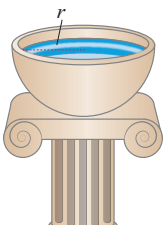


FIGURE 3

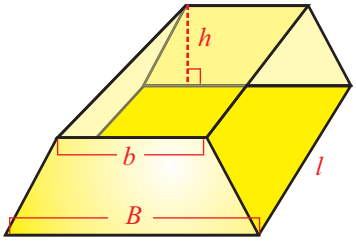


FIGURE 4

c. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$, and the birdbath of which we are to find the volume has the shape of half a sphere. So if we let V stand for the birdbath's volume,

$$V=\left(\frac{1}{2}\right)\left(\frac{4}{3}\pi r^3\right), \text{ or } V=\frac{2}{3}\pi r^3.$$

d. A *right cylinder* is the three-dimensional object generated by extending a plane region along an axis perpendicular to itself for a certain distance. (Such objects are often called prisms when the plane region is a polygon.) The volume of any right cylinder is the product of the area of the plane region and the distance that region has been extended perpendicular to itself. The gold ingot under consideration in this example is a right cylinder based on a trapezoid, as shown in Figure 4. The area of the trapezoid is $\frac{1}{2}(B+b)h$ and the ingot has length l, so its volume is $V=\frac{1}{2}(B+b)hl$. This could also be written as $V=\frac{(B+b)hl}{2}$.

1.3 EXERCISES

PRACTICE

Simplify each of the following expressions, writing your answer with only positive exponents. See Examples 1 and 2.

1. $(-2)^4$ **2.** -2^4 **3.** -3^2 **4.** $(-3)^2$

5. $3^2\cdot 3^2$ **6.** $2^3\cdot 3^2$ **7.** $4\cdot 4^2$ **8.** $(-3)^3$

9. $\frac{8^2}{4^3}$ **10.** $2^2\cdot 2^3$ **11.** $\frac{7^4}{7^5}$ **12.** $n^2\cdot n^5$

13. $\frac{x^5}{x^2}$ **14.** $\frac{y^3\cdot y^8}{y^2}$ **15.** $\frac{3^7}{3^4 s^{-10}}$

Use the properties of exponents to simplify each of the following expressions, writing your answer with only positive exponents. See Examples 1, 2, and 3.

16. $\frac{3t^{-2}}{t^3}$ **17.** $-2y^0$ **18.** $\frac{1}{7x^{-5}}$

19. $9^0x^3y^0$ **20.** $\frac{2n^3}{n^{-5}}$ **21.** $\frac{11^{21}}{11^{19}x^{-7}}$

22. $\frac{x^7y^{-3}z^{12}}{x^{-1}z^9}$ **23.** $\frac{x^4(-x^{-3})}{-y^0}$ **24.** $\frac{s^3}{s^{-2}}$

25. $\dfrac{x^{-1}}{x}$

26. $x^{(y^0)}x^9$

27. $\dfrac{x^2y^{-2}}{x^{-1}y^{-5}}$

28. $\dfrac{s^5y^{-5}z^{-11}}{s^8y^{-7}}$

29. $\dfrac{2^7s^{-3}}{2^3}$

30. $\dfrac{3^{-5}}{\left(3^{-4}x^5y^4\right)^2}$

31. $\dfrac{-9^0\left(x^2y^{-2}\right)^{-3}}{3x^{-4}y}$

32. $\left[\left(2x^{-1}z^3\right)^{-2}\right]^{-1}$

33. $\dfrac{\left(3yz^{-2}\right)^0}{3y^2z}$

34. $\left(12a^2-3b^4\right)^0$

35. $\dfrac{3^{-1}}{\left(3^2xy^2\right)^{-2}}$

36. $\left[9m^2-\left(2n^2\right)^3\right]^{-1}$

37. $\left[\left(12x^{-6}y^4z^3\right)^5\right]^0$

38. $\dfrac{x\left(x^{-2}y^3\right)^3}{\left(2x^4\right)^{-2}y}$

39. $\dfrac{\left(-3a\right)^{-2}\left(bc^{-2}\right)^{-3}}{a^5c^4}$

40. $\left[\left(5m^4n^{-2}\right)^{-1}\right]^{-2}$

41. $\left(9x^{-1}z\right)^2\left(2xy^{-3}\right)^{-1}$

42. $\left(4^{-2}x^5y^{-3}z^4\right)^{-2}$

43. $\left[\left(4a^2b^{-5}\right)^{-1}\right]^{-3}$

44. $\left[\left(2^{-3}m^{-6}n^3\right)^3\right]^{-1}$

45. $\left[\left(3^{-1}x^{-1}y\right)\left(x^2y\right)^{-1}\right]^{-3}$

46. $\left[\dfrac{100^0\left(x^{-1}y^3\right)^{-1}}{x^2y}\right]^{-3}$

47. $\left(5z^6-\left(3x^3\right)^4\right)^{-1}$

48. $\left[\dfrac{y^6\left(xy^2\right)^{-3}}{3x^{-3}z}\right]^{-2}$

Convert each number from scientific notation to standard notation, or vice versa, as indicated. See Example 4.

