Answer Key

Chapter 1: Fundamental Concepts of Algebra

1.1 Exercises

1. a. $19,\ 2^5$ **b.** $19,\ \frac{0}{15},\ 2^5$

c. $19,\ \frac{0}{15},\ 2^5,\ -33$

d. $19,\ -4.3,\ \frac{0}{15},\ 2^5,\ -33$

e. $-\sqrt{3}$ **f.** all except $\frac{15}{0}$

g. $\frac{15}{0}$

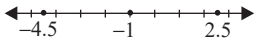
3. a. $|-16|,\ \frac{12}{3},\ \sqrt{4}$

b. $|-16|,\ \frac{12}{3},\ 0,\ \sqrt{4}$

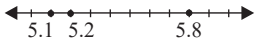
c. $|-16|,\ \frac{12}{3},\ 0,\ \sqrt{4}$

d. all **e.** none **f.** all **g.** none

5.



7.



9. $<, \leq$

11. $<, \leq$

13. $<, \leq$

15. $>, \geq$

17. $>, \geq$

19. $2a + b > c$

21. $9 \geq 7$

23. $x + 5 < 3$

25. $9 \geq 8$

27. $\{3n \mid n \text{ is an integer and } -2 \leq n \leq 3\}$

29. $\{n \mid n \text{ is a prime}\}$

31. $\left\{\frac{1}{n} \middle| n \text{ is an odd integer}\right\}$

33. $(-\infty, 15)$

35. $(2.5, 3.7]$

37. $(-\infty, 4)$

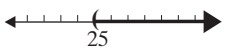
39. $\left(-\frac{1}{2}, \frac{2}{5}\right)$

41. $[0, \infty)$

43.

45.

47.



49. 4

51. $\sqrt{5} - \sqrt{3}$

53. -15

55. 1

57. -1

59. -12

61. 8

63. 6

65. 11

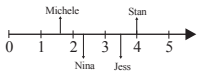
67. $-\frac{2516}{495}$

69. 1

71. $\frac{7414}{999}$

73. $1.08\overline{3}$

75.



77. 20 miles

79. If sugar $= s$,

$\{s \mid 3 \leq s \leq 4\} = [3, 4]$

If walnuts $= w$,

$\left\{w \middle| \frac{1}{2} \leq w \leq \frac{2}{3}\right\} = \left[\frac{1}{2}, \frac{2}{3}\right]$

81. Yes, all whole numbers are also integers, but the negative integers are not whole numbers, so not all integers are whole numbers.

