5.4 EXERCISES

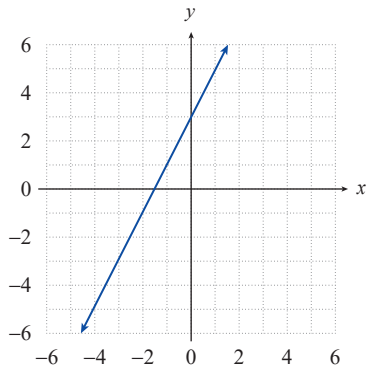
PRACTICE

Graph the inverse of each of the following relations, and state its domain and range. See Example 1.

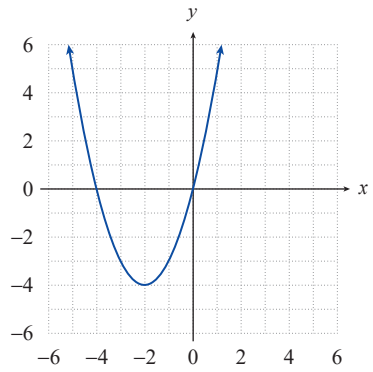
1. $R = \{(-4,2),(3,2),(0,-1),(3,-2)\}$
2. $S = \{(-3,-3),(-1,-1),(0,1),(4,4)\}$
3. $y = x^3$
4. $y = |x| + 2$
5. $x = |y|$
6. $x = -\sqrt{y}$
7. $y = \frac{1}{2}x - 3$
8. $y = -x + 1$
9. $y = \sqrt{x} + 2$
10. $T = \{(4,2),(3,-1),(-2,-1),(2,4)\}$
11. $x = y^2 - 2$
12. $y = 2\sqrt{x}$

Determine if each of the following functions is a one-to-one function. If so, graph the inverse of the function and state its domain and range.

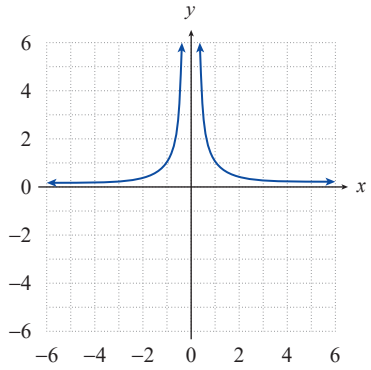
13. $y = 2x + 3$



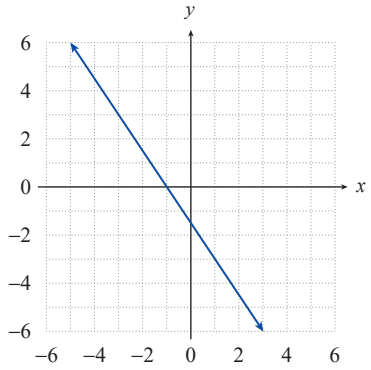
14. $y = x^2 + 4x$



15. $y = \frac{1}{x^2}$



16. $y = \frac{-3x - 3}{2}$



Determine if the following functions have inverse functions. If not, suggest a domain to restrict the function to so that it would have an inverse function (answers will vary). See Example 2.

17. $f(x)=x^2+1$ **18.** $g(x)=(x-2)^3-1$ **19.** $h(x)=\sqrt{x+3}$

20. $s(x)=\dfrac{1}{x^2}$ **21.** $G(x)=3x-5$ **22.** $F(x)=-x^2+5$

23. $r(x)=-\sqrt{x^3}$ **24.** $b(x)=\dfrac{1}{x}$ **25.** $f(x)=x^2-4x$

26. $m(x)=\dfrac{13x-2}{4}$ **27.** $H(x)=|x-12|$ **28.** $p(x)=10-x^2$

Find a formula for the inverse of each of the following functions. If necessary, first restrict the domain of the function. See Examples 3 and 4.