49. -1.76×10^{-5}; convert to standard

50. $-912,000,000$; convert to scientific

51. 0.00000021; convert to scientific

52. 3.2×10^7; convert to standard

53. 5100; convert to scientific

54. –0.000187; convert to scientific

55. 3.1212×10^2; convert to standard

56. 1.934×10^{-4}; convert to standard

57. 0.00000002587; convert to scientific

58. -8.039×10^6; convert to standard

59. There are approximately 31,536,000 seconds in a calendar year. Express the number of seconds in scientific notation.

60. Together, the 46 human chromosomes are estimated to contain some 3.0×10^{9} base pairs of DNA. Express the number of pairs of DNA in standard notation.

61. A particular Italian sports car can be bought new for \$675,000. Express this price in scientific notation.

62. A white blood cell is approximately 3.937×10^{-4} inches in diameter. Express this diameter in standard notation.

63. The probability of winning the lottery with one dollar is approximately 0.0000002605. Express this probability in scientific notation.

Evaluate each expression using the properties of exponents. Use a calculator only to check your final answer. See Example 5.

64. $\left(2.3\times10^{13}\right)\left(2\times10^{12}\right)$

65. $\dfrac{\left(8\times10^{-3}\right)\left(3\times10^{-2}\right)}{2\times10^{5}}$

66. $\left(2\times10^{-13}\right)\left(5.5\times10^{10}\right)\left(-1\times10^{3}\right)$

67. $\dfrac{\left(4\times10^{34}\right)\left(3\times10^{-32}\right)}{24}$

68. $\left(6\times10^{21}\right)\left(5\times10^{-19}\right)\left(5\times10^{4}\right)$

69. $\left(3.2\times10^{7}\right)\left(5\times10^{-4}\right)\left(2\times10^{-10}\right)$

70. $\dfrac{4\times10^{-6}}{\left(5\times10^{4}\right)\left(8\times10^{-3}\right)}$

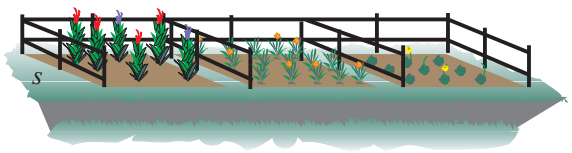
71. $\dfrac{\left(4.6\times10^{12}\right)\left(9\times10^{3}\right)}{\left(1.5\times10^{8}\right)\left(2.3\times10^{-5}\right)}$

Apply the definition of integer exponents to demonstrate the following properties.

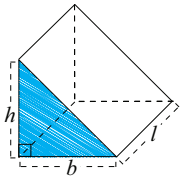
72. $a^{n}\cdot a^{m}=a^{n+m}$ **73.** $\left(a^{n}\right)^{m}=a^{nm}$ **74.** $(ab)^{n}=a^{n}b^{n}$

APPLICATIONS

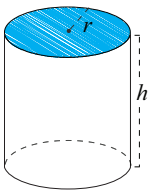
75. A farmer fences in three square garden plots that are situated along a road, as shown. Each square plot has a side length of s, and he doesn't put fence along the roadside. Find an expression, in the variable s, for the amount of fencing used.



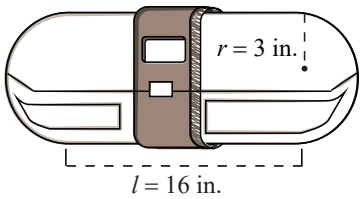
76. The prism shown below is a right triangular cylinder, where the base is a right triangle. Find the volume of the prism in terms of b, h, and l.



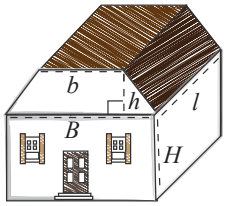
77. Determine the volume of the right circular cylinder shown, in terms of r and h.



78. Matt wants to let people in the future know what life is like today, so he goes shopping for a time capsule. Capacity, along with price and quality, is an important consideration for him. One time capsule he looks at is a right circular cylinder with a hemisphere on each end. Find the volume of the time capsule, given that the length l is 16 inches and the radius r is 3 inches.



79. Bill and Dee are buying a new house. The house is a right cylinder based on a trapezoid atop a rectangular prism. The bases of the trapezoid are $B = 10$ m and $b = 8$ m, and the length of the house is $l = 15$ m. The height of the house up to the bottom of the roof is $H = 3$ m, and the height of the roof is $h = 1$ m. Find the volume of the house.



80. Determine the expression for the volume of water contained in a rectangular swimming pool of length l feet and width w feet, assuming the water has a uniform depth of 6 feet.

81. Determine the expression for the volume of water contained in an above-ground circular swimming pool that has a diameter of 18 feet, assuming the water has a uniform depth of d feet.

82. The floor of a rectangular bedroom measures N feet wide and M feet long. The height of the walls is 7 feet. Find an expression for the number of square feet of wallpaper needed to cover all the walls. (Ignore the presence of doors and windows.)

83. The interior surface of the birdbath in Example 6c needs to be painted with a waterproof (and nontoxic) coating. Determine the expression for the interior surface area.

WRITING & THINKING

84. Give a few examples of instances in which it would be more useful to use scientific notation rather than standard.

85. In February of 2006, the US national debt was approximately 8.2 trillion dollars. How is saying 8.2 trillion similar to scientific notation? How is it different?

86. In your own words, explain why $a^0 = 1$.