1.2 Exercises

1. $3x^2y^3,\ -2\sqrt{x+y},\ 7z$

3. $-2,\ \sqrt{x+y}$

5. $1,\ 8.5,\ -14$

7. $\frac{-5x}{2yz}, -8x^5y^3,\ 6.9z$

9. $\frac{-5}{2}, \frac{1}{y}, \frac{1}{z}, x$

11. 20

13. 8

15. $-\frac{\sqrt{2}}{36} + 2$

17. 4

19. $58 + 6\pi$

21. $\frac{-1}{3}$

23. Commutative

25. Associative

27. Associative

29. Distributive

31. Commutative

33. Multiplicative cancellation; $\frac{1}{5}$

35. Additive cancellation; x

37. Zero-Factor Property

39. Multiplicative cancellation; 6

41. Multiplicative cancellation; $\frac{1}{3}$

43. $\frac{11}{2}$

45. -10

47. 1

49. 70

51. $\frac{103}{6}$

53. $\frac{-144}{5}$

55. $\frac{37}{2}$

57. $23.\overline{66}$

59. 1.64

61. $-\frac{1}{5}\left(\sqrt{3(3+7)-5}\right)^3$

63. $\left(\frac{\sqrt[3]{x-4}}{2}\right)^2$

65. $(-5, 4]$

67. $[3, 4]$

69. $[-\pi, 21)$

71. $(3, 9]$

73. $\mathbb{Z}$

75. $\mathbb{Z}$

77. \$66

79. \$102

81. 2.19 square meters

83. It is the same number you began with. Explanations may vary.

85. Answers may vary. (Ex: Please Excuse My Dear Aunt Sally.)

1.3 Exercises

1. 16 **3.** -9

5. 81 **7.** 64

9. 1 **11.** $\frac{1}{7}$

13. x^3 **15.** $27s^{10}$

17. -2 **19.** x^3

21. $121x^7$ **23.** x

25. $\frac{1}{x^2}$ **27.** x^3y^3

29. $\frac{16}{s^3}$ **31.** $-\frac{y^5}{3x^2}$

33. $\frac{1}{3y^2z}$ **35.** $27x^2y^4$

37. 1 **39.** $\frac{c^2}{9a^7b^3}$

41. $\frac{81y^3z^2}{2x^3}$ **43.** $\frac{64a^6}{b^{15}}$

45. $27x^9$ **47.** $\frac{1}{5z^6-81x^{12}}$

49. -0.0000176

51. 2.1×10^{-7} **53.** 5.1×10^{3}

55. 312.12 **57.** 2.587×10^{-8}

59. 3.1536×10^{7} **71.** 1.2×10^{13}

61. 6.75×10^{5} **75.** $7s$

63. 2.605×10^{-7} **77.** πr^2h

65. 1.2×10^{-9} **79.** 585 m^3

67. 50 **81.** $81\pi d$ ft^3

69. 3.2×10^{-6} **83.** $2\pi r^2$

1.4 Exercises

1. -3 **3.** Not real

5. -2 **7.** -5

9. Not real **11.** $-\frac{3}{5}$

13. $-\frac{1}{2}$ **15.** 2

17. $\frac{2}{5}$ **19.** $3|x|$

21. $\frac{x^2|z|}{2}$ **23.** $x^2y^7z^3$

25. $\frac{ab^4}{3c^2}$ **27.** $\frac{|x^3|y^2}{2}$

29. $\frac{y^6z^5}{2x^7}$ **31.** $\frac{\sqrt[3]{36x^2y^2}}{3y^2}$

33. $-\sqrt{2}-\sqrt{5}$ **35.** $\sqrt{6}+\sqrt{3}$

37. $\frac{x+\sqrt{2x}}{x-2}$ **39.** $\frac{x+2\sqrt{xy}+y}{x-y}$

41. $\frac{y-2\sqrt{y}}{y-4}$ **43.** $\frac{1}{\sqrt{5}+3}$

45. $\frac{9-y}{18-6\sqrt{y}}$ **47.** $\frac{1}{\sqrt{13}-\sqrt{t}}$

49. $\frac{6-y}{6+y-2\sqrt{6y}}$ **51.** $3x\sqrt[3]{2x}$

53. Not possible **55.** 0

57. $4z\sqrt[3]{2z}$

59. 0 **61.** $\left(3x^2-4\right)^2$

63. 27 **65.** n^2

67. $\frac{x^{\frac{4}{5}}}{y^{\frac{5}{3}}}$ **69.** $\frac{1}{125}$

71. $y\sqrt[3]{y^2}$ **73.** $\left(ax^2+by\right)^{\frac{1}{12}}$

75. $a^{\frac{15}{4}}$ **81.** $\sqrt[4]{125}$

77. $x^{\frac{1}{4}}$ **83.** $\sqrt[4]{|y|}$

79. $6^{-\frac{1}{3}}$ **85.** x^3

87. $\sqrt[16]{16{,}807}$

93. $3d^2\sqrt{3}$; 3.326 cm^2

95. 1651 cm^2; no

97. 2.998×10^{8} m/s

99. Because a root is the same as a fractional exponent.

1.5 Exercises

1. Not a polynomial

3. Degree 11; polynomial of four terms

5. Degree 0 monomial

7. Degree 4 binomial

9. Degree 2 trinomial

11. Degree 5 binomial

13. $-x^{13}+7x^{11}-4x^{10}+9$

a. 13 **b.** -1

15. $2s^6-10s^5+4s^3$

a. 6 **b.** 2

17. $9y^6-3y^5+y-2$

a. 6 **b.** 9

19. πz^5+8z^2-2z+1

a. 5 **b.** π

21. $-4x^3y-6y-x^2z$

23. $x^2y+xy^2+6x-6y$

25. $-3ab$

27. xy^2-x^2y-y

29. $3a^3b^3+21a^3b^2+2a^2b^2$
$+14a^2b-3ab^3-21ab^2$

31. $3a^2-2ab-8b^2$

33. $6x^2+33xy-18y^2$

35. $7y^4-34xy^2-5x^2$

37. $6x^3y^3-3x^3y+36x^2y^3+4x^2y^2$
$-18x^2y+24xy^2$

39. $9a^2+6ab+b^2$

41. $4x^2-9y^2$

43. $x^2+4xy+4y^2$

45. $\frac{1}{x^2}-y^2$

47. No; a variable in the denominator is equivalent to a variable with a negative exponent.

49. **a.** Yes; degree = 4; leading coefficient = 2; terms = 4

b. Yes; degree = 3; leading coefficient = 2; terms = 3

1.6 Exercises

1. $m\left(4mn+16m^2+7\right)$

3. $6\left(a-b^2\right)$

5. $2x\left(x^5-7x^2+4\right)$

7. $\left(x^3-y\right)\left(x^3-y-1\right)$

9. $4y^2\left(3y^4-2-4y^3\right)$

11. $\left(a^2+b\right)\left(a-b\right)$

13. $z\left(1+z\right)\left(1+z^2\right)$

15. $\left(n-2\right)\left(x^2+y\right)$

17. $\left(a-5b\right)\left(x+5y\right)$

19. $(2x-11)(2x+11)$

21. $(7a-12b)(7a+12b)$

23. $(5x^2y-3)(5x^2y+3)$

25. $(x-10y)(x^2+10xy+100y^2)$

27. (m^2+5n^3)
$\times(m^4-5m^2n^3+25n^6)$

29. $(3x^2-2y^4z)$
$\times(9x^4+6x^2y^4z+4y^8z^2)$

31. $(4y^2z-3x^4)(4y^2z+3x^4)$

33. $(7y^3+3xz^2)$
$\times(49y^6+21xy^3z^2+9x^2z^4)$

35. $(x+5)(x-3)$

37. $(x-1)^2$

39. $(x-2)^2$

41. $(y+7)^2$

43. $(x+11)(x+2)$

45. $(y-8)(y-1)$

47. $(5a+3)(a-8)$

49. $(x+6)(5x-3)$

51. $(16y-9)(y-1)$

53. $(4a-3)(2a+1)$

55. $(4y-5)(3y-1)$

57. $2x(2x-1)^{\frac{-3}{2}}$

59. $a^{-3}(7a^2-2b)$

61. $2y^{-5}(5y^3-x)$

63. $(5x+7)^{\frac{4}{3}}(5x+6)$

65. $y^{-4}(7y^3+5)$

1.7 Exercises

1. $\frac{2x+1}{x-5}$; $x \neq -3, 5$

3. $x(x-1)$; $x \neq -3$

5. $\frac{x+6}{x+5}$; $x \neq -5, 1$

7. $\frac{1}{x^2-x+1}$; $x \neq -1$

9. $2x+1$; $x \neq -5$

11. $2x-3$; $x \neq -7$

13. $\frac{x^3+9x^2+11x+19}{(x-3)(x+5)}$

15. $\frac{13x}{(x-3)(x+5)}$

17. $\frac{x^3+4x^2-7x+18}{(x+3)(x-3)}$

19. $\frac{x^2+11x+17}{x+3}$

21. $\frac{x+2}{x-6}$

23. $y-1$

25. $(x+2)(2x+3)$

27. $\frac{y-8}{y+8}$

29. $5y^2-2y-3$

31. -6

33. $\frac{x^2+9}{6x-3}$

35. $\frac{2x^2}{x+1}$

37. $\frac{s-r}{r^2s+s}$

39. $\frac{m+n}{mn}$

41. $\frac{x}{y}$

43. x^2y^2

45. $\frac{11x}{7y}$

47. $\frac{5z-3x}{z^2}$

49. $\frac{x-2}{x+2}$

51. $\frac{(z^2-11z+54)(z-9)}{(z-2)}$

53. $\frac{2y^2+5y-4}{y+1}$

1.8 Exercises

1. $5i$

3. $-3i\sqrt{3}$

5. $4i\sqrt{2x}$

7. $i\sqrt{29}$

9. $1-3i$

11. $8-6i$

13. $-5+6i$

15. $16-30i$

17. i

19. -11

21. $40-42i$

23. -9

25. $1+5i$

27. $-1-4i$

29. $7i$

31. $3+i$

33. $-i$

35. $-i$

37. $10-2i$

39. $\frac{14}{37}+\frac{10}{37}i$

41. $\frac{21}{17}-\frac{1}{17}i$

43. $-5+2i\sqrt{6}$

45. 8

47. $-\frac{7}{3}i$

49. $22+10i\sqrt{3}$

51. $6+3j$ ohms

53. $11-2j$ ohms

Chapter 1 Project

1. Alex: 81.7; Ashley: 90.3;
Barron: 81.0; Elizabeth: 83.1;
Gabe: 90.5; Lynn: 82.4

3. The final exam contributes a higher percentage of points to the final grade than does the semester project.

Chapter 1 Review Exercises

1. a. 2^3

b. 2^3, 0

c. $-\sqrt{4}$, 2^3, 0

d. All except $\sqrt{17}$

e. $\sqrt{17}$

f. All

3. $[4,17)$

5. -7

7. 4

9. -1

11. $-\frac{337}{165}$

13. $\frac{x^2}{2y}$, $12.1x$, $-\sqrt{y+5}$

15. $\frac{4\pi}{3}-36$

17. 51

19. Commutative property

21. Zero-Factor Property

23. $[5,8)$

25. $\frac{-t^9}{2s^7}$

27. $\frac{18y^2}{x^4z^5}$

29. 6.952×10^{7}

31. 2.0×10^{-8}

33. 5

35. $5x^{10}$

37. $\frac{-4y}{x^3}$

39. $\frac{2y\sqrt[3]{9x^2y}}{3}$

41. $-\sqrt{2}-\sqrt{6}$

43. $3|x|\sqrt{2xy}-2x\sqrt[3]{2xy}$

45. $\frac{1}{x^{\frac{7}{4}}}$

47. $m^4-5m^3+3m^2+2$

49. $3x^3-4x^2y^3+3xy-4y^4$

51. $(x+3)(x-4)$

53. $(2a+1)(3a-5)$

55. $(6x^3+y)(6x^3-y)$

57. $(2x-5y)(x+3)$

59. $(3x-2y)^{\frac{2}{3}}\left[(3x-2y)^{\frac{2}{3}}-1\right]$

61. $\frac{x+3}{x-3}$, $x\neq0,\pm3$

63. $\frac{-2}{x}$

65. $\frac{b-a}{4a+4b}$

67. $-x-y$

69. 3

71. $5+9i$

73. $4+i$

75. $-\frac{7}{25}+\frac{24}{25}i$

77. $62-16i\sqrt{2}$

Chapter 2: Equations and Inequalities in One Variable

2.1 Exercises

1. $t=-5$

3. $y=-1$

5. $w=-3$

7. $\mathbb{R}$ (Identity)

