Example 5: Roots and Complex Numbers

Simplify the following expressions.

a. $\left(2-\sqrt{-3}\right)^2$

b. $\dfrac{\sqrt{4}}{\sqrt{-4}}$

Solution

a. $$\begin{aligned}\left(2-\sqrt{-3}\right)^2 &= \left(2-\sqrt{-3}\right)\left(2-\sqrt{-3}\right)\\ &= 4-4\sqrt{-3}+\sqrt{-3}\sqrt{-3}\\ &= 4-4i\sqrt{3}+\left(i\sqrt{3}\right)^2\\ &= 4-4i\sqrt{3}-3\\ &= 1-4i\sqrt{3}\end{aligned}$$

Each $\sqrt{-3}$ is converted to $i\sqrt{3}$ before multiplying.

b. $$\begin{aligned}\frac{\sqrt{4}}{\sqrt{-4}} &= \frac{2}{2i}\\ &= \frac{1}{i}\\ &= -i\end{aligned}$$

We simplify each radical before dividing.

We already simplified $\frac{1}{i}$ in Example 4c, so we quickly obtain the correct answer of $-i$.

1.8 EXERCISES

PRACTICE

Evaluate the following square root expressions. See Example 1.

1. $\sqrt{-25}$

2. $\sqrt{-12}$

3. $-\sqrt{-27}$

4. $-\sqrt{-100}$

5. $\sqrt{-32x}$, $x>0$

6. $\sqrt{-x^2}$

7. $\sqrt{-29}$

8. $(-i)^2\sqrt{-64}$

Simplify the following complex expressions. See Examples 2, 3, and 4.

9. $(4-2i)-(3+i)$

10. $(4-i)(2+i)$

11. $(3-i)^2$

12. i^7

13. $(7i-2)+(3i^2-i)$

14. $(3+i)(3-i)$

15. $(5-3i)^2$

16. $(5+i)(2-9i)$

17. i^{13}

18. $(9-4i)(9+4i)$

19. $11i^{314}$

20. i^{132}

21. $(7-3i)^2$

22. $(4-3i)(7+i)$

23. $(3i)^2$

24. $(1+i)+i$

25. $i(5-i)$

26. $i^{11}\left(\frac{6}{i^3}\right)$

27. $(10i^2-9i)+(9+5i)$

28. $(-5i)^3$

29. $i^7\left(\frac{49}{7i^2}\right)$

30. $\frac{1+2i}{1-2i}$

31. $\frac{10}{3-i}$

32. $\frac{i}{2+i}$

33. $\frac{1}{i^9}$

34. $(2+5i)^{-1}$

35. i^{-25}

36. $\frac{1}{i^{27}}$

37. $\frac{52}{5+i}$

38. $(2-3i)^{-1}$

39. $\frac{4i}{5+7i}$

40. i^{-4}

41. $\frac{5+i}{4+i}$

Simplify the following expressions. See Example 5.

42. $(3+\sqrt{-2})^2$

43. $(1+\sqrt{-6})^2$

44. $\frac{\sqrt{18}}{\sqrt{-2}}$

45. $(\sqrt{-32})(-\sqrt{-2})$

46. $(\sqrt{-9})(\sqrt{-2})$

47. $\frac{\sqrt{-98}}{3i\sqrt{-2}}$

48. $(\sqrt{-8})(\sqrt{-2})$

49. $(5+\sqrt{-3})^2$

50. $\frac{\sqrt{-72}}{5i\sqrt{-2}}$

APPLICATIONS

51. Electrical engineers often use j, rather than i, to represent imaginary numbers. This is to prevent confusion with their use of i, which often represents current. Under this convention, assume the impedance of a particular part of a series circuit is $4-3j$ ohms and the impedance of another part of the circuit is $2+6j$ ohms. Find the total impedance of the circuit. (Impedances in series are simply added.)

52. Consider the formula $V=IZ$, where V is voltage (in volts), I is current (in amps), and Z is impedance (in ohms). If you know the current of a circuit is $5-4j$ amps and the impedance is $8+2j$ ohms, find the voltage.

53. If you know the voltage of a circuit is $35+5j$ volts and the current is $3+j$ amps, find the impedance.

WRITING & THINKING

54. Explain why it may be useful to be able to use imaginary numbers in real-world math.

TECHNOLOGY

Use a graphing utility to simplify the following complex expressions.

55. $\dfrac{3-2i}{1+i}$

56. $(3-2i)^4$

57. $\dfrac{2500}{(3+i)^4}$

58. $(2-5i)(3+7i)(1-4i)$

59. $(1+i)^5(3-i)^2$

60. $\dfrac{3+7i}{(2-5i)(1+3i)}$

61. $\dfrac{6+3i}{2-4i}$

62. $(5-3i)^5$

63. $\dfrac{400}{(6+2i)^3}$

64. $(6-3i)(8+i)(7-4i)$

65. $(5-3i)^4(7+2i)^3$

66. $\dfrac{4+3i}{(7-2i)(5+4i)}$