13.5 EXERCISES

PRACTICE

Consider each of the following situations and determine if each is a combination or permutation.

1. double scoop options from 29 ice cream flavors
2. a poker hand from a standard deck
3. a board committee chosen from 15 candidates
4. a seating chart for 24 students

Evaluate the following permutations. See Examples 5 and 6.

5. ${}_4P_2$
6. ${}_{15}P_2$
7. ${}_6P_5$
8. ${}_{19}P_{17}$

Evaluate the following combinations. See Example 7.

9. ${}_6C_4$
10. ${}_4C_2$
11. ${}_{12}C_5$
12. ${}_{21}C_{14}$

Determine how many different arrangements there are of the letters in each of the following words. See Example 8.

13. ABYSS
14. BANANA
15. COLLEGE
16. ALGEBRA
17. MATHEMATICS
18. FIBONACCI

Use the Binomial and Multinomial Theorems to expand each of the following expressions. See Examples 9, 10, and 11.

19. $(3x+y)^5$
20. $(x-2y)^7$
21. $(x-3)^4$
22. $(x^2-y^3)^4$
23. $(6x^2+y)^5$
24. $(4x+5y^2)^6$
25. $(7x^2+8y^2)^4$
26. $(x^3-y^2)^5$
27. $(x+y+z)^2$
28. $(a-2b+c)^3$
29. $(2x+5)^6$
30. $(2x+3y-z)^3$
31. What is the coefficient of the term containing x^3y in the expansion of $(2x+y)^4$?
32. What is the coefficient of the term containing x^4y^3 in the expansion of $(x^2-2y)^5$?
33. Find the first four terms in the expansion of $(x+3y)^{16}$.

34. Find the first three terms in the expansion of $(2x+3)^{13}$.

35. Find the first two terms in the expansion of $\left(3x^{\frac{1}{4}}+5y\right)^{17}$.

36. Find the 11^{th} term in the expansion of $(x+2)^{24}$.

37. Find the 17^{th} term in the expansion of $(2x+1)^{21}$.

38. Find the 9^{th} term in the expansion of $(x-6y)^{12}$.

APPLICATIONS

Use the Multiplication Principle of Counting to answer the following questions.
See Examples 1 and 2.

39. Suppose you write down someone's phone number on a piece of paper, but then accidentally wash it along with your laundry. Upon drying the paper, all you can make out of the number is 42? – 3?7?. How many different phone numbers fit this pattern?

40. How many different 7-digit phone numbers contain no odd digits? (Ignore the fact that certain 7-digit sequences are disallowed as phone numbers.)

41. How many different 7-digit phone numbers do not contain the digit 9? (Ignore the fact that certain 7-digit sequences are disallowed as phone numbers.)

42. A certain combination lock allows the buyer to set any combination of five letters, with repetition allowed, but each of the letters must be A, B, C, D, E, or F. How many combinations are possible?

43. In how many different orders can 15 runners finish a race, assuming there are no ties?

44. How many different 4-letter radio-station names can be made, assuming the first letter must be a K or a W? Assume repetition of letters is allowed.

45. How many different 4-letter radio-station names can be made from the call-letters K, N, I, T, assuming the letter K must appear first? Each of the four letters can be used only once.

46. Three men and three women line up in a row for a photograph, and decide men and women should alternate. In how many different ways can this be done? (Don't forget that the left-most person can be a man or a woman.)

47. How many different ways can a 10-question multiple choice test be answered, assuming every question has 5 possible answers?

48. How many different ways can your 12 favorite novels be arranged in a row?

49. How many different 6-character license plates can be formed if all 26 letters and 10 numerical digits can be used with repetition?

50. How many different 6-character license plates can be formed if all 26 letters and 10 numerical digits can be used without repetition?

51. How many different 6-character license plates can be formed if the first 3 places must be letters and the last 3 places must be numerical digits? (Assume repetition is not allowed.)

52. A box of crayons comes with 8 different colored crayons arranged in a single row. How many different ways can the crayons be ordered in the box?

Express the answer to the following permutation problems using permutation notation $\left({}_nP_k\right)$ and numerically. See Examples 3, 4, and 5.

53. Suppose you have a collection of 30 cherished math books. How many different ways can you choose 12 of them to arrange in a row?

54. In how many different ways can first-place, second-place, and third-place be decided in a 15-person race?

55. Suppose you need to select a user-ID for a computer account, and the system administrator requires that each ID consist of 8 characters with no repetition allowed. The characters you may choose from are the 26 letters of the alphabet (with no distinction between uppercase and lowercase) and the 10 digits. How many choices for a user-ID do you have?

56. How many different 5-letter "words" (they don't have to be actual English words) can be formed from the letters in the word PLASTIC?

57. Seven children rush into a room in which six chairs are lined up in a row. How many different ways can six of the seven children choose a chair to sit in? (The seventh remains standing.) How does the answer differ if there are seven chairs in the room?

