For instance, population growth over a long span of time always seems to show either an increase in the rate of growth or else a tapering off in the rate, neither of which is linear behavior. And the vertex that is present in the graph of any quadratic function also doesn't seem to be something typically seen in graphs of population growth. Is there a better assumption for the shape of the graph? The answer is yes, and we will return to the question of curve fitting several more times as we gain experience with more classes of functions.

4.6 EXERCISES

APPLICATIONS

Construct a mathematical model as appropriate for each of the following situations, and then use your model to answer the accompanying questions.

1. A tinsmith wants to make a small windowsill planter from a $20\text{ cm} \times 60\text{ cm}$ sheet of copper. She'll form it by cutting equally sized squares from each of the four corners of the sheet, folding up the resulting flaps to form the sides of the planter, and then soldering the four vertical edges.

- **a.** Construct a model for the volume of the planter based on the side length of the square cut from each corner, and determine the feasible domain for the model.
- **b.** Is it possible to construct such a planter with a volume of 2000 cm^3 or larger?
- **c.** What is the maximum possible volume, rounded to the nearest whole number?
- **d.** If she wants the ratio of the planter's width to height to be 2, what will be the ratio of its length to width?

2. Suppose the tinsmith of the previous problem wants to construct a similar planter (out of a sheet of copper with dimensions to be determined) but with ratios of height to width to length of $1:2:3$.

- **a.** Construct a model for the volume of such a planter based on the height.
- **b.** What must be the ratio of the width to length of the original sheet of copper?
- **c.** Rounding to the nearest whole number, what minimum height of such a planter will have a volume of 2000 cm^3 or larger?

3. A car dealership wants to try out a new leasing arrangement, which will allow a buyer to trade a car back in to the dealership for a certain amount of credit at any time throughout the first three years of ownership. For a car in good condition, the arrangement will value the car at $\frac{1}{3}$ the original price at the end of three years, and will depreciate the value of the car linearly over the course of the three years.

- **a.** Construct a model for the value at time t (in years) of a car with initial purchase price P, for $0 \le t \le 3$.
- **b.** At what point in time does a car have half its original value?
- **c.** What is the value of a car at the end of the first year?

4. Picture a person standing a certain distance away from a streetlamp at night, with the streetlamp casting a shadow of the person.

a. Find a model for the length s of the shadow of a person of height h cast by a 15-foot-tall streetlamp d feet away.
b. For a 5-foot-tall person, express the length s as a function of d.
c. How does the shadow of a 5-foot-tall person compare to that of a 6-foot-tall person when both are standing 20 feet from the streetlamp?
d. How far away does the 6-foot-tall person have to be for the person's shadow to be 6 feet long?

5. Consider a point on an arbitrary nonvertical line in the plane.

a. Find a model for the distance d between a point on the line $y = mx + b$ and the origin.
b. What form does the model take for lines that are horizontal?
c. What form does the model take for lines that pass through the origin?
d. What form does the model take for a line that passes through the origin and has slope $\sqrt{3}$?

6. a. Find a model for the product of two nonnegative numbers whose sum is 10.
b. What are the largest and smallest possible products?

7. a. Find a model for the sum of the cubes of two nonnegative numbers whose sum is 10.
b. Expressing the sum of the cubes as a function of one variable, graph the sum.
c. What are the largest and smallest possible sums?

8. a. Find a model for the Body Mass Index (BMI) of a person, given that BMI varies directly as a person's weight in pounds and inversely as the square of the person's height in inches.
b. With the additional information that a 6 ft tall person weighing 175 lb has a BMI of 23.73, what is the BMI of someone 5 ft, 6 in. tall weighing 140 lb?
c. How much weight would the 5 ft, 6 in. tall person from part b. need to gain or lose to have a BMI of 22?
d. What is the BMI for a person with the same weight but two inches taller than the 5 ft, 6 in. person?

9. a. Use the gravitational-attraction model of Example 3 to determine how far away from the surface of Earth a person would have to be to feel the force of attraction felt on the moon (about one-sixth that felt on Earth).
b. How does this distance compare to Earth's radius?
c. Why do astronauts appear to be weightless when they are in orbit so much closer to Earth?

10. a. Find a model for the area of sheet metal that must be used to make a cylindrical can, including both top and bottom, in terms of the can's radius r and height h.
b. Modify the model to be a function of r only, given that the volume of the can is to be 1000 cm^3 (1 liter).
c. Estimate the value of r that will minimize the area of metal required.