29. $f(x)=x^{\frac{1}{3}}-2$ **30.** $g(x)=4x-3$ **31.** $r(x)=\dfrac{x-1}{3x+2}$

32. $s(x)=\dfrac{1-x}{1+x}$ **33.** $F(x)=(x-5)^3+2$ **34.** $G(x)=\sqrt[3]{3x-1}$

35. $V(x)=\dfrac{x+5}{2}$ **36.** $W(x)=\dfrac{1}{x}$ **37.** $h(x)=x^{\frac{3}{5}}-2$

38. $A(x)=(x^3+1)^{\frac{1}{5}}$ **39.** $J(x)=\dfrac{2}{1-3x}$ **40.** $k(x)=\dfrac{x+4}{3-x}$

41. $h(x)=x^7+6$ **42.** $F(x)=\dfrac{3-x^5}{-9}$ **43.** $r(x)=\sqrt[5]{2x}$

44. $P(x)=(2+3x)^3$ **45.** $f(x)=3(2x)^{\frac{1}{3}}$ **46.** $q(x)=(x-2)^2+2$

47. $f(x)=(x-3)^2+2$ **48.** $f(x)=|x+2|+3$ **49.** $f(x)=(x+1)^4-2$

In each of the following exercises, verify that $f(f^{-1}(x))=x$ and that $f^{-1}(f(x))=x$. See Example 5.

50. $f(x)=\dfrac{3x-1}{5}$ and $f^{-1}(x)=\dfrac{5x+1}{3}$

51. $f(x)=\sqrt[3]{x+2}-1$ and $f^{-1}(x)=(x+1)^3-2$

52. $f(x)=\dfrac{2x+7}{x-1}$ and $f^{-1}(x)=\dfrac{x+7}{x-2}$

53. $f(x)=x^2,\ x\geq 0$ and $f^{-1}(x)=\sqrt{x}$

54. $f(x)=2x-3$ and $f^{-1}(x)=\dfrac{x+3}{2}$

55. $f(x)=\sqrt[3]{x+1}$ and $f^{-1}(x)=x^3-1$

56. $f(x)=\dfrac{1}{x}$ and $f^{-1}(x)=\dfrac{1}{x}$

57. $f(x)=\dfrac{x-5}{2x+3}$ and $f^{-1}(x)=\dfrac{3x+5}{1-2x}$

58. $f(x)=(x-2)^2,\ x\geq 2$ and $f^{-1}(x)=\sqrt{x}+2,\ x\geq 0$

59. $f(x)=\dfrac{1}{1+x}$ and $f^{-1}(x)=\dfrac{1-x}{x}$

Match the following functions with the graphs of the inverses of the functions.

60. $f(x)=x^3$

61. $f(x)=x-5$

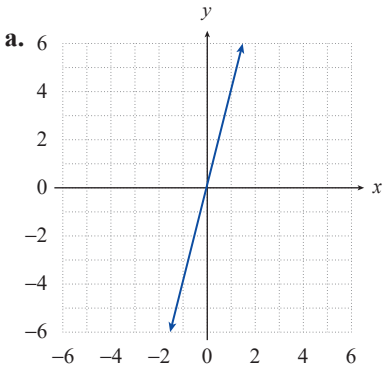
62. $f(x)=\sqrt{x-4}$

63. $f(x)=x^2,\ x\geq 0$

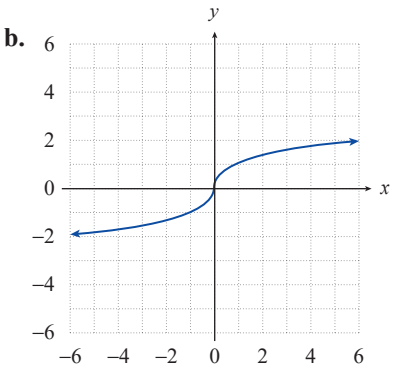
64. $f(x)=\dfrac{x}{4}$

65. $f(x)=\sqrt[3]{x+1}$

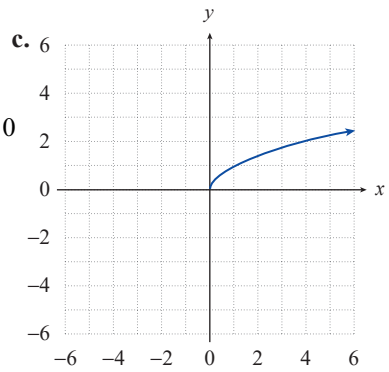
a.



