

We also again select a test point within each of the four intervals defined by the zeros, but instead of computing the exact value of  $p(x)$  at each test point, we simply determine its sign. For instance, using the test point  $x = -3$  in the interval  $(-\infty, -2)$ , we know that  $p(-3)$  is positive because  $(-3+2) < 0$ ,  $(-3-1)^2 > 0$ ,  $(-3-3) < 0$  and the product of a negative factor, a positive factor, and a negative factor is positive. In Figure 17, a representative test point for each interval is shown, followed by a string of “+” and “-” signs indicating the sign of each factor of  $p(x)$  at the test point. Finally, the sign of  $p(x)$  overall on the interval is shown.

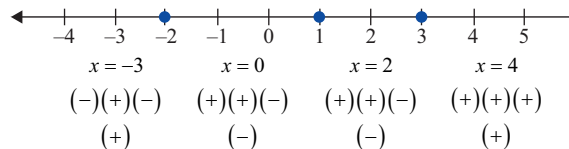


FIGURE 17

The last step of the method is to use the sign chart to identify the interval(s) on which  $p(x) \leq 0$ . Since the inequality is not strict and the two intervals on which  $p(x)$  is nonpositive overlap at  $x = 1$ , the solution is  $[-2, 3]$ .

## 6.1 EXERCISES

### PRACTICE

Verify that the given values of  $x$  solve the corresponding polynomial equations. See Example 1.

1.  $9x^2 - 4x = 2x^3 + 15; x = -1$

2.  $x^2 - 4x = -13; x = 2 - 3i$

3.  $x^2 + 13 = 4x; x = 2 + 3i$

4.  $3x^3 + (5 - 3i)x^2 = (2 + 5i)x - 2i; x = i$

5.  $9x^2 - 4x = 2x^3 + 15; x = 3$

6.  $9x^2 - 4x = 2x^3 + 15; x = \frac{5}{2}$

7.  $3x^3 + (5 - 3i)x^2 = (2 + 5i)x - 2i; x = -2$

8.  $x^5 - 10x^4 - 80x^2 = 32 - 80x - 40x^3; x = 2$

9.  $4x^5 - 8x^4 - 12x^3 = 16x^2 - 25x - 69; x = 3$

10.  $x^2 - 4x - 12 = 0; x = 6$

11.  $23x^7 - 12x^5 = 63x^4 - 3x^2; x = 0$

12.  $x^2 + 74 = 10x; x = 5 + 7i$

13.  $4x^2 + 32x + (8 + i)x^3 = -8; x = 2i$

14.  $8x - 17 = x^2; x = 4 - i$

15.  $(5 - 3i)x - 3x = 4 - 6i; x = 2$

16.  $x^6 - x^5 + 7x^4 + x^3 - 9x = -1; x = 1$

17.  $6x^7 - 3x^5 = 3x^4 - 6x^2; x = -1$

Determine if the given values of  $x$  are solutions of the corresponding polynomial equations. See Example 1.

18.  $16x = x^3 + x^2 + 20; x = -5$

19.  $x^4 - 13x^2 + 12 = -x^3 + x; x = -1$

20.  $x^4 - 3x^3 - 10x^2 = 0; x = 2$

21.  $4x^5 - 216x^2 = 36x^3 - 24x^4; x = -6$

22.  $x^3 - 8ix + 30 = 15x + 2x^2 + 16i; x = -i$

23.  $x^3 - 7x^2 + 4x - 28 = 0; x = 2i$

Solve the following polynomial equations by factoring and/or using the quadratic formula, making sure to identify all the solutions. See Sections 2.3 and 2.4 for review, if necessary.

24.  $x^3 - x^2 - 6x = 0$

25.  $x^2 - 2x + 5 = 0$

26.  $x^4 + x^2 - 2 = 0$

27.  $2x^2 + 5x = 3$

28.  $9x^2 = 6x - 1$

29.  $x^4 - 8x^2 + 15 = 0$

30.  $x^3 - x^2 = 72x$

31.  $x^2 + 5x = -\frac{25}{4}$

32.  $2x^2 + 5 = 11x$

33.  $x^4 - 8x^3 + 25x^2 = 0$

34.  $x^4 - 13x^2 + 36 = 0$

35.  $x^4 + 7x^2 = 8$

For each of the following polynomials, determine the degree and the leading coefficient; then determine the behavior of the graph as  $x \rightarrow \pm\infty$ .

