13.1 EXERCISES

PRACTICE

Determine if each sequence is finite or infinite.

1. The sequence of odd numbers

2. $2, 4, 6, 8, 10, 12, \ldots$

3. $1, 3, 5, 7, 9, 11$

4. $3, 9, 27, 81, 243, \ldots$

5. The sequence of the days of the week

6. $1, 0, 1, 0, 1, 0, 1, 0, \ldots$

7. $1, 2, 3, 4, 5, 6, 7, \ldots$

8. The sequence of letters in the alphabet

9. The sequence of the number of ants in a colony recorded daily

Determine the first five terms of each sequence whose n^{th} term is defined as follows. See Examples 1 and 2.

10. $a_n = 7n - 3$

11. $a_n = -3n + 5$

12. $a_n = (-2)^n$

13. $a_n = \dfrac{3n}{n+2}$

14. $a_n = \dfrac{(-1)^n}{n^2}$

15. $a_n = \dfrac{(-1)^{n+1} 2^n}{3^n}$

16. $a_n = \left(-\dfrac{1}{3}\right)^{n-1}$

17. $a_n = \dfrac{n^2}{n+1}$

18. $a_n = \dfrac{(n-1)^2}{(n+1)^2}$

19. $a_n = \dfrac{n(n+1)}{2}\cos(n\pi)$

20. $a_n = (-2)^n + n$

21. $a_n = (-n+4)^3 - 1$

22. $a_n = \dfrac{2n^2}{3n-2}$

23. $a_n = (-1)^n \sqrt{n}$

24. $a_n = \dfrac{2^n}{n^2}$

25. $a_n = 4n - 3$

26. $a_n = -5n + 15$

27. $a_n = 2^{n-2}$

28. $a_n = 3^{-n-2}$

29. $a_n = (3n)^n$

30. $a_n = \sqrt[2^n]{64}$

31. $a_n = \dfrac{5n}{n+3}$

32. $a_n = \dfrac{n^2}{n+2}$

33. $a_n = \dfrac{n^2 + n}{2}$

34. $a_n = (-1)^n n$

35. $a_n = \dfrac{(n+1)^2}{(n-1)^2}$

36. $a_n = n^2 + n$

37. $a_n = \dfrac{2n-1}{3n}$

38. $a_n = \sqrt{3n} + 1$

39. $a_n = -(n-1)^2$

40. $a_n = (n-1)(n+2)(n-3)$

41. $a_1 = 2$ and $a_n = (a_{n-1})^2$ for $n \geq 2$

42. $a_1 = -2$ and $a_n = 7a_{n-1} + 3$ for $n \geq 2$

43. $a_1 = 1$ and $a_n = na_{n-1}$ for $n \geq 2$

44. $a_1 = -1$ and $a_n = -a_{n-1} - 1$ for $n \geq 2$

45. $a_1 = 2$ and $a_n = \sqrt{(a_{n-1})^2 + 1}$ for $n \geq 2$

46. $a_n = n\sin\left(\frac{n\pi}{2}\right)$ **47.** $a_n = n^3 \sin\left(\frac{n\pi}{2}\right)$ **48.** $a_n = 2^n \cos(n\pi)$

Find a possible formula for the general n^{th} term of each sequence. Answers may vary. See Example 3.

49. $5, 12, 19, 26, 33, \ldots$

50. $-2, 4, -8, 16, -32, \ldots$

51. $-1, 2, -6, 24, -120, \ldots$

52. $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \ldots$

53. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

54. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

55. $-34, -25, -16, -7, 2, \ldots$

56. $\frac{3}{14}, \frac{2}{15}, \frac{1}{16}, 0, -\frac{1}{18}, \ldots$

57. $\frac{1}{4}, \frac{1}{2}, 1, 2, 4, \ldots$

58. $-1, -6, -11, -16, -21, \ldots$

59. $\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}, \ldots$

60. $1, 4, 15, 64, 325, \ldots$

Translate each expanded sum that follows into summation notation, and vice versa. Then evaluate the sum. See Examples 4 and 5.

