

11.4 EXERCISES

PRACTICE

Find the $x'y'$ -coordinates of each point for the given rotation angle θ . See Example 1.

1. $(8, 6)$, $\theta = 30^\circ$

2. $(-5, 1)$, $\theta = \frac{\pi}{3}$

3. $\left(\frac{-1}{2}, \frac{-1}{8}\right)$, $\theta = \frac{\pi}{4}$

4. $(2.7, 5)$, $\theta = 15^\circ$

5. $(13, -4)$, $\theta = 78^\circ$

6. $\left(\frac{12}{\sqrt{18}}, \frac{240}{\sqrt{1152}}\right)$, $\theta = 45^\circ$

7. $\left(3.65, \frac{3}{8}\right)$, $\theta = \frac{\pi}{6}$

8. $\left(\frac{3 + \sqrt{48}}{-(2\sqrt{12} + 3)}, \frac{\sqrt{4096}}{8\sqrt{25} \cdot \frac{1}{5}\sqrt{64}}\right)$, $\theta = \frac{\pi}{2}$

Use the discriminant to determine whether the equation of the given conic represents an ellipse, a parabola, or a hyperbola. See Example 4.

9. $2x^2 - 3xy + 2y^2 - 2x = 0$

10. $3x^2 + 7xy + 5y^2 - 6x + 7y + 15 = 0$

11. $3x^2 + 8xy + 4y^2 - 7 = 0$

12. $5x^2 + 6xy - 3y^2 - 9 = 0$

13. $-2x^2 - 8xy + 2y^2 + 2y + 5 = 0$

14. $3x^2 - 6xy + 3y^2 + 3x - 9 = 0$

15. $x^2 - xy + 4y^2 + 2x - 3y + 1 = 0$

16. $x^2 - 4xy + 4y^2 + 2x + 3y - 1 = 0$

Use the discriminant to classify each of the following conic sections. Then determine the angle θ that will allow you to convert the equation and eliminate the xy -term. Finally, sketch the graph of the conic section. See Examples 2, 3, and 4.

17. $xy = 2$

18. $xy - 4 = 0$

19. $x^2 + 2xy + y^2 - x + y = 0$

20. $7x^2 + 5\sqrt{3}xy + 2y^2 = 14$

21. $22x^2 + 6\sqrt{3}xy + 16y^2 - 49 = 276$

22. $2\sqrt{3}x^2 - 6xy + \sqrt{3}x - 9y = 0$

23. $34x^2 + 8\sqrt{3}xy + 42y^2 = 1380$

24. $xy + x - 4y = 6$

Sketch the graphs of the following conic sections. See Examples 2, 3, and 4.

25. $x^2 + 6xy + y^2 = 18$

26. $x^2 - 4xy + 3y^2 = 12$

27. $9x^2 + 14xy - 9y^2 = 15$

28. $36x^2 - 19xy + 8y^2 = 72$

29. $40x^2 + 20xy + 10y^2 + (2\sqrt{2} - 6)x - (4\sqrt{2} + 8)y = 90$

30. $72x^2 + 19xy + 4y^2 = 20$

31. $48x^2 + 15xy + 7y^2 = 28$

32. $72x^2 + 18xy - 9y^2 = 14$

Match the equation with its corresponding graph.

