LORAN can actually determine the location of the ship by performing the same computations for another pair of simultaneous signals sent out from two additional transmitters, located at A' and B'. This defines a second hyperbola, and the ship must be at a point where the two hyperbolas intersect, as shown in Figure 10.

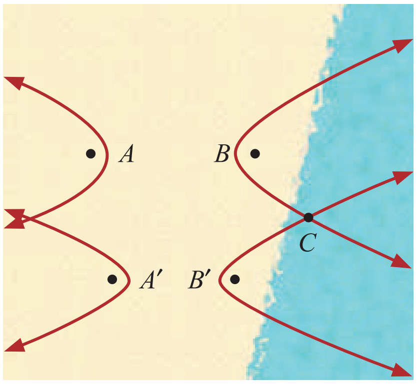


FIGURE 10: Two Sets of Transmitters

11.3 EXERCISES

PRACTICE

Sketch the graphs of the following hyperbolas, using asymptotes as guides. Determine the coordinates of the foci in each case. See Examples 1 and 2.

1. $\frac{(x+3)^2}{4}-\frac{(y+1)^2}{9}=1$

2. $\frac{(y-2)^2}{25}-\frac{(x+2)^2}{9}=1$

3. $4y^2-x^2-24y+2x=-19$

4. $x^2-9y^2+4x+18y-14=0$

5. $9x^2-25y^2=18x-50y+241$

6. $9x^2-16y^2+116=36x+64y$

7. $\frac{x^2}{16}-\frac{(y-2)^2}{4}=1$

8. $\frac{(y-1)^2}{9}-(x+3)^2=1$

9. $9y^2-25x^2-36y-100x=289$

10. $9x^2+18x=4y^2+27$

11. $9x^2-16y^2-36x+32y-124=0$

12. $x^2-y^2+6x-6y=4$

13. $\frac{(y-2)^2}{64}-\frac{(x+7)^2}{49}=1$

14. $\frac{(y-4)^2}{49}-\frac{(x+2)^2}{16}=1$

15. $\frac{(x+1)^2}{64}-\frac{(y+7)^2}{4}=1$

16. $\frac{(x+10)^2}{16}-\frac{(y+8)^2}{25}=1$

Find the center, foci, and vertices of the hyperbola that each equation describes.

17. $\frac{(x+3)^2}{4}-\frac{(y-2)^2}{9}=1$

18. $\frac{(y-2)^2}{16}-\frac{(x+1)^2}{9}=1$

19. $3(x-1)^2-(y+4)^2=9$

20. $(y-2)^2-2(x-4)^2=4$

21. $(x+2)^2-5(y-1)^2=25$

22. $6(y+2)^2-(x+1)^2=12$

23. $2x^2 + 12x - y^2 - 2y + 9 = 0$

24. $y^2 - 9x^2 + 6y + 72x - 144 = 0$

25. $x^2 - 4y^2 - 2x = 0$

26. $4y^2 - x^2 + 32y + 2x + 47 = 0$

27. $4x^2 - y^2 - 64x + 10y + 167 = 0$

28. $4x^2 - 9y^2 - 36y - 72 = 0$

Match the following equations to their graphs.