58. At a meeting of 17 people, a president, vice president, secretary, and treasurer are to be chosen. How many different ways can these positions be filled?

59. Given 26 building blocks, each with a different letter of the alphabet printed on it, how many different 3-letter "words" can be formed?

Express the answer to the following combination problems using combination notation $\left({}_nC_k\right)$ and numerically. See Example 7.

60. In many countries, it is not uncommon for quite a few political parties to have their representatives in power. Suppose a committee composed of 10 Conservatives, 13 Liberals, 6 Greens, and 4 Socialists decides to form a subcommittee consisting of 3 Conservatives, 4 Liberals, 2 Greens and 1 Socialist. How many different such subcommittees can be formed?

61. A trade union asks its members to select 3 people, from a slate of 7, to serve as representatives at a national meeting. How many different sets of 3 can be chosen?

62. Many lottery games are set up so that players select a subset of numbers from a larger set and the winner is the person whose selection matches that chosen by some random mechanism. The order of the numbers is irrelevant. How many choices of six numbers can be made from the numbers 1 through 49?

63. How many different lines can be drawn through a set of nine points in the plane, assuming that no three of the points are collinear? (Points are said to be *collinear* if a single line containing them can be drawn.)

64. Suppose you are taking a 10-question True-False test, and you are guessing that the professor has arranged it so that five of the answers are True and five are False. How many different ways are there of marking the test with five True answers and five False answers?

65. A caller in a Bingo game draws 5 marked ping pong balls from a basket of 75 and calls the numbers out to the players. How many different combinations are possible assuming that the order is irrelevant?

Use the techniques seen in this section, to answer the following questions.

66. How many different ways are there of choosing five cards from a standard 52-card deck and arranging them in a row? How many different five-card hands can be dealt from a standard 52-card deck?

67. Suppose you have 10 Physics texts, 8 Computer Science texts, and 13 Math texts. How many different ways can you select 4 of each to take with you on vacation?

68. Suppose you have 10 Physics texts, 8 Computer Science texts, and 13 Math texts. How many different ways can you select 4 of each and then arrange them in a row on a shelf, so that the books are grouped by discipline?

69. A certain ice cream store has four different kinds of cones and 28 different flavors of ice cream. How many different single-scoop ice cream cones is it possible to order at this ice cream store?

70. If a local pizza shop has three different types of crust, two different kinds of sauce, and five different toppings, how many different one topping pizzas can be ordered?

71. A man has 8 different shirts, 4 different shorts, and 3 different pairs of shoes. How many different outfits can the man choose from?

72. A couple wants to have three children. They want to know the different possible gender outcomes for birth order. How many different birth orders are possible?

73. A student has to make out his schedule for classes next fall. He has to take a math class, a science class, an elective, a history class, and an English class. There are three math classes to choose from, two science classes, four electives, three history classes, and four English classes. How many different schedules could the student have?

74. A basketball team has 12 different people on the team. The team consists of three point guards, two shooting guards, one weak forward, three power forwards, and three centers. How many different starting line-ups are possible? (The starting line-up will consist of one player in each of the 5 positions.)

75. How many 5-letter strings can be formed using the letters V, W, X, Y, and Z, if the same letter cannot be repeated?

76. If at the racetrack nine greyhounds are racing against each other, how many different first, second, and third place finishes are possible?

77. A basketball tryout has four distinct positions available on the team. If 25 people show up for tryouts, how many different ways can the four positions be filled?

78. If a trumpet player is practicing eight different pieces of music, in how many different orders can he play his pieces of music?

79. A pizza place has 12 total toppings to choose from. How many different 4-topping pizzas can be ordered?

80. How many different 5-digit numbers can be formed using each of the numbers 6, 8, 1, 9, and 4?

81. If eight cards are chosen randomly from a deck of 52, how many possible groups of eight can be chosen?

82. A baseball team has 15 players on the roster and abatting line-up consists of 9 players. How many different batting line-ups are possible?

83. How many different ways can two red balls, one orange ball, one black ball, and three yellow balls be arranged?

WRITING & THINKING

Pascal's triangle is a triangular arrangement of binomial coefficients, the first few rows of which appear as follows.

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \\ & & & & \vdots & & & & \end{array}$$

Each number (aside from those on the perimeter of the triangle) is the sum of the two numbers diagonally adjacent to it in the previous row. Pascal's triangle is a useful way of generating binomial coefficients, with the n^{th} row containing the coefficients of a binomial raised to the $(n-1)^{\text{th}}$ power. It can also be used to suggest useful relationships between binomial coefficients. Prove each of the following such relationships algebraically.

84. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ (Note that this is a restatement of how Pascal's triangle is formed.)

85. $\binom{n}{k} = \binom{n}{n-k}$

86. $\binom{n}{0} = \binom{n}{n} = 1$

87. $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$

(**Hint:** Use the Binomial Theorem on $(x+y)^n$ for a convenient choice of x and y.)