9. $\varnothing$ (Contradiction)

11. $m=7$

13. $x=3.7$

15. $x=1.05$

17. $y=-5$

19. $\mathbb{R}$ (Identity)

21. $\mathbb{R}$ (Identity)

23. $x=3$

25. $\varnothing$ (Contradiction)

27. $y=-\frac{1}{3},-3$

29. $x=\frac{1}{3}$

31. $x=-311, 420$

33. $x=-\frac{4}{5}, 2$

35. $\varnothing$ (Contradiction)

37. $x=-2, 2$

39. $x=5$

41. $x=-\frac{1}{2}$

43. $x=\frac{1}{4}$

45. $x=\frac{1}{7}$

47. $r=\frac{C}{2\pi}$

49. $a=\frac{v^2-v_0^{\,2}}{2x}$

51. $F=\frac{9}{5}C+32$

53. $h=\frac{A-2lw}{2w+2l}$

55. $m=\frac{2K}{v^2}$

57. $\frac{19}{3}$ hours, or 6 hours and 20 minutes

59. 13.5 miles

61. \$390

63. 2 gallons 44%, 1 gallon 50%

65. 24 child tickets, 15 adult tickets

67. 7.5%

69. 26 feet by 26 feet

71. 53, 55, and 57

73. 36.4%

75. $x\approx0.72$

77. $x\approx13.11$

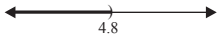
2.2 Exercises

1. $\{-9,3.14,-2.83,1,-3,4\}$

3. $\{-2.83,1,-3\}$

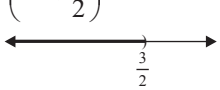
5. $(-\infty,-3]$

7. $(-\infty,4.8)$

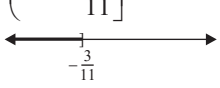


9. $(-\infty,2.25)$

11. $\left(-\infty,\frac{3}{2}\right)$

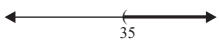


13. $\left(-\infty,-\frac{3}{11}\right]$



15. $(7,\infty)$

17. $(35,\infty)$



19. $(-3,\infty)$

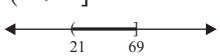
21. $(-0.11,\infty)$

23. $(1,5]$

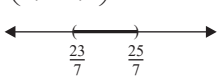
25. $(-10,6]$

27. $[-8,-2)$

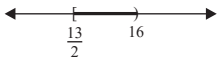
29. $(21,69]$



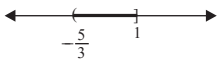
31. $\left(\frac{23}{7},\frac{25}{7}\right)$



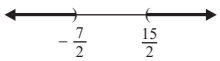
33. $\left[\frac{13}{2},16\right)$



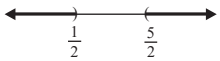
35. $\left(-\frac{5}{3},1\right]$



37. $\left(-\infty,-\frac{7}{2}\right)\cup\left(\frac{15}{2},\infty\right)$

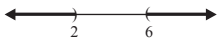


39. $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{5}{2},\infty\right)$



41. $\varnothing$

43. $(-\infty,2)\cup(6,\infty)$



45. $\varnothing$

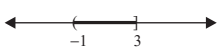
47. $\varnothing$

49. $[-4,0]$

51. $(-\infty,\infty)$

53. $(3,15)$

55. $(-1,3]$



57. $(-\infty,\infty)$

59. $[-2,3)$

61. $[73,113]$ for an A, $(113,115)$ for an A+.

