Solution

a. Since $f(c)-f(b)>0$ and $c-b>0$, $\dfrac{f(c)-f(b)}{c-b}>0.$ That is, the average rate of change of *f* is positive on $[b,c]$.

b. Since $f(b)-f(a)<0$ but $b-a>0$, $\dfrac{f(b)-f(a)}{b-a}<0.$ That is, the average rate of change of *f* is negative on $[a,b]$.

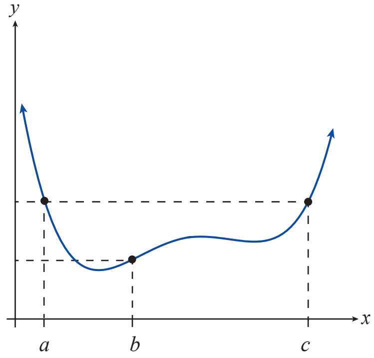

FIGURE 16

c. Since $f(c)=f(a)$, $\dfrac{f(c)-f(a)}{c-a}=0.$ That is, the average rate of change of *f* is zero on $[a,c]$.

5.2 EXERCISES

PRACTICE

Determine if each of the following relations is a function. If so, determine whether it is even, odd, or neither. Also determine if it has *y*-axis symmetry, *x*-axis symmetry, origin symmetry, or none of these symmetries, and then sketch the graph of the relation. See Example 1.

1. $f(x)=|x|+3$ **2.** $g(x)=x^3$ **3.** $h(x)=x^3-1$

4. $w(x)=\sqrt[3]{x}$ **5.** $x=-y^2$ **6.** $3y-2x=1$

7. $x+y=1$ **8.** $F(x)=(x-1)^2$ **9.** $x=y^2+1$

10. $x=2|y|$ **11.** $g(x)=\dfrac{x^2}{5}-5$ **12.** $s(x)=\left[\!\left[x+\dfrac{1}{2}\right]\!\right]$

13. $m(x)=\sqrt[3]{x}-1$ **14.** $xy=2$ **15.** $x+y^2=3$

For each of the following functions, find the open intervals of monotonicity where the function is increasing, decreasing, or constant. See Examples 2 and 3.

16. $f(x)=(x+3)^2$ **17.** $g(x)=-|x-2|$ **18.** $h(x)=\dfrac{1}{x-1}$

19. $H(x)=\dfrac{1}{(x+3)^2}$ **20.** $G(x)=\sqrt{x+1}$ **21.** $F(x)=-2$

22. $p(x)=-30|x-1|$ **23.** $q(x)=(4-x)^2+1$

24. $r(x)=\dfrac{(x-7)^4}{-2}+4$ **25.** $P(x)=\begin{cases}(x+3)^2 & \text{if } x<-1\\ 1 & \text{if } x\geq -1\end{cases}$

26. $Q(x)=\begin{cases}|x-1| & \text{if } x\leq 3\\ 5-x & \text{if } x>3\end{cases}$

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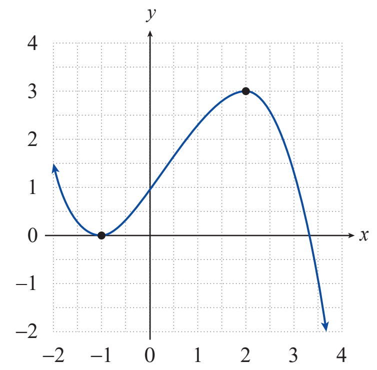
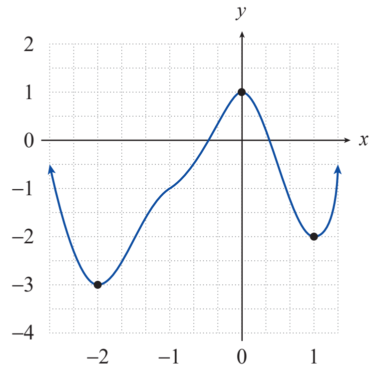
Using the graph of each of the following functions determine **a.** the locations and types of the local extrema, and **b.** the values of the local extrema. See Example 4.

27.

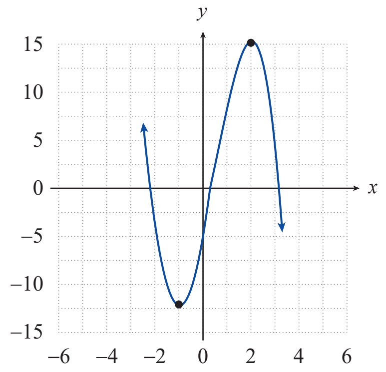


28.

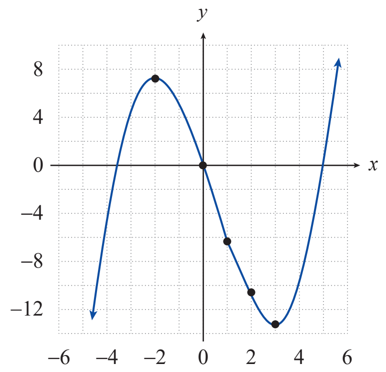


Using the graph and given formula for each of the following functions, determine **a.** the locations and types of the local extrema, and **b.** the values of the local extrema.