36.  $p(x) = 2x^4 - 3x^3 - 6x^2 - x - 23$

37.  $j(x) = 4x^7 + 5x^5 + 12$

38.  $r(x) = (3x + 5)(x - 2)(2x - 1)(4x - 7)$

39.  $h(x) = -6x^5 + 2x^3 - 7x$

40.  $g(x) = (x - 5)^3(2x + 1)(-x - 1)$

41.  $f(x) = -2(x + 4)(x - 4)(x^2)$

For each of the following polynomial functions, determine the behavior of its graph as  $x \rightarrow \pm\infty$  and identify the  $x$ - and  $y$ -intercepts. Use this information to sketch the graph of each polynomial. See Example 2.

42.  $f(x) = (x - 3)(x + 2)(x + 4)$

43.  $g(x) = (3 - x)(x + 2)(x + 4)$

44.  $f(x) = (x - 2)^2(x + 5)$

45.  $h(x) = -(x + 2)^3$

46.  $r(x) = x^2 - 2x - 3$

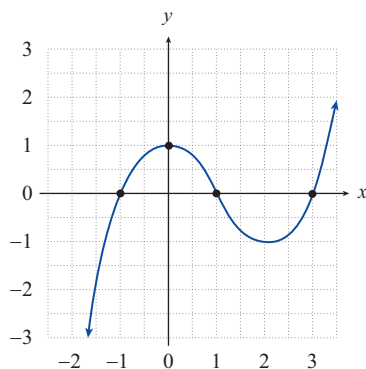
47.  $s(x) = x^3 + 3x^2 + 2x$

48.  $f(x) = -(x - 2)(x + 1)^2(x + 3)$

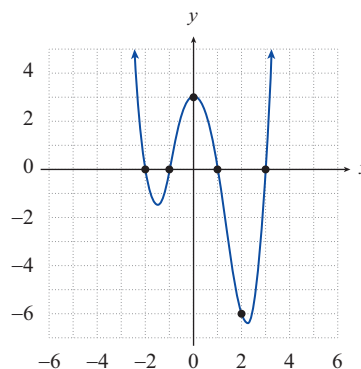
49.  $g(x) = (x - 3)^5$

Find the polynomial of lowest possible degree that corresponds to the given graph. See Example 3.

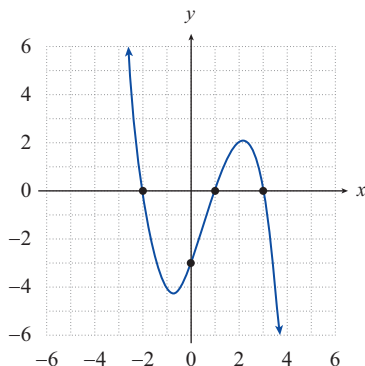
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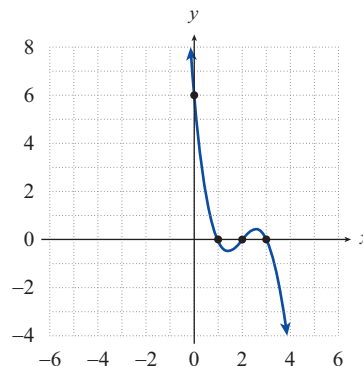
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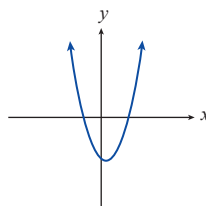
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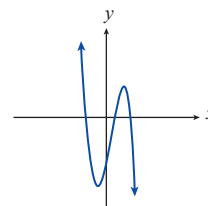
In Exercises 54 through 59, use the behavior as  $x \rightarrow \pm\infty$  and the intercepts to match each polynomial with its graph.

54.  $g(x) = (x+1)^2(x-3)^2$

a.

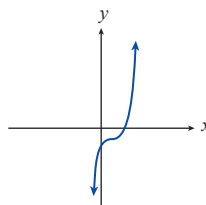


b.

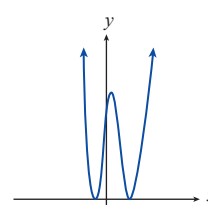


55.  $h(x) = 1 - (x+2)^2$

c.



d.

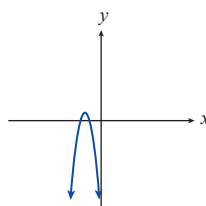


56.  $f(x) = (x-1)(x+2)(3-x)$

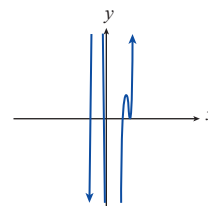
57.  $r(x) = x^2 - x - 6$

58.  $s(x) = (x-1)^3 - 2$

e.



f.