63. $(1140,1600]$

2.3 Exercises

1. $\left\{\frac{3}{2},-1\right\}$

3. $\{7\}$

5. $\left\{\frac{-3}{2},-3\right\}$

7. $\{-3,1\}$

9. $\{2\}$

11. $\{3,11\}$

13. $\{0,6\}$

15. $\left\{\frac{3}{8}\right\}$

17. $\{17,19\}$

19. $\left\{\frac{1}{2}\pm\sqrt{2}\right\}$

21. $\{-5,9\}$

23. $\left\{\frac{6}{5},\,6\right\}$

25. $\{-5,-3\}$

27. $\left\{-5,\,\frac{3}{2}\right\}$

29. $\{-9,-1\}$

31. $\{-13,5\}$

33. $\{-16,-6\}$

35. $\left\{-\frac{4}{3},1\right\}$

37. $\{0.17\pm0.86i\}$

39. $\left\{0,\,\frac{2}{3}\right\}$

41. $\left\{-\frac{17}{7},3\right\}$

43. Two complex conjugate solutions

45. Two real solutions

47. $\{-4.5\pm4.5i\}$

49. $\{-14,-6\}$

51. $\{2,14\}$

53. $\left\{\frac{-1\pm\sqrt{7}}{2}\right\}$

55. $\{-3,9\}$

57. $\{-16,12\}$

59. $\{-7,-6\}$

61. $\{0,6\}$

63. $\{-1,2\}$

65. $\left\{1,2,\frac{3\pm\sqrt{17}}{2}\right\}$

67. $\left\{\pm3,\pm\sqrt{7}\right\}$

69. 4.5 seconds

71. 4.8 seconds

75. $\left(3x-1-\sqrt{5}\right)\left(3x-1+\sqrt{5}\right)$

77. $(5x-1-i)(5x-1+i)$

2.4 Exercises

1. $\{-3,4\}$

3. $\{8,13\}$

5. $\left\{\pm\sqrt{2},\,\pm i\sqrt{5}\right\}$

7. $\left\{1\pm2i,1\pm\sqrt{3}\right\}$

9. $\left\{\frac{1}{8},\,27\right\}$

11. $\{\pm2i,\pm3\}$

13. $\{-1,\pm2,\,3\}$

15. $\left\{1,\,-\frac{8}{27}\right\}$

17. $\{-1,-2,-3\}$

19. $\{\pm1,\,3\}$

21. $\left\{-\frac{5}{2},\,0,\,3\right\}$

23. $\{\pm2,\pm5i\}$

25. $\left\{\pm 2, -\frac{6}{5}\right\}$

27. $\left\{\pm\frac{3}{2}, \pm\frac{3i}{2}\right\}$

29. $\left\{-\frac{5}{2}, 0, \frac{4}{7}\right\}$

31. $\left\{-\frac{4}{3}, \frac{2\pm 2i\sqrt{3}}{3}\right\}$

33. $\left\{-3, \frac{3\pm 3i\sqrt{3}}{2}\right\}$

35. $\left\{\frac{5}{2}\right\}$

37. $\{1\}$

39. $\{4\}$

41. $\{0, 2, 3\}$

43. $\left\{-\frac{1}{5}, \frac{1}{7}\right\}$

45. $\left\{-1, 0, \frac{2}{5}\right\}$

47. $\left\{\frac{8}{3}\right\}$

49. $\left\{-3, -\frac{13}{4}\right\}$

51. $b = -4$, $c = -12$, and $d = 0$

53. $a = 1$, $c = -36$, and $d = -144$

55. $a = 15$, $b = -16$, and $c = -5$

2.5 Exercises

1. $\left\{-\frac{3}{2}\right\}$

3. $\left\{3\pm\sqrt{10}\right\}$

5. $\left\{-3\pm\sqrt{6}\right\}$

7. $\{-2\}$

9. $\varnothing$

11. $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

13. $\left\{\frac{5}{2}, \frac{7}{2}\right\}$

15. $\left\{1, -\sqrt{5}, 2+\sqrt{5}\right\}$

17. $\frac{35}{12}$ hours, or 2 hours and 55 minutes

19. 7.5 hours

21. 20 weeks

23. 45 minutes

2.6 Exercises

1. $\{0\}$

3. $\varnothing$

5. $\{1\}$

7. $\left\{\frac{2}{3}\right\}$

9. $\varnothing$

11. $\left\{\frac{29}{8}\right\}$

13. $\{6\}$

15. $\varnothing$

17. $\{-2, 1\}$

19. $\{1\}$

21. $\{10\}$

23. $\{4\}$

25. $\{2\}$

27. $\{-32\}$

29. $\left\{\pm\frac{125}{343}\right\}$

31. $\{-2, 5\}$

33. $\{7, 10\}$

35. $a = \pm\sqrt{c^2 - b^2}$

37. $m = \frac{k}{\omega^2}$

39. $v = \pm\sqrt{\frac{Fr}{m}}$

41. $h = \pm\sqrt{\frac{w}{23}}$

43. $c = \pm\sqrt{\frac{2gm}{r}}$

45. $b = \pm\sqrt{c^2 - a^2}$

47. $a = \sqrt[3]{\frac{uP^2}{4\pi^2}}$

Chapter 2 Project

1. Scenario 1: \$472.97
Scenario 2: \$550.32
Scenario 3: \$450.77
Scenario 4: \$458.19
Scenario 5: \$802.54

3. Scenario 1: \$4402.56
Scenario 2: \$1311.52
Scenario 3: \$3136.96
Scenario 4: \$3893.12
Scenario 5: \$760.96

Chapter 2 Review Exercises

1. $\varnothing$ (Contradiction)

3. $x = 6.25$

5. $x = 3, 4$

7. $z = -\frac{10}{7}, 0$

9. $x = -3, 4$

11. $c = \frac{2A}{h} - b$

13. $C = \frac{5}{9}(F - 32)$

15. \$85

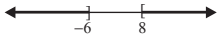
17. $(4,\infty)$

19. $(1,\infty)$

21. $[-7,4)$

23. $(-5,-1)$

25. $(-\infty,-6]\cup[8,\infty)$



27. $(-\infty,4]\cup[10,\infty)$

29. $\left\{-\frac{2}{5},3\right\}$

31. $\{2\pm 3i\}$

33. $\left\{4\pm\sqrt{2}\right\}$

35. $\left\{3\pm i\sqrt{7}\right\}$

37. $\left\{-4,\frac{5}{2}\right\}$

39. $\left\{\frac{19\pm\sqrt{701}}{17}\right\}$

41. $\left\{\pm 1,\pm\sqrt{2}\right\}$

43. $\{-6,4\}$

45. $\left\{\pm\sqrt{2},4\right\}$

47. $\{1,\pm 2i\}$

49. $\{-1,0,4\}$

51. $\left\{\frac{3}{2},2\right\}$

53. $b=-2$ and $c=-8$

55. -5

57. $\{0,3\}$

59. $\{-5\}$

61. $\{-4\}$

63. $\{2\}$

65. $\left\{-\frac{3}{2},5\right\}$

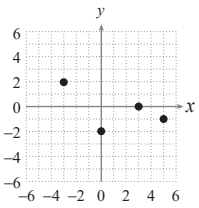
67. $\{3\}$

69. $r=\sqrt{\frac{3V}{\pi h}}$

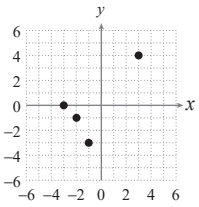
Chapter 3: Equations and Inequalities in Two Variables

3.1 Exercises

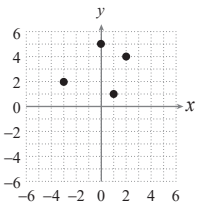
1.



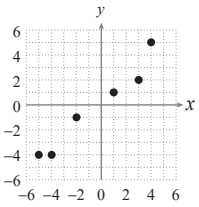
3.



5.



7.



9. III

11. IV

13. Positive x-axis

15. III

17. IV

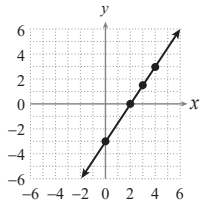
19. II

21. IV

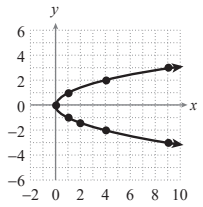
23. I

25. Negative y-axis

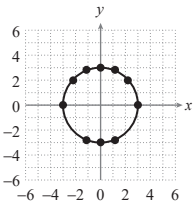
27. $\left\{(0,-3),(2,0),\left(3,\frac{3}{2}\right),(4,3)\right\}$



29. $\left\{(0,0),(1,\pm 1),(4,\pm 2),(9,\pm 3),\left(2,-\sqrt{2}\right)\right\}$



31. $\left\{(0,\pm 3),(\pm 3,0),\left(-1,\pm 2\sqrt{2}\right),\left(1,\pm 2\sqrt{2}\right),\left(\pm\sqrt{5},2\right)\right\}$



33. $\sqrt{34},\ \left(\frac{-7}{2},\frac{1}{2}\right)$

35. $\sqrt{58},\ \left(\frac{3}{2},\frac{7}{2}\right)$

37. $2\sqrt{2},(-1,-1)$

39. $4\sqrt{34},(3,-8)$

41. $10,(1,-6)$

43. $3\sqrt{13},\ \left(2,\frac{1}{2}\right)$

45. $10\sqrt{2},\ (3,3)$

47. $x=2$ or 18

49. $x=10, y=1$

51. 12

53. $2\sqrt{29}+\sqrt{26}+5\sqrt{2}$

55. 54

57. 1.25 kilometers

59. **a.** 249.19 meters

b. $\left(\frac{133}{2},\frac{709}{2}\right)$

61. area $= \dfrac{15}{2}$

63. area $= 25$

65. area $= 17$

67. area $= 48$

69. $x = [-5, 6]; y = [-8, 9]$

71. $x = [-3, 6]; y = [-4, 5]$

73. $x = [-6, 8]; y = [-9, 7]$

3.2 Exercises

1. $(x+4)^2 + (y+3)^2 = 25$

3. $(x-7)^2 + (y+9)^2 = 9$

5. $x^2 + y^2 = 6$

7. $\left(x-\sqrt{5}\right)^2 + \left(y-\sqrt{3}\right)^2 = 16$

9. $(x-7)^2 + (y-2)^2 = 4$

11. $(x+3)^2 + (y-8)^2 = 2$

13. $(x-4)^2 + (y-8)^2 = 10$

15. $x^2 + y^2 = 85$

17. $\left(x+\dfrac{7}{2}\right)^2 + \left(y-\dfrac{17}{2}\right)^2 = \dfrac{53}{2}$

19. $(x+6)^2 + \left(y-\dfrac{3}{2}\right)^2 = \dfrac{125}{4}$

21. $\left(x+\dfrac{13}{2}\right)^2 + (y+7)^2 = \dfrac{365}{4}$

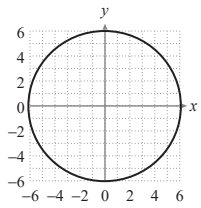
23. $(x-4)^2 + (y-3)^2 = 25$

25. $(x-2)^2 + y^2 = 4$

27. $(x-2)^2 + (y-4)^2 = 49$

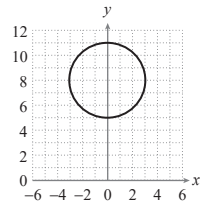
29. $(x+3)^2 + (y+2)^2 = 64$

31.



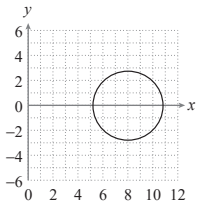
$(0,0),\ r = 6$

33.



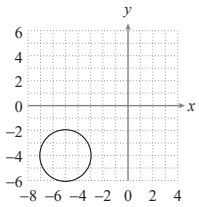
$(0,8),\ r = 3$

35.