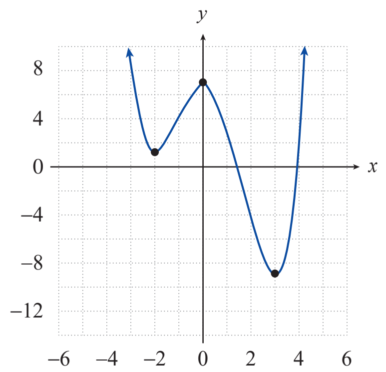
29. $f(x) = -2x^3 + 3x^2 + 12x - 5$



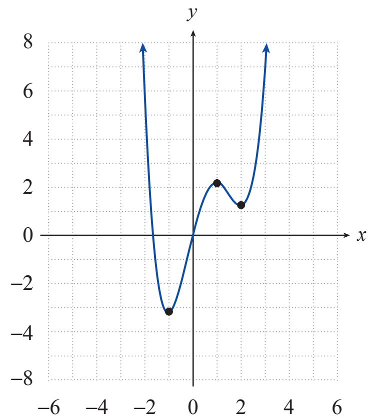
30. $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 6x$



31. $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - 3x^2 + 7$



32. $f(x) = \frac{x^4}{2} - \frac{4x^3}{3} - x^2 + 4x$



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For each of the following functions, determine **a.** the locations and types of the local extrema, and **b.** the values of the local extrema. A sketch of the graph may be helpful.

33. $f(x)=(x-5)^2+2$

34. $f(x)=-(x+1)^2+3$

35. $f(x)=-x^2-4x-3$

36. $f(x)=x^2-10x+27$

37. $f(x)=5|x-3|-2$

38. $f(x)=-|x+1|+2$

For each given function and interval, determine the average rate of change of the function over the interval. See Example 5.

39. $f(x)=x^3-2x$; $[1,3]$

40. $f(x)=3x+17$; $[-2,0]$

41. $f(x)=x^2-5x+3$; $[2,5]$

42. $f(x)=-x^3+x^2-7$; $[-1,1]$

43. $f(x)=\sqrt{x}$; $[2,4]$

44. $f(x)=-x^2+3x-1$; $[-2,1]$

45. $f(x)=x^2-3$; $[c,c+h]$

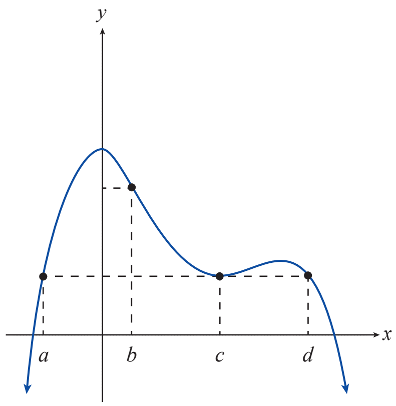
46. $f(x)=-3x^2+2x-5$; $[c,c+h]$

47. $f(x)=\frac{1}{x}$; $[3,4]$

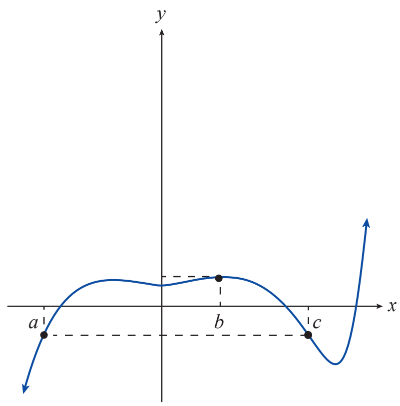
48. $f(x)=\frac{2}{x+1}$; $[-3,-2]$

Use each given graph of a function and the marked locations on the *x*-axis as endpoints to determine intervals over which the average rate of change of the function is **a.** positive, **b.** negative, **c.** zero. See Example 6.

49.



50.



APPLICATIONS

51. During the summer months, the water level of a garden pool varies as water is added and as it evaporates. On May 1st the pool was 3.4 feet deep. After a steady and linear increase due to rain, the depth had increased to 4.9 feet on June 1st. By July 1st the water level had decreased linearly to 4.2 feet. Knowing that the pool would be covered for the winter, the owner filled the pool (in an essentially linear fashion) until it reached 5 feet on August 1st. Graph the water level as a function of time and determine the open intervals of monotonicity.

52. The profit made by a hot dog vendor is given by the function

$$P(x)=\begin{cases}2x-3 & \text{if } x\geq 0 \text{ and } x<7\\ \frac{1}{4}x^2 & \text{if } x\geq 7\end{cases}$$

where x is the number of hot dogs sold. Graph the profit function and determine the open intervals of monotonicity.

53. The cost incurred by a newspaper stand is given by the function

$$C(x)=\begin{cases}-2\sqrt{x}+8 & \text{if } x\geq 0 \text{ and } x<3\\ -x+8 & \text{if } x\geq 3\end{cases}$$

where x is the number of newspapers sold. Graph the cost function and determine the open intervals of monotonicity.

WRITING & THINKING

54. Determine the average rate of change of the function $f(x)=5x-19$ over each of the following intervals: $[-3,-2],[1,7],[c,c+h]$. What do you conclude from your calculations?

55. Let $f(x)=mx+b$, where m and b are both unspecified constants. Determine the average rate of change of f over several different intervals of your choice. What do you conclude from your calculations?

56. Let $f(x)=3x^2-7x+2$. Find the difference quotient of f at c with increment h. What happens to this difference quotient as the increment h becomes very small?

57. Let $f(x)=px^2+qx+r$, where p, q, and r are unspecified constants. Find the difference quotient of f at c with increment h. What happens to this difference quotient as the increment h becomes very small?

58. What can be deduced about the average rate of change of a function if the function is increasing over the interval?

59. What can be deduced about the monotonicity of a function over an interval if the function's average rate of change is positive over the interval?

60. What can be deduced about the monotonicity of a function over an interval if the function's average rate of change is zero over the interval?

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