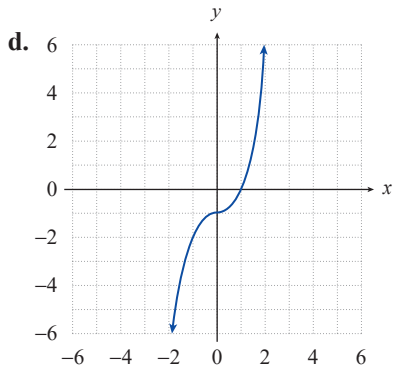
b.



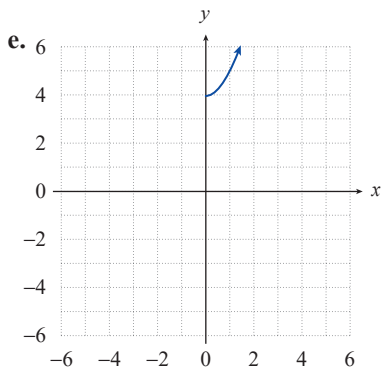
c.



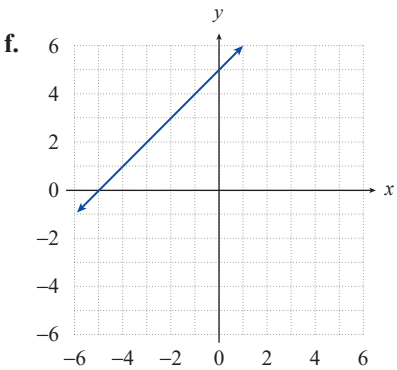
d.



e.



f.



APPLICATIONS

An inverse function can be used to encode and decode words and sentences by assigning each letter of the alphabet a numerical value (A = 1, B = 2, C = 3, ..., Z = 26).
Example: Use the function $f(x) = x^2$ to encode the word PRECALCULUS. The encoded message would be 256 324 25 9 1 144 9 441 144 441 361. The word can then be decoded by using the inverse function $f^{-1}(x) = \sqrt{x}$. The inverse values are 16 18 5 3 1 12 3 21 12 21 19 which translates back to the word PRECALCULUS. Encode or decode the following words using the numerical values A = 1, B = 2, C = 3, ..., Z = 26.

66. Encode the message SANDY SHOES using the function $f(x) = 4x - 3$.

67. Encode the message WILL IT RAIN TODAY using the function $f(x) = 8x$.

68. The following message was encoded using the function $f(x) = 8x - 7$. Decode the message.
41 137 65 145 9 33 33 169 113 89 89 33 193 9 1 89 89 1 105 25 57 113 137 145 33 145 57 113 33 145

69. The following message was encoded using the function $f(x) = 5x + 1$. Decode the message.
91 26 66 26 66 11 26 91 126 76 106 91 96 106 71 11 61 76 16 56

70. The following message was encoded using the function $f(x) = x^3$. Decode the message.
27 1 8000 27 512 1 12167 1 10648 125

71. The following message was encoded using the function $f(x) = -3 - 5x$. Decode the message.
–13 –28 –8 –18 –43 –33 –108 –73 –48 –73 –103 –43 –28 –98 –108 –73

TECHNOLOGY

A graphing utility can be used to verify the inverse of a function. Use a graphing utility to graph each of the following functions and its inverse in the same viewing window. Determine the domain and range of the inverse.

72. $f(x) = \sqrt{x+5}$ and $f^{-1}(x) = x^2 - 5$ **73.** $f(x) = x^3 - 1$ and $f^{-1}(x) = \sqrt[3]{x+1}$

74. $f(x) = \dfrac{2x+1}{x-1}$ and $f^{-1}(x) = \dfrac{x+1}{x-2}$ **75.** $f(x) = \dfrac{4}{\sqrt{x}}$ and $f^{-1}(x) = \dfrac{16}{x^2}$

76. $f(x) = -\sqrt{x^2 - 16}, x \geq 4$ and $f^{-1}(x) = \sqrt{x^2 + 16}$

77. $f(x) = x^2 + 3, x \geq 0$ and $f^{-1}(x) = \sqrt{x-3}$