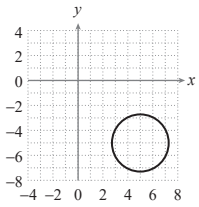
$(8,0),\ r = 2\sqrt{2}$

37.



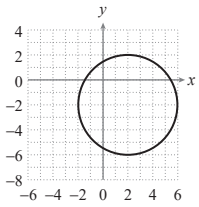
$(-5,-4),\ r = 2$

39.



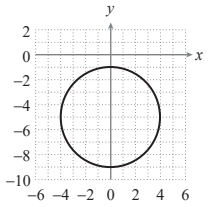
$(5,-5),\ r = \sqrt{5}$

41.



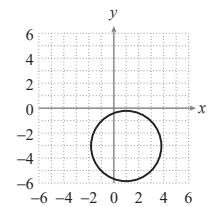
$(2,-2),\ r = 4$

43.



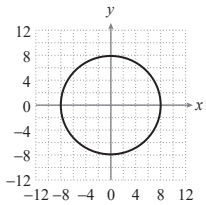
$(0,-5),\ r = 4$

45.



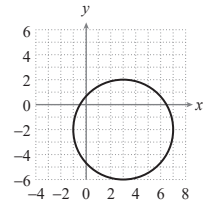
$(1,-3),\ r = 2\sqrt{2}$

47.



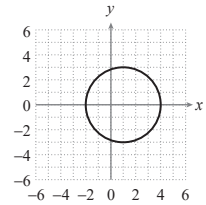
$(0,0),\ r = 8$

49.



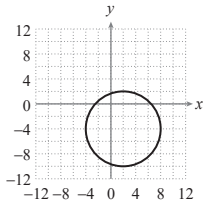
$(3,-2),\ r = 4$

51.



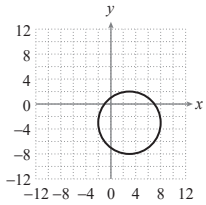
$(1,0),\ r = 3$

53.



$(2,-4),\ r = 6$

55.



$(3,-3),\ r = 5$

3.3 Exercises

1. Yes

3. No

5. No

7. No

9. Yes

11. Yes

13. Yes

15. No

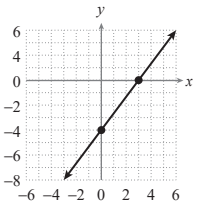
17. No

19. No

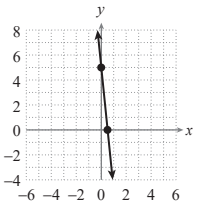
21. No

23. Yes

25.



27.



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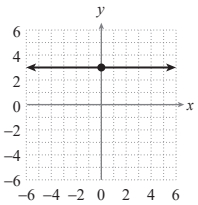

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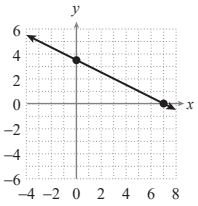

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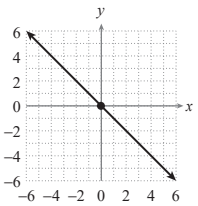

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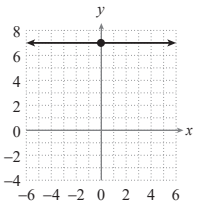

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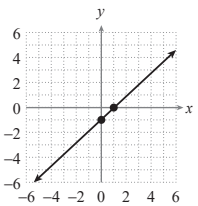

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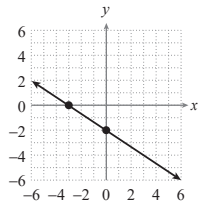


41. e

43. c

45. f

47. $a = P - b - c$

49. $j = 24{,}000 + 9b$;

$b = \dfrac{j - 24{,}000}{9}$; Yes

3.4 Exercises

1. -4

3. 0

5. Undefined

7. $\dfrac{2}{3}$

9. $\dfrac{1}{6}$

11. -7

13. -3

15. $-\dfrac{9}{13}$

17. $-\dfrac{1}{4}$

19. 0

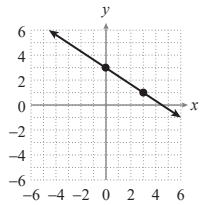
21. Undefined

23. 2

25. $\dfrac{7}{6}$

27. $-\dfrac{5}{2}$

29.



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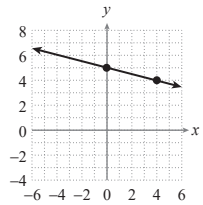
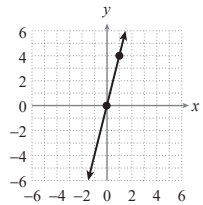


33.



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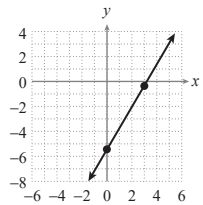


37. $y = \dfrac{3}{4}x - 3$

39. $y = -\dfrac{5}{2}x - 7$

41. $y = -5x - 9$

43. $3x - 2y = 3$

45. $y = 5$

47. $10x - y = 31$

49. $3x + y = 26$

51. $4x + 3y = 5$

53. $x = 2$

55. $y = -1$

57. $2x + 7y = 52$

59. $y = 5$

61. $15x - 8y = 0$

63. c

65. e

67. d

69. **a.** \$2225

b. \$2100

c. \$0.25

71. \$325

3.5 Exercises

1. $y = 4x + 9$

3. $y = 3x - 11$

5. $y = -9$

7. $y = x$

9. $y = \frac{7}{6}x + \frac{53}{6}$

11. Yes

13. Yes

15. Yes

17. No

19. No

21. No

23. No

25. No

27. Yes

29. No

31. $y = -\frac{1}{3}x - 1$

33. $y = 7$

35. $y = -\frac{1}{4}x - \frac{3}{4}$

37. $y = x + 3$

39. $y = -3x + 28$

41. No

43. No

45. No

47. No

49. Yes

51. No

53. No

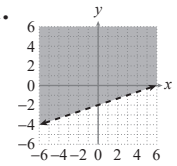
55. No

57. Yes

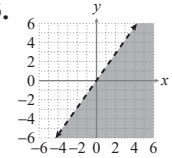
59. $41\frac{2}{3}$ ft

3.6 Exercises

1.



3.



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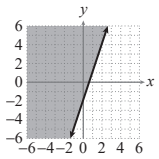

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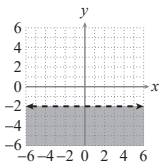

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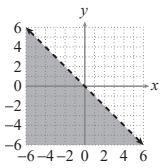

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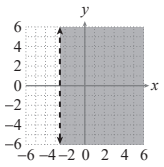

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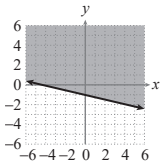

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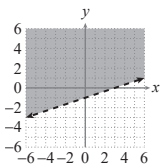

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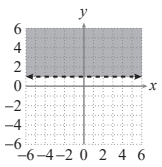

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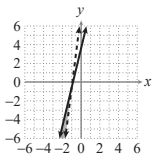

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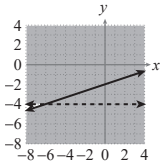

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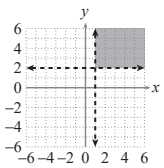

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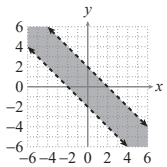

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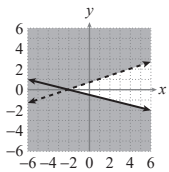

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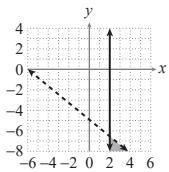

