Example 4: Solving Equations by Factoring

Solve the equation $x^{-2}-7x^{-1}+12=0$ by factoring.

Solution

While the given equation is not quadratic, it certainly bears a strong resemblance to one that is. In fact, if we make the substitution $y=x^{-1}$, we obtain the quadratic equation

$$y^2-7y+12=0,$$

which we can solve by factoring the trinomial as $(y-3)(y-4)$. This gives us $y=3$ or $y=4$. Translating this back to the variable x, we have $x^{-1}=3$ or $x^{-1}=4$, so $x=\frac{1}{3}$ or $x=\frac{1}{4}$.

2.4 EXERCISES

PRACTICE

Solve the following quadratic-like equations. See Example 1.

1. $(x-1)^2+(x-1)-12=0$
2. $(z-8)^2-7(z-8)+12=0$
3. $(y-5)^2-11(y-5)+24=0$
4. $(x^2-1)^2+(x^2-1)-12=0$
5. $(x^2+1)^2+(x^2+1)-12=0$
6. $(x^2-13)^2+(x^2-13)-12=0$
7. $(x^2-2x+1)^2+(x^2-2x+1)-12=0$
8. $2y^{\frac{2}{3}}+y^{\frac{1}{3}}-1=0$
9. $2x^{\frac{2}{3}}-7x^{\frac{1}{3}}+3=0$
10. $(x^2-6x)^2+4(x^2-6x)-5=0$
11. $(y^2-5)^2+5(y^2-5)-36=0$
12. $(x^2+7)^2+8(x^2+7)+12=0$
13. $(t^2-t)^2-8(t^2-t)+12=0$
14. $2x^{\frac{1}{2}}-5x^{\frac{1}{4}}+2=0$
15. $3x^{\frac{2}{3}}-x^{\frac{1}{3}}-2=0$
16. $y^{\frac{1}{2}}-5y^{\frac{1}{4}}+6=0$
17. $(z^2+4z)^2+7(z^2+4z)+12=0$
18. $5y^{\frac{2}{3}}+33y^{\frac{1}{3}}+18=0$

Solve the following polynomial equations by factoring. See Example 2.

19. $a^3-3a^2=a-3$
20. $2x^3+x^2+2x+1=0$
21. $2x^3-x^2=15x$
22. $x^4+5x^2-36=0$
23. $y^4+21y^2-100=0$
24. $y^3+8=0$
25. $5s^3+6s^2-20s=24$
26. $8a^3-27=0$
27. $16a^4=81$
28. $6x^3+8x^2=14x$
29. $14x^3+27x^2-20x=0$
30. $5z^3+28z^2=49z$
31. $27x^3+64=0$
32. $x^3-4x^2+x=4$
33. $x^3+27=0$

Solve the following equations by factoring. See Examples 3 and 4.

34. $3x^{\frac{11}{3}} + 2x^{\frac{8}{3}} - 5x^{\frac{5}{3}} = 0$

35. $(x-3)^{\frac{-1}{2}} + 2(x-3)^{\frac{1}{2}} = 0$

36. $(y-6)^{\frac{-5}{2}} + 7(y-6)^{\frac{-3}{2}} = 0$

37. $y^{-2} - 2y^{-1} + 1 = 0$

38. $2x^{\frac{13}{5}} - 5x^{\frac{8}{5}} + 2x^{\frac{3}{5}} = 0$

39. $(2x-5)^{\frac{1}{3}} - 3(2x-5)^{\frac{-2}{3}} = 0$

40. $x^{-4} - 13x^{-2} + 36 = 0$

41. $y^{\frac{7}{2}} - 5y^{\frac{5}{2}} + 6y^{\frac{3}{2}} = 0$

42. $(t+4)^{\frac{2}{3}} + 2(t+4)^{\frac{8}{3}} = 0$

43. $y^{-2} - 2y^{-1} - 35 = 0$

44. $x^{\frac{11}{2}} - 6x^{\frac{9}{2}} + 9x^{\frac{7}{2}} = 0$

45. $5y^{\frac{11}{3}} + 3y^{\frac{8}{3}} - 2y^{\frac{5}{3}} = 0$

46. $5y^{\frac{12}{5}} - 43y^{\frac{7}{5}} + 24y^{\frac{2}{5}} = 0$

47. $(3x-3)^{\frac{-1}{3}} - 5(3x-3)^{\frac{-4}{3}} = 0$

48. $x^{-2} + 8x^{-1} + 15 = 0$

49. $(y+3)^{\frac{2}{5}} + 4(y+3)^{\frac{7}{5}} = 0$

WRITING & THINKING

50. Find b, c, and d so the equation $x^3 + bx^2 + cx + d = 0$ has solutions of -3, -1, and 5.

51. Find b, c, and d so the equation $x^3 + bx^2 + cx + d = 0$ has solutions of -2, 0, and 6.

52. Find b and c so the equation $x^3 + bx^2 + cx = 0$ has solutions of 0, 1, and -7.

53. Find a, c, and d so the equation $ax^3 + 4x^2 + cx + d = 0$ has solutions of -4, 6, and -6.

54. Find a, b, and d so the equation $ax^3 + bx^2 + 3x + d = 0$ has solutions of -3, $-\frac{1}{2}$, and 0.

55. Find a, b, and c so the equation $ax^3 + bx^2 + cx + 6 = 0$ has solutions of $-\frac{3}{5}$, $\frac{2}{3}$, and 1.