33. $3x^2 + 2xy + y^2 - 10 = 0$

35. $xy - 1 = 0$

37. $x^2 - y^2 - 16 = 0$

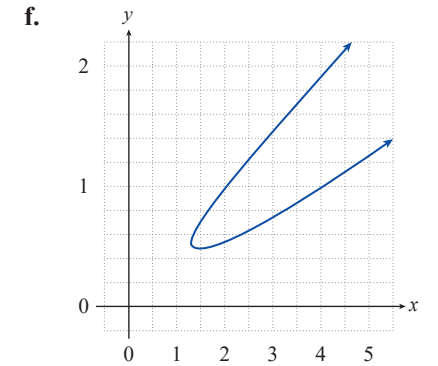
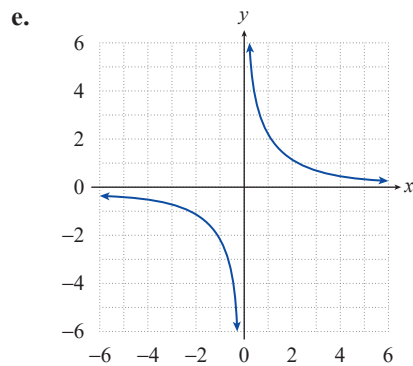
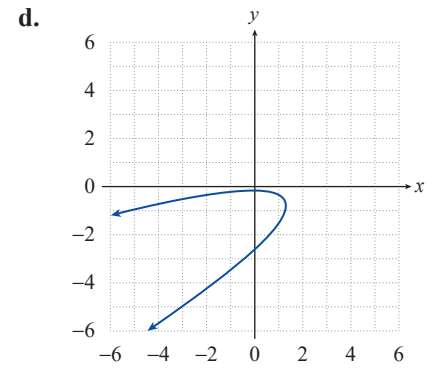
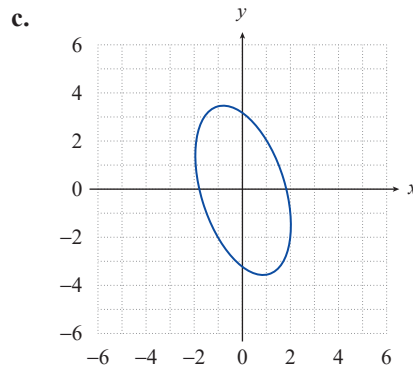
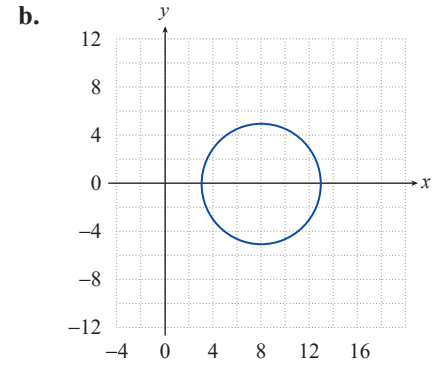
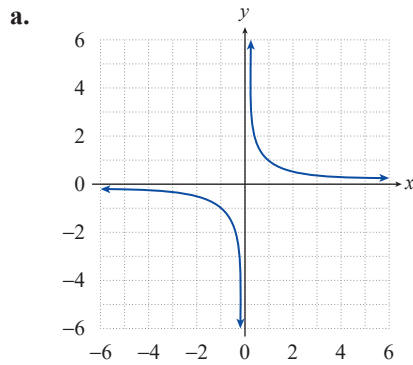
39. $4xy - 9 = 0$

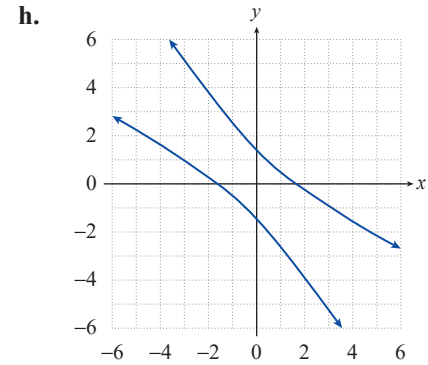
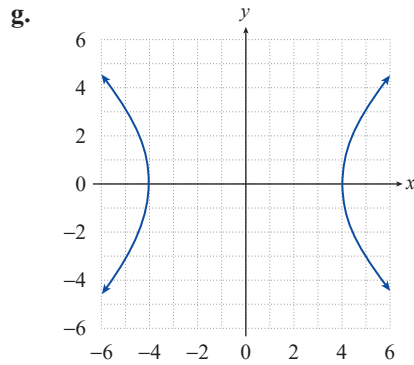
34. $x^2 - 4xy + 4y^2 + 5\sqrt{5}y + 1 = 0$

36. $x^2 + y^2 - 16x + 39 = 0$

38. $3x^2 + 8xy + 4y^2 - 7 = 0$

40. $x^2 - 6xy + 9y^2 - 2y + 1 = 0$





 **WRITING & THINKING**

41. You have just used the rotation of axes to rotate the x - and y -axes until they were parallel to the axes of the conic. The resulting equation in the $x'y'$ -plane is of the form

$$A'x'^2 + Bx'y' + E'y' + F' = 0.$$

What is wrong with the resulting equation?

42. What must the angle of rotation θ be if the coefficients of x^2 and y^2 are equal and $B \neq 0$? Support your answer.
43. Using the equation $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$,
- show that the rotation invariant $F = F'$ is true,
 - show that the rotation invariant $A + C = A' + C'$ is true, and
 - show that the rotation invariant $B^2 - 4AC = B'^2 - 4A'C'$ is true.