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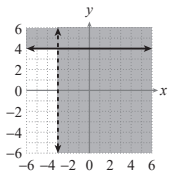

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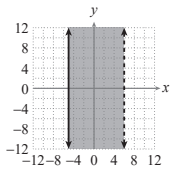

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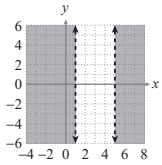

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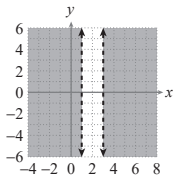

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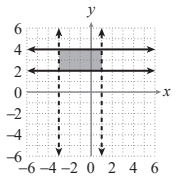

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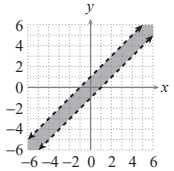

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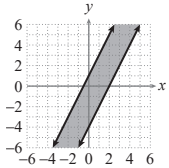

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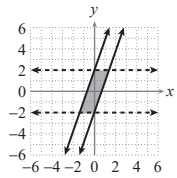


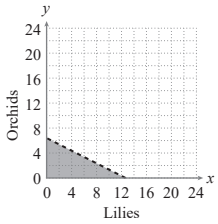
47. h

49. b

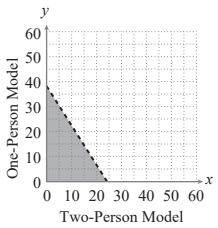
51. g

53. c

55. $12x + 22y < 150$



57. $73x + 46y < 1750$



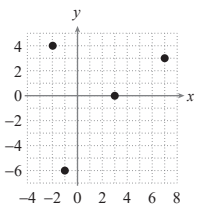
Chapter 3 Project

1. Shelbyville Tower: approx. 47.5 ft; Brockton Tower: 75 ft; Springfield Tower: approx. 60.6 ft; Ogdenville Tower: approx. 109.5 ft

3. No; attachment point: 29.4 ft from the top

Chapter 3 Review Exercises

1.



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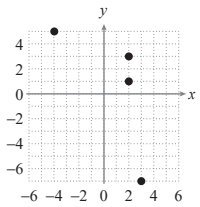
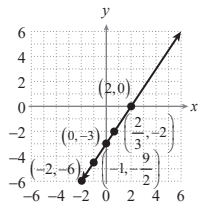


5. Positive x-axis

7.



$(2,0), (0,-3), \left(-1,-\frac{9}{2}\right), \left(\frac{2}{3},-2\right), (-2,-6)$

9. a. $\sqrt{2}$ **b.** $\left(\frac{5}{2},-\frac{13}{2}\right)$

11. a. $2\sqrt{13}$ **b.** $(-5,3)$

13. 2

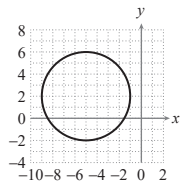
15. 24

17. $\left(x-\sqrt{5}\right)^2+\left(y+\sqrt{2}\right)^2=16$

19. $(x-2)^2+(y+1)^2=20$

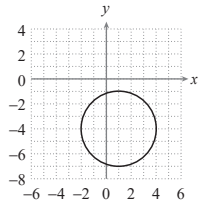
21. Center: $(-3,1)$; Radius: $2\sqrt{2}$

23.



$r = 4$; $(h,k) = (-5,2)$

25.



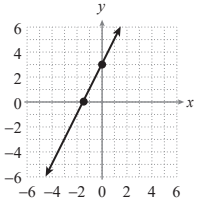
$r = 3$; $(h,k) = (1,-4)$

27. No

29. Yes

31. No

33.



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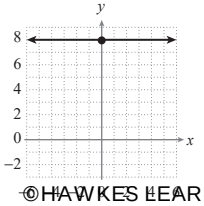

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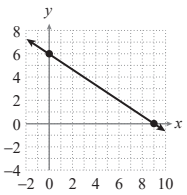
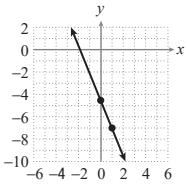


39. 12

41. Undefined

43.



45. $x - y = 5$

47. $y = \frac{5}{9}x - 2$

49. $9x - 2y = 31$

51. $W = 0.08s + 2800$

53. Perpendicular

55. $y = 3x + 10$

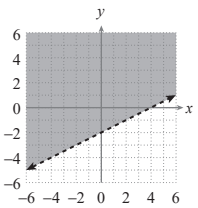
57. $y = 2x - 3$

59. $y = -\frac{4}{3}x + 6$

61. $x = 7$

63. Yes

65.



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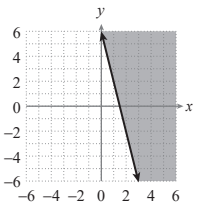

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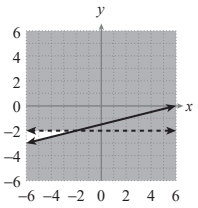
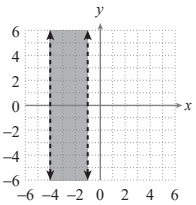
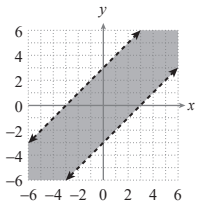
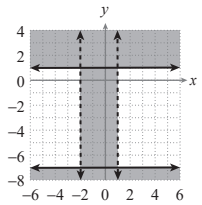


71. 

73. 

75. 

77. $3x+4y\geq 1500$

Chapter 4: Relations, Functions, and Their Graphs

4.1 Exercises

1. Dom $=\{-2\}$,
Ran $=\{5,3,0,-9\}$

3. Dom $=\{\pi,-2\pi,3,1\}$,
Ran $=\{2,4,0,7\}$

5. Dom $=\mathbb{Z}$,
Ran = even integers

7. Dom $=\mathbb{Z}$,
Ran $=\{\ldots,-2,1,4,\ldots\}$

9. Dom = Ran $=\mathbb{R}$

11. Dom $=[0,\infty)$, Ran $=\mathbb{R}$

13. Dom $=\mathbb{R}$, Ran $=\{-1\}$

15. Dom $=\{0\}$, Ran $=\mathbb{R}$

17. Dom $=[-3,1]$, Ran $=[0,4]$

19. Dom $=[0,3]$, Ran $=[1,5]$

21. Dom $=[-1,3]$, Ran $=[-4,3]$

23. Dom = All males with siblings,
Ran = All people who have brothers

25. Not a function;
$(-2,5)$ and $(-2,3)$

27. Function

29. Not a function;
$(6,-1)$ and $(6,4)$

31. Not a function;
$(-1,0)$ and $(-1,4)$

33. Function

35. Function

37. Function

39. Not a function;
$(-1,-1)$ and $(-1,1)$

41. Function

43. Function

45. Not a function;
$(1,-2)$ and $(1,2)$

47. $f(x)=-6x^2+2x$,
$f(-1)=-8$

49. $f(x)=\dfrac{-x+10}{3}$,
$f(-1)=\dfrac{11}{3}$

51. $f(x)=-2x-10$,
$f(-1)=-8$

53. 1

55. 3

57. $x=-3,2,3$

59. **a.** 10
b. x^2+x-2
c. $2ax+3a+a^2$
d. x^4+3x^2

61. **a.** 8
b. $3x-1$
c. $3a$
d. $3x^2+2$

63. **a.** -2
b. $-6x+16$
c. $-6a$
d. $-6x^2+10$

65. **a.** $i-3$
b. $\sqrt{2-x}-3$
c. $\sqrt{1-x-a}-\sqrt{1-x}$
d. $\sqrt{1-x^2}-3$

67. $2x+h-5$

69. $\dfrac{-1}{(x+h+2)(x+2)}$

71. $5(2x+h)$

73. 2

75. $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

77. Dom = Cod = Ran $=\mathbb{R}$

79. Dom = Cod = Ran $=\mathbb{Z}$

81. Dom = Cod $=\mathbb{N}$,
Ran $=\{6,7,8,\ldots\}$

83. $[1,\infty)$

85. $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$

87. $\mathbb{R}$

89. $\left(-\infty,\dfrac{1}{3}\right)\cup\left(\dfrac{1}{3},\infty\right)$

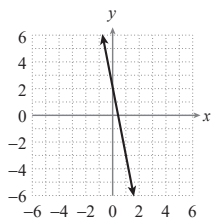
91. $(-\infty,2)\cup(2,\infty)$

93. $[-6,\infty)$

95. $(-\infty,0)\cup(0,\infty)$

97. A function is a special relation in which every element of the domain is paired with exactly one element of the range.