29. $\dfrac{x^2}{9} - y^2 = 1$

30. $y^2 - \dfrac{x^2}{4} = 1$

31. $x^2 - \dfrac{(y-3)^2}{4} = 1$

32. $\dfrac{(x-3)^2}{4} - \dfrac{(y+1)^2}{9} = 1$

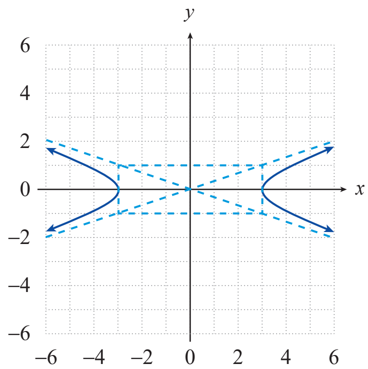
33. $(y+2)^2 - \dfrac{(x-2)^2}{4} = 1$

34. $\dfrac{x^2}{9} - \dfrac{(y+2)^2}{4} = 1$

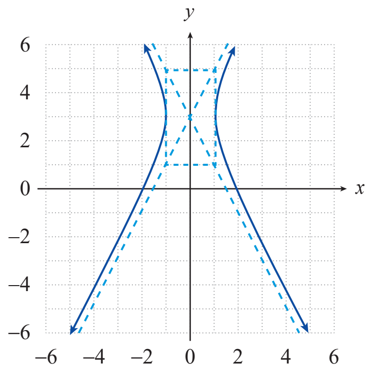
35. $\dfrac{y^2}{4} - (x-1)^2 = 1$

36. $x^2 - y^2 = 1$

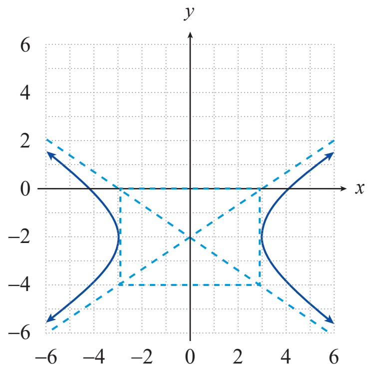
a.



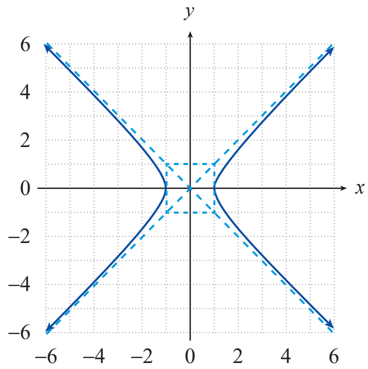
b.



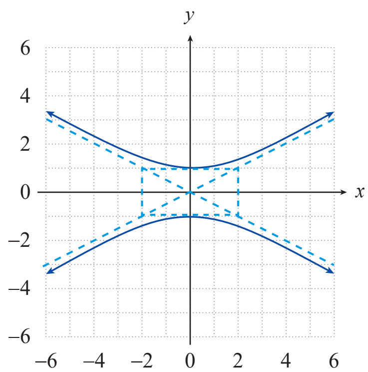
c.



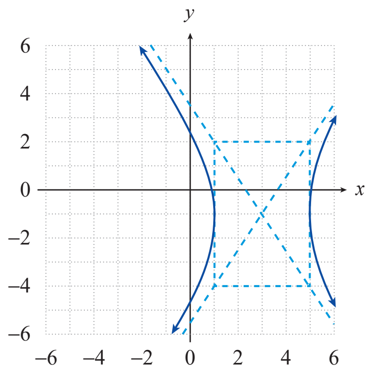
d.



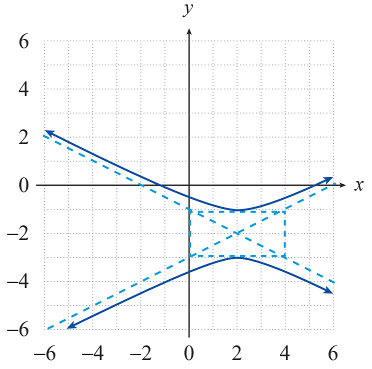
e.



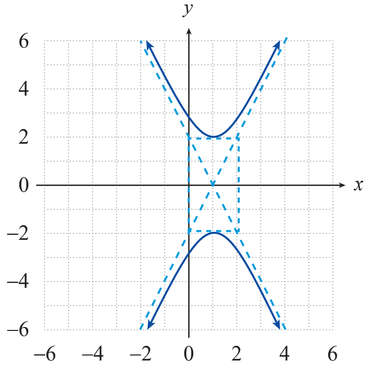
f.



g.



h.



Find the equation, in standard form, for the hyperbola with the given properties or with the given graph. See Example 3.

37. Foci at $(-3,0)$ and $(3,0)$ and vertices at $(-2,0)$ and $(2,0)$.

38. Foci at $(1,5)$ and $(1,-1)$ and vertices at $(1,3)$ and $(1,1)$.

39. Asymptotes of $y=\pm 2x$ and vertices at $(0,-1)$ and $(0,1)$.

40. Asymptotes of $y=\pm(x-2)+1$ and vertices at $(-1,1)$ and $(5,1)$.

41. Foci at $(2,4)$ and $(-2,4)$ and asymptotes of $y=\pm 3x+4$.

42. Foci at $(-1,3)$ and $(-1,-1)$ and asymptotes of $y=\pm(x+1)+1$.

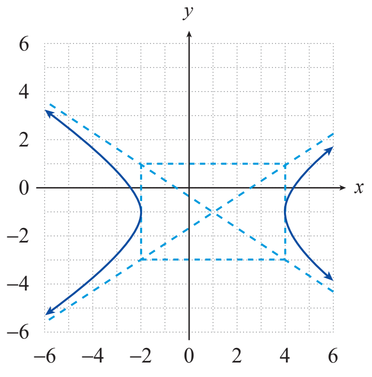
43. Foci at $(2,5)$ and $(10,5)$ and vertices at $(3,5)$ and $(9,5)$.

