest compounded continuously. Ben opens his account with \$8700 while Casey opens her account with \$3100.

a. How long will it take Ben's account to reach \$10,000?
b. How long will it take Casey's account to reach \$10,000?
c. How much money will be in Ben's account after the time found in part b.?

81. Cesium-137 has a half-life of approximately 30 years. How long would it take for 160 grams of cesium-137 to decay to 159 grams?

82. A chemist, running tests on an unknown sample from an illegal waste dump, isolates 50 grams of what he suspects is a radioactive element. In order to help identify the element, he would like to know its half-life. He determines that after 40 days only 44 grams of the original element remains. What is the half-life of this mystery element?

TECHNOLOGY

The dollar-cost averaging application in this section concluded with the equation

$$x^{n+1} - \left(\frac{A}{P} + 1\right)x + \frac{A}{P} = 0,$$

where A represents the total accumulation in an account in which amount P has been invested every month for n months. A graphing utility can be used to solve the equation for x when given known values for A, P, and n. Then the equivalent monthly compounded annual rate of interest r can be found by solving the equation

$$x = 1 + \frac{r}{12}.$$

For instance, if a monthly investment of $P = 100$ dollars for $n = 12$ months results in an accumulation of $A = 1300$ dollars, the first step is to solve the following equation.

$$x^{12+1} - \left(\frac{1300}{100} + 1\right)x + \frac{1300}{100} = 0 \quad \Rightarrow \quad x^{13} - 14x + 13 = 0$$

As noted, $x = 1$ will always be one solution to the dollar-cost averaging equation, but the second positive solution is the one we seek. The second solution will lie between 0 and 1 if the equivalent rate r is negative and will be larger than 1 if r is positive. In this case, since \$1200 has been invested and the total accumulation after 12 months is \$1300, we know r is positive, and a graphing utility tells us that the second solution is indeed greater than 1; specifically, $x \approx 1.01225$. Solving $1.01225 = 1 + \frac{r}{12}$ for r gives us $r \approx 0.147$, or $r \approx 14.7\%$.

Use a graphing utility to determine the equivalent monthly compounded interest rate *r* in each of the following scenarios, where *P* represents the amount invested each month and *A* is the accumulated value at the end of *n* months.

83. $n = 24$, $P = \$50.00$, and $A = \$1275.00$

84. $n = 120$, $P = \$100.00$, and $A = \$15{,}000.00$

85. $n = 12$, $P = \$50.00$, and $A = \$590.00$

86. $n = 24$, $P = \$75.00$, and $A = \$2000.00$