4.2 Exercises

1. 

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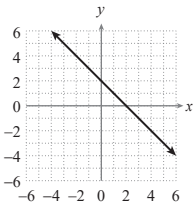

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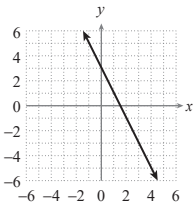

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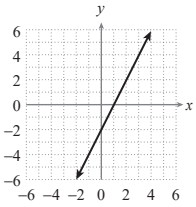

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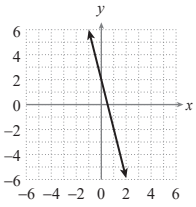

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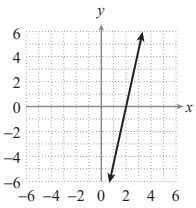

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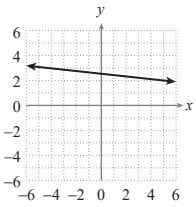

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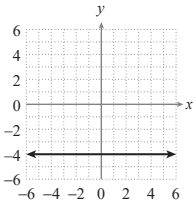


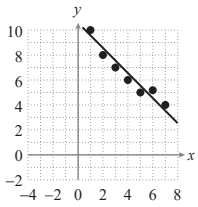
17. b

19. a

21. $f(x) = -x + 3$

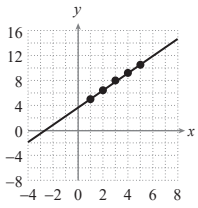
23. $f(x) = 2x - 3$

25. a. $y = -0.93x + 10.14$



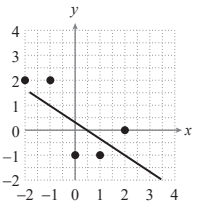
b. $r \approx -0.969$

27. a. $y = 1.36x + 3.82$



b. $r \approx 0.998$

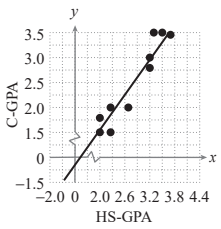
29. a. $y = -0.7x + 0.4$



b. $r \approx -0.730$

31. a. $y = 1.20x - 0.85$

b.

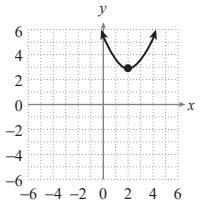


c. $r \approx 0.951$

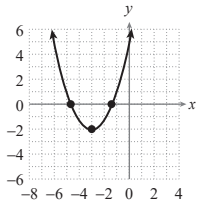
33. Neither. They are equal because the correlation is the absolute value.

4.3 Exercises

1. Vertex: $(2,3)$; no x-int.

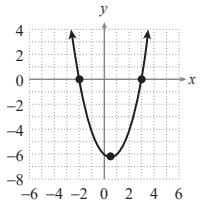


3. Vertex: $(-3,-2)$;
x-int.: $x = -3 \pm \sqrt{2}$

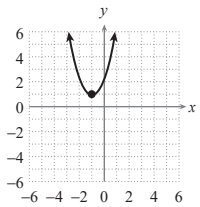


5. Vertex: $\left(\frac{1}{2}, -\frac{25}{4}\right)$;

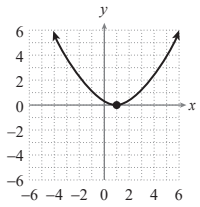
x-int.: $x = -2, 3$



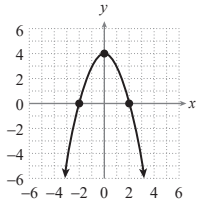
7. Vertex: $(-1,1)$;
no x-int.



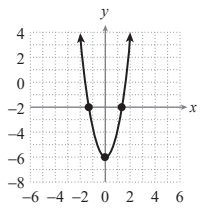
9. Vertex: $(1,0)$;
x-int.: $x = 1$



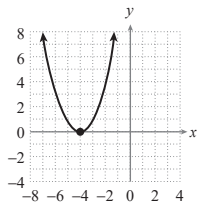
11. Vertex: $(0,4)$; x-int.: $x = -2, 2$



13. Vertex: $(0,-6)$;
x-int.: $x=\pm\frac{\sqrt{6}}{2}$



15. Vertex: $(-4,0)$; x-int.: $x=-4$



17. b

19. d

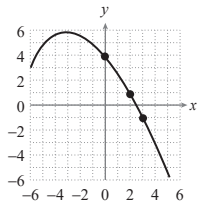
21. a. $f(x)=-\frac{1}{2}(x+2)(x-1)$

b. $\left(-\frac{1}{2},\frac{9}{8}\right)$

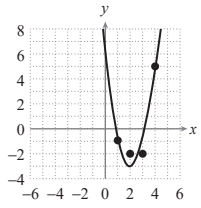
23. a. $f(x)=3(x+1)(x-3)$

b. $(1,-12)$

25. $f(x)=-0.167x^2-1.17x+4$



27. $f(x)=2x^2-8.2x+5.5$



29. a. $f(x)=x^2-2x+2$

b. $(1,1)$

31. a. $f(x)=-x^2+2x-3$

b. $(1,-2)$

33. 5 and 5

35. $(2,1)$

37. The dimensions should be 5 inches by 10 inches by 10 feet

39. 49 people; \$2401

41. 12 and 24

43. 375 units

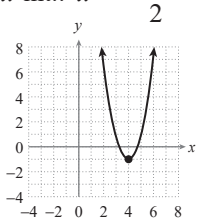
45. 25 sets of golf clubs

47. 112 feet

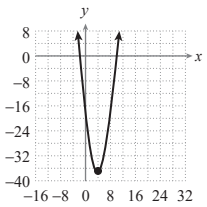
49. 164 feet

51. $h(t)=-15.88t^2+60.17t+7$; the maximum height of the baseball was approximately 64 feet.

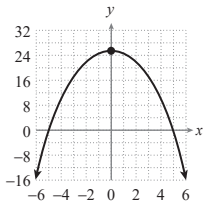
53. Vertex: $(4,-1)$;
x-int.: $x=\frac{8\pm\sqrt{2}}{2}$



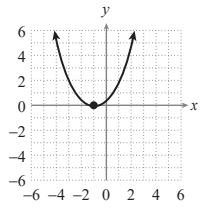
55. Vertex: $(4,-36)$;
x-int.: $x=-2,10$



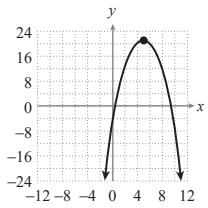
57. Vertex: $(0,25)$;
x-int.: $x=-5,5$



59. Vertex: $(-1,0)$;
x-int.: $x=-1$

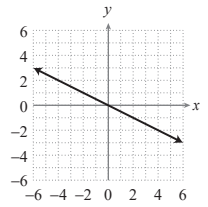


61. Vertex: $(5,21)$;
x-int.: $x=5\pm\sqrt{21}$



4.4 Exercises

1.