59.  $f(x) = (x-3)^2(4x+1)(x+2)(x-2)$

Match each of the following functions to the appropriate description.

60.  $z(x) = (x-1)(x+2)(4-x)$  a. cubic curve increasing as  $x \rightarrow \infty$ , has  $x$ -intercepts of 0,  $-1$ , and  $-2$ , and crosses the  $y$ -axis at 0.
61.  $r(x) = x^2 - 6x - 7$  b. parabola that opens up, has  $x$ -intercepts at 6 and  $-1$ , crosses the  $y$ -axis at  $-6$ .
62.  $s(x) = x^3 + 3x^2 + 2x$  c. cubic curve increasing as  $x \rightarrow \infty$ , has  $x$ -intercepts of 0,  $-1$ , and  $-4$ , and crosses the  $y$ -axis at 0.
63.  $g(x) = (x-1)(x+4)(3-x)$  d. parabola that opens up, has  $x$ -intercepts at 7 and  $-1$ , crosses the  $y$ -axis at  $-7$ .
64.  $s(x) = x^3 + 5x^2 + 4x$  e. cubic curve decreasing as  $x \rightarrow \infty$ , has  $x$ -intercepts at 1, 4, and  $-2$ , crosses the  $y$ -axis at  $-8$ .
65.  $s(x) = x^2 - 5x - 6$  f. cubic curve decreasing as  $x \rightarrow \infty$ , has  $x$ -intercepts of 1, 3, and  $-4$ , and crosses the  $y$ -axis at  $-12$ .

Solve the following polynomial inequalities. See Examples 5 and 6.

66.  $x^2 - x - 6 \leq 0$  67.  $x^2 > x + 6$
68.  $(x+2)^2(x-1)^2 > 0$  69.  $x^3 + 3x^2 + 2x < 0$
70.  $(x-2)(x+1)(x+3) \geq 0$  71.  $(x-1)(x+2)(3-x) \leq 0$
72.  $-x^3 - x^2 + 30x > 0$  73.  $(x^2 - 1)(x-4)(x+5) \leq 0$
74.  $x^4 + x^2 > 0$  75.  $4x^2 < 6x + 4$
76.  $x^2(x+4)(x-3) > 0$  77.  $(x-3)(x+4)(2-x) > 0$

### APPLICATIONS

For Exercises 78–83, use the fact that profit is equal to revenue minus cost.

78. A small start-up skateboard company projects that the cost per month of manufacturing  $x$  skateboards will be  $C(x) = 10x + 300$ , and the revenue per month from selling  $x$  skateboards will be  $r(x) = -x^2 + 50x$ . For what value(s) of  $x$  will the company break even or make a profit?
79. A manufacturer has determined that the revenue from the sale of  $x$  cordless telephones is given by  $r(x) = -x^2 + 15x$ . The cost of producing  $x$  telephones is  $C(x) = 135 - 17x$ . For what value(s) of  $x$  will the company break even or make a profit?

80. The revenue from the sale of  $x$  fire extinguishers is estimated to be  $r(x) = 9 - x^2$ . The total cost of producing  $x$  fire extinguishers is  $C(x) = 209 - 33x$ . For what value(s) of  $x$  will the company break even or make a profit?
81. A manufacturer has determined that the cost and revenue of producing and selling  $x$  telescopes are  $C(x) = 253 - 7x$  and  $r(x) = 27x - x^2$ , respectively. For what value(s) of  $x$  will the company break even or make a profit?
82. A company that produces and sells compact refrigerators has found that the revenue from the sale of  $x$  compact refrigerators is  $r(x) = -x^2 + 30x - 370$ . The cost function is given by  $C(x) = 6 - 25x$ . For what value(s) of  $x$  will the company break even or make a profit?
83. An electronics company is deciding whether or not to begin producing phones. The company must determine if a profit can be made on the phones. The profit function is modeled by the equation  $P(x) = x + 0.27x^2 - 0.0015x^3 - 300$ , where  $x$  is the number of phones produced in hundreds. Given this equation, how many phones must the company produce to make a profit?
84. The population of sea lions on an island is represented by the function  $L(m) = 110m^2 - 0.35m^4 + 750$ , where  $m$  is the number of months the sea lions have been observed on the island. Given this information, how many more months will there be sea lions on the island?
85. The population of mosquitoes in a city in Florida is modeled by the function  $M(w) = 200w^2 - 0.01w^4 + 1200$ , where  $w$  is the number of weeks since the town began spraying for mosquitoes. How many weeks will it take for all the mosquitoes to die?