44. Foci at $(7,4)$ and $(7,-4)$ and vertices at $(7,1)$ and $(7,-1)$.

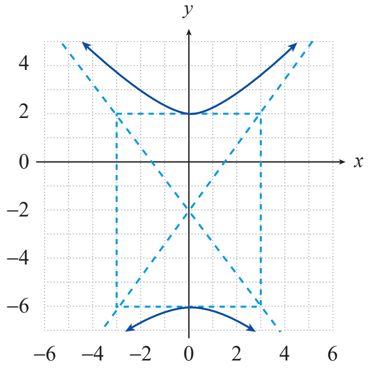
45. Asymptotes of $y=\pm(2x+8)+3$ and vertices at $(-6,3)$ and $(-2,3)$.

46. Asymptotes of $y=\pm\frac{4}{3}x-3$ and vertices at $(0,-7)$ and $(0,1)$.

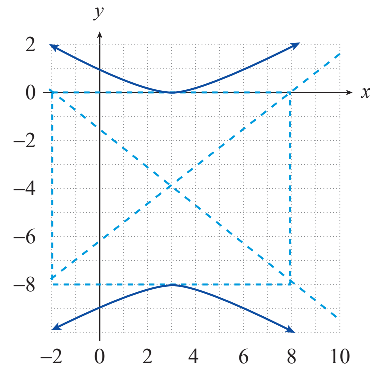
47.



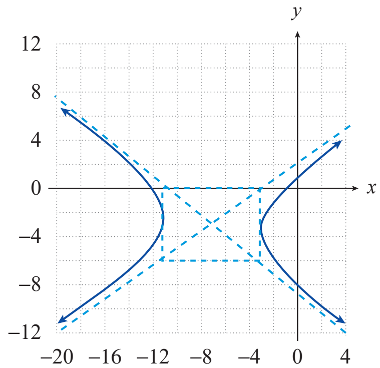
48.



49.

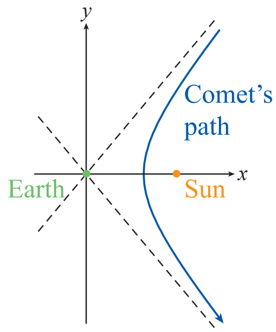


50.



APPLICATIONS

51. As mentioned in this section, some comets trace one branch of a hyperbola through the solar system, with the sun at one focus. Suppose a comet is spotted that appears to be headed straight for Earth, as shown in the figure. As the comet gets closer, however, it becomes apparent that it will pass between Earth, which lies at the center of the hyperbolic path of the comet, and the sun. In the end, the closest the comet comes to Earth is 60,000,000 miles. Using an estimate of 94,000,000 miles for the distance from Earth to the sun, and positioning Earth at the origin of a coordinate system, find the equation for the path of the comet.



52. Suppose two LORAN radio transmitters are 26 miles apart. A ship at sea receives signals sent simultaneously from the two transmitters and is able to determine that the difference between the distances from the ship to each of the transmitters is 24 miles. By positioning the two transmitters on the y-axis, each 13 miles from the origin, find the equation for the hyperbola that describes the set of possible locations for the ship.

TECHNOLOGY

Use a graphing utility to graph the following equations.

53. $x^2 - 6y^2 = 15$

54. $4y^2 - 9x^2 = 18$

55. $3y^2 - 18x^2 = 36$

56. $x^2 - 6 = 3y^2$

57. $(y+2)^2 - 20 = 5x^2$

58. $(x+5)^2 = 3(y-2)^2 + 15$

59. $x^2 - 2y^2 = 4x + 12y + 26$

60. $2x^2 - y^2 + 12x + 2y = 3$

61. $5x^2 - y^2 + 20x = 10y + 25$

62. $x^2 - 5y^2 = 14x + 20y - 4$