Example 6: Evaluating Logarithmic Expressions

Evaluate the following logarithmic expressions.

a. $\ln\left(\sqrt[3]{e}\right)$ **b.** $\log 1000$

c. $\ln(4.78)$ **d.** $\log(10.5)$

Solution

a. $\ln\left(\sqrt[3]{e}\right) = \ln\left(e^{\frac{1}{3}}\right) = \frac{1}{3}$ No calculator is necessary for this problem, just an application of an elementary property of logarithms.

b. $\log 1000 = \log\left(10^3\right) = 3$ Again, no calculator is required.

c. $\ln(4.78) \approx 1.564$ This time, a calculator is needed, and only an approximate answer can be given. Be sure to use the correct logarithm.

d. $\log(10.5) \approx 1.021$ Again, we must use a calculator, though we can say beforehand that the answer should be only slightly larger than 1, as $\log 10 = 1$ and 10.5 is only slightly larger than 10.

7.3 EXERCISES

PRACTICE

Write the following equations in logarithmic terms.

1. $625 = 5^4$ **2.** $216 = 6^3$ **3.** $x^3 = 27$

4. $b^2 = 3.2$ **5.** $4.2^3 = C$ **6.** $1.3^2 = V$

7. $4^x = 31$ **8.** $16^{2x} = 215$ **9.** $(4x)^{\sqrt{3}} = 13$

10. $e^x = \pi$ **11.** $2^{e^x} = 11$ **12.** $4^e = N$

Write the following logarithmic equations as exponential equations.

13. $\log_3 81 = 4$ **14.** $\log_2\left(\frac{1}{8}\right) = -3$ **15.** $\log_b 4 = \frac{1}{2}$

16. $\log_y 9 = 2$ **17.** $\log_2 15 = b$ **18.** $\log_5 8 = d$

19. $\log_5 W = 12$ **20.** $\log_7 T = 6$ **21.** $\log_\pi(2x) = 4$

22. $\log_{\sqrt{3}}(2\pi) = x$ **23.** $\ln 2 = x$ **24.** $\ln(5x) = 3$

Sketch the graphs of the following functions. State their domain and range. See Examples 2 and 3.

25. $f(x) = \log_3(x-1)$

26. $g(x) = \log_5(x+2) - 1$

27. $r(x) = \log_{\frac{1}{2}}(x-3)$

28. $p(x) = 3 - \log_2(x+1)$

29. $q(x) = \log_3(2-x)$

30. $s(x) = \log_{\frac{1}{3}}(5-x)$

31. $h(x) = \log_7(x-3) + 3$

32. $m(x) = \log_{\frac{1}{2}}(1-x)$

33. $f(x) = \log_3(6-x)$

34. $p(x) = 4 - \log(x+3)$

35. $s(x) = -\log_{\frac{1}{3}}(-x)$

36. $g(x) = \log_5(2x) - 1$

Match the graph of the appropriate equation to the logarithmic function.

37. $f(x) = \log_2(x-1)$

38. $f(x) = \log_2(2-x)$

39. $f(x) = \log_2(-x)$

40. $f(x) = \log_2(x-3)$

41. $f(x) = 1 - \log_2 x$

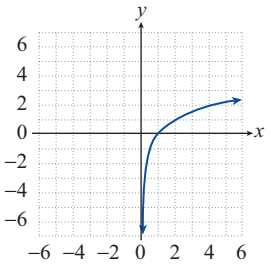
42. $f(x) = -\log_2 x$

43. $f(x) = -\log_2(-x)$

44. $f(x) = \log_2 x$

45. $f(x) = \log_2(x+3)$

a.



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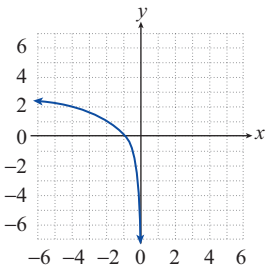

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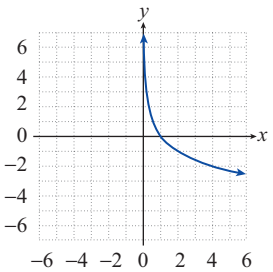

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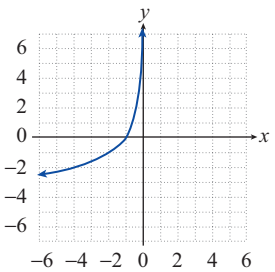
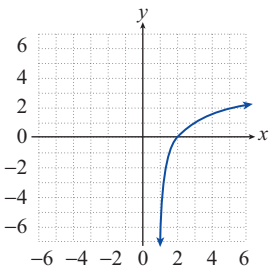
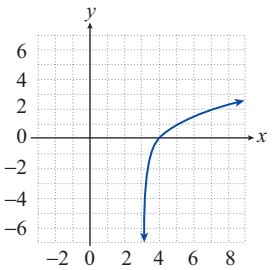


e.



f.