11. **a.** Find a model for the surface area of a cube of side length x in terms of the cube's volume.

b. What is the surface area of a cube that has a volume of 1000 mm^3?

12. **a.** Find a model for the surface area of a sphere of radius r in terms of the sphere's volume.

b. What is the surface area of a sphere that has a volume of $\frac{500}{3}\text{ mm}^3$?

13. Maria plans to build a rectangular dog run with an area of 1800 ft^2 at the edge of her property. She wants to use a solid fence that costs $\$6/\text{ft}$ for the side that will sit on the edge of her property, but is willing to use fencing that only costs $\$2/\text{ft}$ for the other three sides.

a. Find a model for the total cost of the fencing.

b. Estimate the length of solid fence that will minimize her total cost.

14. *Restaurante Caro* frequently offers a special prix fixe meal and has been charging \$150 per person for the event. At that price, they've been averaging 30 customers each time. Their marketing firm has convinced them that they'll gain a customer for every dollar they lower the cost of the event, and conversely lose a customer for every dollar they raise the cost. Their fixed cost per event is \$1500 and preparing each customer's meal costs an additional \$20.

a. Find the cost, revenue, and profit functions for these prix fixe events.

b. What are the break-even points in terms of customers served?

c. Is there a number of customers that will maximize their profit?

d. If so, what price per person should they charge?

15. A ceramicist made 6 bowls of a certain style and sold them for \$12 each, just breaking even at that price. Each time she starts up her kiln and makes any number of the bowls, she has a fixed cost of \$36, and there is an additional cost in materials to make each bowl. Polling the six customers who bought the bowls, she learned that only half would have bought the same bowl at a price of \$21.

a. Assuming a linear relationship between price per bowl and the number of bowls sold, and given the information she has, find the revenue, cost, and profit functions for selling a certain number of bowls.

b. Is there another break-even point for her product?

c. Is there an ideal number of bowls she should make in each production run?

d. If so, what should she charge per bowl and what maximum profit can she expect?

Interpolate and extrapolate, as appropriate, to answer the questions with the given data. Use a graphing utility to answer questions marked with 📈.

16. Use the linear and quadratic functions of best fit modeling the US population data in Tables 1 and 2 of this section to answer the following questions.

a. How do the interpolated populations for 1955 compare in the two models?
b. What is the extrapolated linear-model population for the year 1800? How do you interpret this result?
c. What is the calculated quadratic-model population for the year 1800? How does this compare to the actual population in 1800 on which the quadratic model is based?
d. Imagine tracing the quadratic model population far back in time, before the US actually existed as a country. What is the extrapolated quadratic model population in the year 1000? How do you interpret this result?

17. 📈 The table below shows the height at half-second intervals of a rock that breaks free from the top of a 200-foot-tall cliff and falls without obstruction to the river below.

Time t (in seconds)	Height (in feet)
0	200
0.5	196
1.0	184
1.5	164
2.0	136
2.5	100
3.0	56

a. Graph the heights (either by hand or with a graphing utility) and estimate the time the rock hits the water.
b. Find the linear function of best fit that models the height of the rock, and graph the function along with the given heights. By the linear model, what is the extrapolated time when the rock hits the water? What is the calculated linear-model height of the rock at time $t = 0$?
c. Find the quadratic function of best fit that models the height of the rock, and graph the function along with the given heights. By the quadratic model, what is the extrapolated time when the rock hits the water? What is the calculated quadratic-model height of the rock at time $t = 0$?
d. Which model appears to be more appropriate?

18. Karen bought a puppy and has been tracking its weight at the end of each month since it was born. She was told by the dog's breeder that the dog should have an adult weight somewhere between 40 and 45 pounds.

End of Month	Weight (in pounds)
1	3
2	5
3	8
4	11
5	17
6	22
7	28
8	32
9	35
10	38
11	40
12	42

a. Find the linear function of best fit that models the dog's weight, and graph the function along with the given weights. By the linear model, what is the extrapolated weight at the end of the second year? How do you interpret this result?

b. Find the quadratic function of best fit that models the dog's weight, and graph the function along with the given weights. By the quadratic model, what is the extrapolated weight at the end of the second year? How do you interpret this result?