

But does  $p(x) \rightarrow -\infty$  as  $x \rightarrow \infty$ ? No, if we multiply out, the leading term of this polynomial would be  $x^3$ , which has a positive leading coefficient. To fix this, we multiply the entire polynomial by  $-1$ .

$$\begin{aligned} p(x) &= -(x+3)(x-2)(x-5) \\ &= -x^3 + 4x^2 + 11x - 30 \end{aligned}$$

- b. Once again,  $p(x)$  is a product of linear factors, identified by the required zeros.

$$p(x) = (x+5)(x+2)(x-1)(x-3)$$

Our second condition is that the  $y$ -intercept must be  $(0,15)$ . If we substitute  $x = 0$ , we see that  $p(0) = (5)(2)(-1)(-3) = 30$ , so the  $y$ -intercept is  $(0,30)$ . To fix this, we might try subtracting 15 from the polynomial.

$$p(x) \stackrel{?}{=} (x+5)(x+2)(x-1)(x-3) - 15$$

But, we cannot do this because it causes  $-5$ ,  $-2$ ,  $1$  and  $3$  to no longer be zeros! Instead, we multiply  $p(x)$  by  $\frac{1}{2}$ .

$$\begin{aligned} p(x) &= \frac{1}{2}(x+5)(x+2)(x-1)(x-3) \\ &= \frac{1}{2}x^4 + \frac{3}{2}x^3 - \frac{15}{2}x^2 - \frac{19}{2}x + 15 \end{aligned}$$

## 6.2 EXERCISES

### PRACTICE

Use polynomial long division to rewrite each of the following fractions in the form

$q(x) + \frac{r(x)}{d(x)}$ , where  $d(x)$  is the denominator of the original fraction,  $q(x)$  is the quotient, and  $r(x)$  is the remainder. See Examples 1 through 3.

1.  $\frac{6x^4 - 2x^3 + 8x^2 + 3x + 1}{2x^2 + 2}$

2.  $\frac{5x^2 + 9x - 6}{x + 2}$

3.  $\frac{x^3 - 6x^2 + 12x - 10}{x^2 - 4x + 4}$

4.  $\frac{7x^5 - x^4 + 2x^3 - x^2}{x^2 + 1}$

5.  $\frac{4x^3 - 6x^2 + x - 7}{x + 2}$

6.  $\frac{x^3 + 2x^2 - 4x - 8}{x - 3}$

7.  $\frac{3x^5 + 18x^4 - 7x^3 + 9x^2 + 4x}{3x^2 - 1}$

8.  $\frac{9x^5 - 10x^4 + 18x^3 - 28x^2 + x + 3}{9x^2 - x - 1}$

9.  $\frac{2x^5 - 5x^4 + 7x^3 - 10x^2 + 7x - 5}{x^2 - x + 1}$

10.  $\frac{14x^5 - 2x^4 + 27x^3 - 3x^2 + 9x}{2x^3 + 3x}$

11. 
$$\frac{x^4 + x^2 - 20x - 8}{x - 3}$$

13. 
$$\frac{9x^3 + 2x}{3x - 5}$$

15. 
$$\frac{2x^2 + x - 8}{x + 3}$$

17. 
$$\frac{2x^3 - 3ix^2 + 11x + (1 - 5i)}{2x - i}$$

19. 
$$\frac{3x^3 + ix^2 + 9x + 3i}{3x + i}$$

12. 
$$\frac{2x^5 - 3x^2 + 1}{x^2 + 1}$$

14. 
$$\frac{-4x^5 + 8x^3 - 2}{2x^3 + x}$$

16. 
$$\frac{5x^5 + x^4 - 13x^3 - 2x^2 + 6x}{x^3 - 2x}$$

18. 
$$\frac{9x^3 - (18 + 9i)x^2 + x + (-2 - i)}{x - 2 - i}$$

20. 
$$\frac{35x^4 + (14 - 10i)x^3 - (7 + 4i)x^2 + 2ix}{7x - 2i}$$

Use synthetic division to determine if the given value for  $c$  is a zero of the corresponding polynomial. If not, determine  $p(c)$ . See Example 4.