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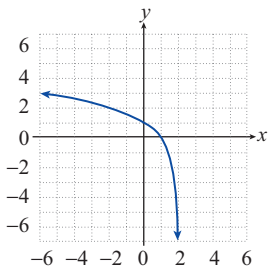

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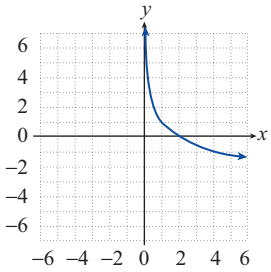
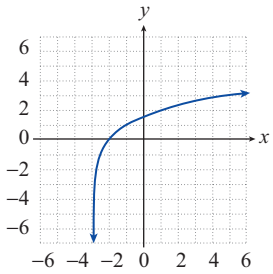


i.



Evaluate the following logarithmic expressions without the use of a calculator. See Examples 4 and 6.

46. $\log_7\left(\sqrt{7}\right)$

47. $\log_{\frac{1}{2}} 4$

48. $\log_9\left(\frac{1}{81}\right)$

49. $\log_3 27$

50. $\log_{27} 3$

51. $\log_9\left(\frac{1}{3}\right)$

52. $\log_{27} 9$

53. $\log_{\frac{1}{16}}\left(\frac{1}{8}\right)$

54. $\log_3\left(\log_{27} 3\right)$

55. $\ln\left(e^{2.89}\right)$

56. $\log\left(0.0001\right)$

57. $\log_a\left(a^{\frac{5}{3}}\right)$

58. $\ln\left(\frac{1}{e}\right)$

59. $\log\left(\log\left(10^{10}\right)\right)$

60. $\log_3 1$

61. $\ln\left(\sqrt[5]{e}\right)$

62. $\log_{\frac{1}{16}} 4$

63. $\log_8\left(4^{\log 1000}\right)$

Use the elementary properties of logarithms to solve the following equations. See Example 5.

64. $\log_{16} x = \frac{3}{4}$

65. $\log_{16}\left(x^{\frac{1}{2}}\right) = \frac{3}{4}$

66. $\log_{16} x = -\frac{3}{4}$

67. $\log_5\left(5^{\log_3 x}\right) = 2$

68. $\log_a\left(a^{\log_b x}\right) = 0$

69. $\log_3\left(9^{2x}\right) = -2$

70. $\log_{\frac{1}{3}}\left(3^x\right) = 2$

71. $\log_7\left(3x\right) = -1$

72. $4^{\log_3 x} = 0$

73. $\log\left(x^{10}\right) = 10$

74. $\log_x\left(\log_{\frac{1}{2}}\left(\frac{1}{4}\right)\right) = 1$

75. $6^{\log_x\left(e^2\right)} = e$

Hint: Note that $\log_a b = \log_{a^2}\left(b^2\right)$. This follows from the fact that $\log_a b = y \Leftrightarrow b = a^y \Leftrightarrow b^2 = a^{2y} = \left(a^2\right)^y \Leftrightarrow \log_{a^2}\left(b^2\right) = y$.

Solve the following logarithmic equations, using a calculator if necessary to evaluate the logarithms. See Examples 5 and 6. Express your answer either as a fraction or a decimal rounded to two decimal places.

76. $\log(3x) = 2.1$

77. $\log(x^2) = -2$

78. $\ln(x + 1) = 3$

79. $\ln(2x) = -1$

80. $\ln(e^x) = 5.6$

81. $\ln(\ln(x^2)) = 0$

82. $\log 19 = 3x$

83. $\log(e^x) = 5.6$

84. $\log_9(2x - 1) = 2$

85. $\log(\log(x - 2)) = 1$

86. $\log(300^{\log x}) = 9$