CHAPTER 13 REVIEW EXERCISES

Section 13.1

Determine the first five terms of each sequence whose n^{th} term is defined as follows.

1. $a_n = (-3)^n$

2. $a_n = (-1)^n \sqrt[3]{n}$

3. $a_1 = -3, a_{n-1} = a_n + 1$ for $n \geq 2$

4. $a_n = \dfrac{n!}{n^n}$

Find a possible formula for the general n^{th} term of each sequence. Answers may vary.

5. $-7, -1, 5, 11, 17, \ldots$

6. $\dfrac{1}{2}, \dfrac{3}{4}, \dfrac{9}{8}, \dfrac{27}{16}, \dfrac{81}{32}, \ldots$

7. $0, 3, 8, 15, 24, 35, \ldots$

8. $\dfrac{3}{2}, \dfrac{5}{3}, \dfrac{7}{4}, \dfrac{9}{5}, \dfrac{11}{6}, \ldots$

9. $-2, -4, -12, -48, -240, \ldots$

10. $2, 6, 12, 20, 30, \ldots$

Translate each expanded sum that follows into summation notation, and vice versa. Then evaluate the sum.

11. $\sum_{i=3}^{8} (-2i + 3)$

12. $\sum_{i=2}^{7} (-2)^{i-1}$

13. $8 + 27 + 64 + \cdots + 343$

14. $\sum_{i=1}^{6} (2i - 3)$

15. $\sum_{i=1}^{5} -4(2^i)$

16. $8 + 18 + 32 + \cdots + 200$

Find a formula for the n^{th} partial sum S_n of each series. If the series is finite, determine the sum. If the series is infinite, determine if it converges or diverges, and if it converges, determine the sum.

17. $\sum_{i=1}^{80} \left(\dfrac{1}{i+1} - \dfrac{1}{i+2} \right)$

18. $\sum_{i=1}^{\infty} \left(\dfrac{1}{i+1} - \dfrac{1}{i+2} \right)$

19. $\sum_{i=1}^{\infty} (3^i - 3^{i+1})$

Determine the first five terms of each generalized Fibonacci sequence.

20. $a_1 = -2,\ a_2 = 5,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

21. $a_1 = -10,\ a_2 = -12,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

Section 13.2

Find the explicit formula for the general n^{th} term of each arithmetic sequence.

22. $5, 2, -1, -4, -7, \ldots$

23. $a_2 = 14$ and $a_4 = 19$

24. $a_7 = -43$ and $d = -9$

25. $a_1 = 2, a_4 = 11$

26. $a_9 = \dfrac{13}{2}, d = \dfrac{3}{4}$

27. $-5, 4, 13, 22, 31, \ldots$

Use the given information about each arithmetic sequence to answer the question.

28. Given that $a_1 = -2$ and $a_4 = -20$, what is a_{25}?

29. Given that $a_3 = 17$ and $a_7 = 29$, what is a_{89}?

30. In the sequence $8, 19, 30, \ldots$, which term is 668?

31. In the sequence $6, 1, -4, \ldots$, which term is -169?

32. In the sequence $\frac{8}{3}, \frac{10}{3}, 4, \ldots$, which term is $\frac{56}{3}$?

Find the value of the partial sum of each arithmetic sequence.

33. $\sum_{i=1}^{97} (2i - 7)$

34. $\sum_{i=1}^{60} (-4i + 3)$

35. Sylvia suspects that she has an ant infestation in her apartment. The first day she noticed them, she saw 10 ants in her kitchen. Each day she notices 4 more ants than on the previous day. If she doesn't call an exterminator, how many ants would she see on the fifteenth day?

Section 13.3

Find the explicit formula for the general n^{th} term of each geometric sequence.

36. $2, 8, 32, 128, 512, \ldots$

37. $3, \frac{3}{5}, \frac{3}{25}, \frac{3}{125}, \frac{3}{625}, \ldots$

38. $18, -6, 2, -\frac{2}{3}, \frac{2}{9}, \ldots$

39. $a_1 = 6$ and $a_4 = 384$

40. $a_2 = 20$ and $a_6 = 320$

41. $a_1 = 8$ and $a_4 = \frac{1}{8}$

Given the two terms of a geometric sequence, find the common ratio and first five terms of the sequence.