21.  $p(x) = 32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x + 2; c = 1$

22.  $p(x) = 32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x + 2; c = \frac{1}{2}$

23.  $p(x) = 12x^4 - 7x^3 - 32x^2 - 7x + 6; c = 2$

24.  $p(x) = 12x^4 - 7x^3 - 32x^2 - 7x + 6; c = 1$

25.  $p(x) = 12x^4 - 7x^3 - 32x^2 - 7x + 6; c = \frac{1}{3}$

26.  $p(x) = 2x^2 - (3 - 5i)x + (3 - 9i); c = -2$

27.  $p(x) = 8x^4 - 2x + 6; c = 1$

28.  $p(x) = x^4 - 1; c = 1$

29.  $p(x) = x^5 + 32; c = -2$

30.  $p(x) = 3x^5 + 9x^4 + 2x^2 + 5x - 3; c = -3$

31.  $p(x) = 2x^2 - (3 - 5i)x + (3 - 9i); c = -3i$

32.  $p(x) = x^2 - 6x + 13; c = 2$

33.  $p(x) = x^2 - 6x + 13; c = 3 - 2i$

34.  $p(x) = 3x^3 - 13x^2 - 28x - 12; c = -2$

35.  $p(x) = 3x^3 - 13x^2 - 28x - 12; c = 6$

36.  $p(x) = 2x^3 - 8x^2 - 23x + 63; c = 2$

37.  $p(x) = 2x^3 - 8x^2 - 23x + 63; c = 5$

38.  $p(x) = x^4 - 3x^3 - 3x^2 + 11x - 6; c = 1$

39.  $p(x) = x^4 - 3x^3 - 3x^2 + 11x - 6; c = -2$

40.  $p(x) = x^4 - 3x^3 - 3x^2 + 11x - 6; c = 3$

Use synthetic division to rewrite each of the following fractions in the form  $q(x) + \frac{r(x)}{d(x)}$ , where  $d(x)$  is the denominator of the original fraction,  $q(x)$  is the quotient, and  $r(x)$  is the remainder. See Example 5.

41. 
$$\frac{x^3 + x^2 - 18x + 9}{x + 5}$$

42. 
$$\frac{-2x^5 + 4x^4 + 3x^3 - 7x^2 + 3x - 2}{x - 2}$$

43. 
$$\frac{x^8 + x^7 - 3x^3 - 3x^2 + 3}{x + 1}$$

44. 
$$\frac{x^8 - 5x^7 - 3x^3 + 15x^2 - 2}{x - 5}$$

45. 
$$\frac{4x^3 - (16 + 4i)x^2 + (14 + 4i)x + (-6 - 2i)}{x - 3 - i}$$

46. 
$$\frac{x^6 - 2x^5 + 2x^4 + 4x^2 - 8x + 8}{x - 1 + i}$$

47. 
$$\frac{x^5 - 3x^4 + x^3 - 5x^2 + 18}{x - 2}$$

48. 
$$\frac{x^5 - 3x^4 + x^3 - 5x^2 + 18}{x - 3}$$

49. 
$$\frac{x^4 + (i - 1)x^3 + (1 - i)x^2 + ix}{x + i}$$

50. 
$$\frac{x^6 + 8x^5 + x^3 + 8x^2 - 14x - 112}{x + 8}$$

51. 
$$\frac{2x^3 - 10ix^2 + 5x + (8 - 3i)}{x - 3i}$$

52. 
$$\frac{4x^5 - 6x^4 + 10x^3 - 4x^2 - 4x}{x - 1}$$

### WRITING & THINKING

Construct a polynomial function with the stated properties. See Example 6.

53. Second-degree, zeros of  $-4$  and  $3$ , and goes to  $-\infty$  as  $x \rightarrow -\infty$ .
54. Third-degree, zeros of  $-2$ ,  $1$ , and  $3$ , and a  $y$ -intercept of  $-12$ .
55. Second-degree, zeros of  $2 - 3i$  and  $2 + 3i$ , and a  $y$ -intercept of  $-13$ .
56. Third-degree, zeros of  $1 - i$ ,  $2 + i$ , and  $-1$ , and a leading coefficient of  $-2$ .
57. Fourth-degree and a single  $x$ -intercept of  $3$ .
58. Second-degree, zeros of  $-\frac{3}{4}$  and  $2$ , and a  $y$ -intercept of  $6$ .
59. Fourth-degree, zeros of  $-3$ ,  $-2$ , and  $1$ , and a  $y$ -intercept of  $18$ .
60. Third-degree, zeros of  $1$ ,  $2$ , and  $3$ , and passes through the point  $(4, 12)$ .

### APPLICATIONS

61. A box company makes a variety of boxes, all with volume given by the formula  $x^3 + 10x^2 + 31x + 30$ . If the height is given by  $x + 3$ , what is the formula for the area of the base?