61. $\sum_{i=1}^{7}(3i-5)$ **62.** $\sum_{i=1}^{5}-3i^2$ **63.** $1+8+27+\cdots+216$

64. $1+4+7+\cdots+22$ **65.** $\sum_{i=3}^{10}5i^2$ **66.** $9+16+25+\cdots+81$

67. $\sum_{i=1}^{6}-3(2)^i$ **68.** $\sum_{i=6}^{13}(i+3)(i-10)$ **69.** $9+27+81+\cdots+19{,}683$

Find a formula for the n^{th} partial sum S_n of each series. If the series is finite, determine the sum. If the series is infinite, determine if it converges or diverges, and if it converges, determine the sum. See Example 6.

70. $\sum_{i=1}^{100}\left(\frac{1}{i+3}-\frac{1}{i+4}\right)$ **71.** $\sum_{i=1}^{\infty}\left(\frac{1}{i+3}-\frac{1}{i+4}\right)$ **72.** $\sum_{i=1}^{\infty}\left(2^i - 2^{i-1}\right)$

73. $\sum_{i=1}^{15}\left(2^i - 2^{i-1}\right)$ **74.** $\sum_{i=1}^{49}\left(\frac{1}{2i}-\frac{1}{2i+2}\right)$ **75.** $\sum_{i=1}^{\infty}\left(\frac{1}{2i}-\frac{1}{2i+2}\right)$

76. $\sum_{i=1}^{100}\ln\left(\frac{i}{i+1}\right)$ (**Hint:** Make use of a property of logarithms to rewrite the sum.)

77. $\sum_{i=1}^{\infty}\ln\left(\frac{i}{i+1}\right)$ (**Hint:** Make use of a property of logarithms to rewrite the sum.)

78. $\sum_{i=1}^{30}\left(\frac{1}{2i+5}-\frac{1}{2i+7}\right)$ **79.** $\sum_{i=1}^{\infty}\left(\frac{1}{3i+1}-\frac{1}{3i+4}\right)$ **80.** $\sum_{i=1}^{65}\ln\left(\frac{i}{i+1}\right)$

Determine the first five terms of each generalized Fibonacci sequence.

81. $a_1 = 4,\ a_2 = 7,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

82. $a_1 = -9,\ a_2 = 1,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

83. $a_1 = 10,\ a_2 = 20,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

84. $a_1 = -17,\ a_2 = 13,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

85. $a_1 = 13,\ a_2 = -17,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

Determine the first five terms of each recursively defined sequence.

86. $a_1 = 2,\ a_2 = -3,$ and $a_n = 3a_{n-1} + a_{n-2}$ for $n \geq 3$

87. $a_1 = 1,\ a_2 = -3,$ and $a_n = a_{n-1}a_{n-2}$ for $n \geq 3$

88. $a_1 = 3,\ a_2 = 1,$ and $a_n = \left(a_{n-2}\right)^{a_{n-1}}$ for $n \geq 3$

APPLICATIONS

89. Suppose you buy one cow and a number of bulls. In year one, your cow gives birth to a female calf and continues to bear another female calf every year for the rest of her life. Assuming that every calf born is female, that each cow begins calving in her third year (at age two), and that your cows never die, determine the number of cows (do not count the bulls) you will have at the end of the 14$^{\text{th}}$ year.

90. You borrow \$638 to buy a new car stereo. You plan to pay this sum back with monthly payments of \$74. The interest rate on your loan is 6% compounded monthly (recall that's 0.5% per month). Let A_n be the amount you owe at the end of the n^{th} month. Find a recursive sequence to represent A_n. Use this sequence to find the amount owed after 4 months and the amount owed after 6 months. How many months will it take to pay off your loan?

WRITING & THINKING

91. Beginning with yourself, create a sequence describing the number of biological predecessors you have in each of the past 7 generations of your family.

92. The Fibonacci sequence is quite prevalent in nature. Do some research on your own to find an occurrence in nature (other than population growth) of the Fibonacci sequence.