b. $\dfrac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}} = \dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$

$= \dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}} \cdot \dfrac{x^2y^2}{x^2y^2}$

$= \dfrac{xy^2-x^2y}{y^2-x^2}$

$= \dfrac{xy\cancel{(y-x)}}{\cancel{(y-x)}(y+x)}$

$= \dfrac{xy}{y+x}$

This expression is a complex rational expression, a fact that is more clear once we rewrite the terms that have negative exponents as fractions.

The LCD in this case is x^2y^2, so we multiply top and bottom by this and factor the resulting polynomials.

We cancel the common factor of $(y-x)$ to obtain the final simplified expression.

1.7 EXERCISES

PRACTICE

Simplify the following rational expressions, indicating which real values of the variable must be excluded. See Example 1.

1. $\dfrac{2x^2+7x+3}{x^2-2x-15}$ **2.** $\dfrac{x^2+5x-6}{x^3+2x^2-3x}$ **3.** $\dfrac{x^3+2x^2-3x}{x+3}$ **4.** $\dfrac{x^2-4x+4}{x^2-4}$

5. $\dfrac{x^2+5x-6}{x^2+4x-5}$ **6.** $\dfrac{2x^2+7x-15}{x^2+3x-10}$ **7.** $\dfrac{x+1}{x^3+1}$ **8.** $\dfrac{x^3+x}{3x^2+3}$

9. $\dfrac{2x^2+11x+5}{x+5}$ **10.** $\dfrac{x^4-x^3}{x^2-3x+2}$ **11.** $\dfrac{2x^2+11x-21}{x+7}$ **12.** $\dfrac{8x^3-27}{2x-3}$

Add or subtract the rational expressions, as indicated, and simplify your answer. See Example 2.

13. $\dfrac{x-3}{x+5}+\dfrac{x^2+3x+2}{x-3}$ **14.** $\dfrac{x^2-1}{x-2}-\dfrac{x-1}{x+1}$

15. $\dfrac{x+2}{x-3}-\dfrac{x-3}{x+5}-\dfrac{1}{x^2+2x-15}$ **16.** $\dfrac{x+1}{x-3}+\dfrac{x^2+3x+2}{x^2-x-6}-\dfrac{x^2-2x-3}{x^2-6x+9}$

17. $\dfrac{x^2+1}{x-3}+\dfrac{x-5}{x+3}$ **18.** $\dfrac{x-37}{(x+3)(x-7)}+\dfrac{3x+6}{(x-7)(x+2)}-\dfrac{3}{x+3}$

19. $\dfrac{x^2+2x-35}{x-5}+\dfrac{x-4}{x+3}$ **20.** $\dfrac{y+2}{y-2}+\dfrac{y-6}{y+4}+\dfrac{4}{y^2+2y-8}$

21. $\dfrac{x+2}{x-6}+\dfrac{x^2+5x+6}{x^2-3x-18}-\dfrac{x^2-4x-12}{x^2-12x+36}$

22. $\dfrac{y^2+2}{y+3}-\dfrac{y-4}{y-3}$

Multiply or divide the rational expressions, as indicated, and simplify your answer. See Example 3.

23. $\dfrac{y-2}{y+1}\cdot\dfrac{y^2-1}{y-2}$

24. $\dfrac{a^2-3a-4}{a-2}\div\dfrac{a^2-2a-8}{a-2}$

25. $\dfrac{2x^2-5x-12}{x-3}\cdot\dfrac{x^2-x-6}{x-4}$

26. $\dfrac{z^2+2z+1}{2z^2+3z+1}\cdot\dfrac{2z^2-5z-3}{z+1}$

27. $\dfrac{y^2-11y+24}{y+6}\div\dfrac{y^2+5y-24}{y+6}$

28. $\dfrac{y^2+8y+16}{5y^2+22y+8}\cdot\dfrac{5y^2-13y-6}{y+4}$

29. $\dfrac{5y^2-27y-18}{y-5}\cdot\dfrac{y^2-6y+5}{y-6}$

30. $\dfrac{4z^2+20z-56}{z^2-8z+12}\div\dfrac{5z^2+43z+56}{15z^2-66z-144}$

31. $\dfrac{3b^2+9b-84}{b^2-5b+4}\div\dfrac{5b^2+37b+14}{-10b^2+6b+4}$

32. $\dfrac{3x^2-x-10}{x-1}\cdot\dfrac{x^2-1}{6x^2+x-15}\div\dfrac{x^2-x-2}{2x^2+5x-12}$

Simplify the complex rational expressions. See Example 4.

33. $\dfrac{\frac{3}{x}+\frac{x}{3}}{2-\frac{1}{x}}$

34. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}$

35. $\dfrac{6x-6}{3-\frac{3}{x^2}}$

36. $\dfrac{x^{-2}-y^{-2}}{y-x}$

37. $\dfrac{\frac{1}{r}-\frac{1}{s}}{r+\frac{1}{r}}$

38. $\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{y^3}-\frac{1}{xy^2}}$

39. $\dfrac{\frac{m}{n}-\frac{n}{m}}{m-n}$

40. $\dfrac{\frac{1}{y}-\frac{1}{x+3}}{\frac{1}{x}-\frac{y}{x^2+3x}}$

41. $\dfrac{x+y^{-1}}{x^{-1}+y}$

42. $\dfrac{1+xy}{x^{-2}-y^2}$

43. $\dfrac{x^2-y^2}{y^{-2}-x^{-2}}$

44. $\dfrac{xy^{-1}+\left(\frac{x}{y}\right)^{-1}}{x^{-2}+y^{-2}}$

45. $\dfrac{\frac{1}{7y}+\frac{1}{x-2}}{\frac{1}{11x}+\frac{7y}{11x^2-22x}}$

46. $\dfrac{8z+8}{2-\frac{2}{z^2}}$

47. $\dfrac{25x^{-2}-9z^{-2}}{\frac{5z+3x}{x^2}}$

48. $\dfrac{\frac{3y}{5}-\frac{5}{3y}}{3-\frac{5}{y}}$

Perform the indicated operations on the following rational expressions, and simplify your answer.

49. $\left(\dfrac{x^2-3x}{x^2+6x-27}-\dfrac{2}{x+9}\right)\cdot\dfrac{x+9}{x+2}$

50. $\dfrac{2y(y-1)}{y^2+6y-16}\div\dfrac{2}{y+8}-\dfrac{2}{y-2}$

51. $\left(\dfrac{z^2-17z+30}{z^2+2z-8}+\dfrac{6}{z-2}\right)\div\dfrac{1}{z^2-5z-36}$

52. $\dfrac{y+3}{2y+18}+\dfrac{y^2+2y+4}{y^2+3y-54}\cdot\dfrac{y-6}{y+3}$

53. $\dfrac{y^2+2y-15}{y+1}\cdot\left(\dfrac{y^2+3y+4}{y^2+3y-10}+\dfrac{y+4}{y+5}\right)\div\dfrac{y-3}{y-2}$

54. $\dfrac{y+6}{y-3}\left(\dfrac{y+5}{y-3}+\dfrac{y-3}{y+6}-\dfrac{y^2+4}{y^2+3y-18}\right)$