42. $a_1 = 4$ and $a_4 = 108$

43. $a_4 = \frac{5}{3}$ and $a_6 = \frac{20}{27}$

Use the given information about each geometric sequence to answer the question.

44. Given that $a_2 = \frac{3}{5}$ and $a_4 = \frac{1}{15}$, what is the common ratio r?

45. Given that $a_1 = 3$ and $a_4 = -24$, what is the common ratio r?

46. Given that $a_5 = -16$ and $a_6 = -4$, what is a_{11}?

Each of the following sums is a partial sum of a geometric sequence. Use this fact to evaluate the sums.

47. $\sum_{i=3}^{9} 3\left(\frac{1}{2}\right)^i$

48. $5 + 10 + \cdots + 20,480$

Determine if each of the following infinite geometric series converges. If so, find the sum.

49. $\sum_{i=0}^{\infty} -3\left(\frac{3}{4}\right)^i$

50. $\sum_{i=1}^{\infty} \left(-\frac{5}{4}\right)^i$

51. $\sum_{i=1}^{\infty} \frac{2}{5}\left(\frac{5}{7}\right)^i$

Section 13.4

Use the Principle of Mathematical Induction to prove the following statements.

52. $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$

53. $5+8+11+\cdots+(3n+2)=\frac{n(3n+7)}{2}$

54. $1\cdot 3+2\cdot 4+3\cdot 5+\cdots+n(n+2)=\frac{n(n+1)(2n+7)}{6}$

55. For all natural numbers n, 11^n-7^n is divisible by 4.

56. For all natural numbers n, 7^n-1 is divisible by 3.

Section 13.5

Use the Multiplication Principle of Counting and the permutation and combination formulas to answer the following questions.

57. A license plate must contain 4 numerical digits followed by 3 letters. If the first digit cannot be 0 or 1, how many different license plates can be created?

58. How many different 7-digit phone numbers do not contain the digits 6 or 7?

59. In how many different orders can the letters in the word "aardvark" be arranged?

60. In how many different ways can first place, second place, and third place be awarded in a 10-person shot put competition?

61. At a meeting of 21 people, a president, vice president, secretary, treasurer, and recruitment officer are to be chosen. How many different ways can these positions be filled?

62. A college admissions committee selects 4 out of 12 scholarship finalists to receive merit-based financial aid. How many different sets of 4 recipients can be chosen?

Use the Binomial and Multinomial Theorems to expand each of the following expressions.

63. $(1-2y)^5$

64. $(x+2)^7$

65. $(5x^2-2y)^5$

66. $(x+2y+z)^3$

Section 13.6

Apply the formulas for the probabilities of intersection and union to the following sets and determine **a.** $P(E\cap F)$ and **b.** $P(E\cup F)$. Let $n(S)$ equal the size of the sample space.

67. $n(S)=9,\ E=\{3,5,7\},\ F=\{1,2,3,4\}$

68. $n(S)=6,\ E=\{A,B\},\ F=\{X,Y,Z\}$

69. $n(S)=7,\ E=\{\alpha,13\},\ F=\{\alpha,\beta,13,14\}$

70. $n(S)=8,\ E=\{a,4,m,7\},\ F=\{m,3,s\}$

71. $n(S)=10,\ E=\{3,4,X,Y,5,Z\},\ F=\{5,6,7\}$

72. A card is drawn from a standard 52-card deck. Find the probability of drawing
- **a.** a seven or a club.
- **b.** a face card but not a red queen.
- **c.** a black three or a spade.

73. What is the probability of being dealt a five-card hand (from a standard 52-card deck) that contains only face cards?

74. There is a 10% chance of rain each individual day for an entire week. What is the probability that it will rain at least once during this seven-day period?