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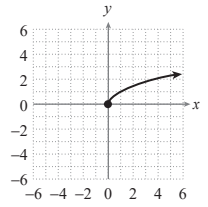

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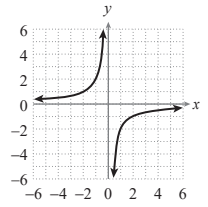

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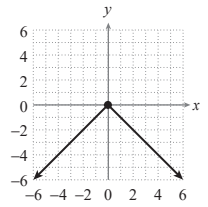

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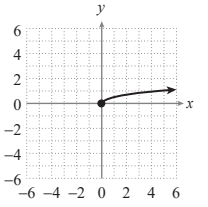

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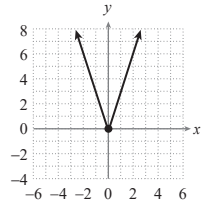
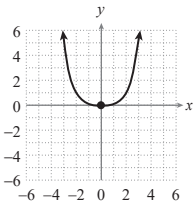
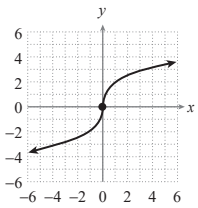
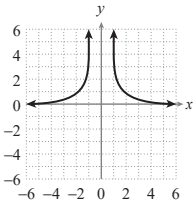
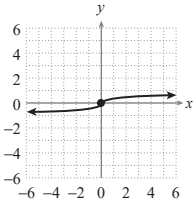
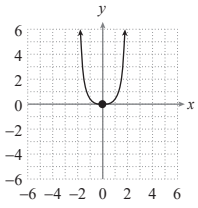


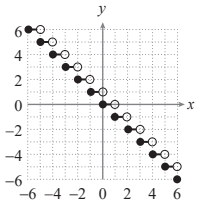
13. 

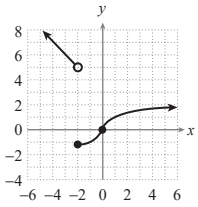
15. 

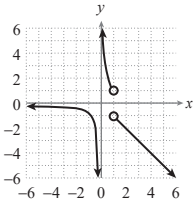
17. 

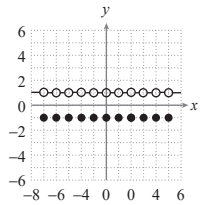
19. 

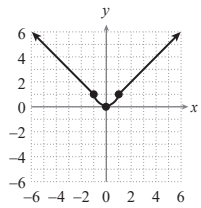
21. 

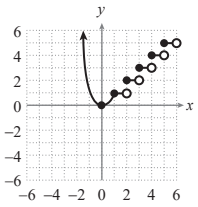
23. 

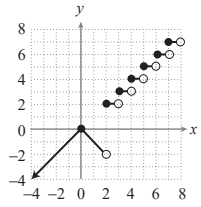
25. 

27. 

29. 

31. 

33. 

35. 

37. j

39. a

41. i

43. e

45. f

4.5 Exercises

1. $A = kbh$

3. $W = \dfrac{k}{d^2}$

5. $r = \dfrac{k}{t}$

7. $x = ky^3z^2$

9. $y = 18\sqrt{5}$

11. $y = 60\sqrt[3]{2}$

13. $y = 0.75$

15. $y = 0.0024$

17. $z = 112$

19. $z = 48$

21. $a = 10\sqrt{3}$

23. $a = 36$

25. $a = 108$

27. 256 feet

29. 20.60

31. 6.7 meters

33. 1.25 centimeters

35. 34.54 inches

37. 164.7872 in.^{2}

39. 9 watts

41. \$43

43. 17.28 ohms

45. 19.66 inches

47. 210 cubic inches

49. $P(\sigma,\varepsilon) = \dfrac{\sigma^2}{2\varepsilon}$

4.6 Exercises

1. a. If x represents the side length of each square cut from a corner, $V(x) = (60-2x)(20-2x)x = 4x^3 - 160x^2 + 1200x$ for $0 \le x \le 10$.

b. Yes; for example, $V(4) = 2496$ cm^3.

c. Approximately 2525 cm^3, obtained by letting $x = 4.5$ cm.

d. A height of 5 cm will lead to a width of 10 cm and consequently a length of 50 cm, so the ratio of length to width will be 5:1.

3. a. $V(t) = P\left(1 - \dfrac{2t}{9}\right)$ for $0 \le t \le 3$

b. $\dfrac{9}{4}$ years, or 2 years and 3 months

c. $V(1) = \dfrac{7}{9}P$

5. a. $d = \sqrt{(1+m^2)x^2 + 2bmx + b^2}$

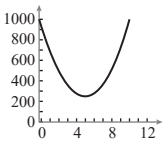
b. $d = \sqrt{x^2 + b^2}$

c. $d = \sqrt{(1+m^2)}\ |x|$

d. $d = 2\ |x|$

7. a. $s(x) = 30x^2 - 300x + 1000$ for $0 \le x \le 10$

b.



c. 1000, 250

9. a. 9.25×10^6

b. 1.45 times, or 145%

c. They aren't weightless, but instead in free fall. Astronauts fall toward Earth at the same rate as the craft they're in, making it appear as though they're floating.

11. a. $A = 6V^{\frac{2}{3}}$

b. $A = 600 \text{ mm}^2$

13. a. $C = 8x + 4y = 8x + \dfrac{7200}{x}$, where x is the length of fence along the boundary

b. 30 ft

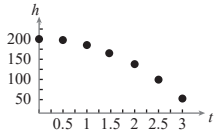
15. a. $R(x) = x(30 - 3x)$,
$C(x) = 36 + 6x$,
$P(x) = -3x^2 + 24x - 36$

b. 2

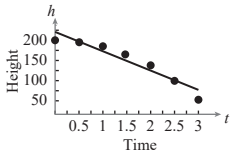
c. 4

d. \$18 for \$12 maximum profit

17. a.

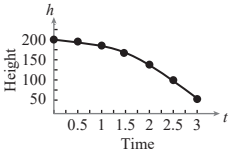


b.



$h(t) = -48t + 220$;
approximately 4.58 s; 220 ft

c.

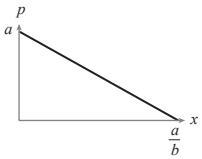


$h(t) = -16t^2 + 200$;
approximately 3.54 s; 200 ft

d. The quadratic model is more accurate.

Chapter 4 Project

1. a.



b. p is a linear decreasing function of x.

c. $x = 0$ corresponds to $p = a$, so a price of a corresponds to no sales.

d. $p = 0$ corresponds to $x = \dfrac{a}{b}$, which is the maximum feasible sales figure.

e. $R(x) = xp = x(a - bx)$
$= -bx^2 + ax$

f. R is a quadratic function.

g. The graph of R is a downward-opening parabola.

Chapter 4 Review Exercises

1. Dom $= \{-2, -3\}$,
Ran $= \{-9, -3, 2, 9\}$; No

3. Dom $= \mathbb{R}$,
Ran $= \{2\}$; Yes

5. Dom $= \mathbb{R}$,
Ran $= \mathbb{R}$; Yes

7. Dom $= [0, \infty)$,
Ran $= [4, \infty)$; Yes

9. Dom $= \{-2, 4\}$,
Ran $= \{-1, 5\}$; Yes

11. $f(x) = 3\sqrt{x + 11} - 4$;
$f(-2) = 5$

13. 4

15. $x = -1, \quad x = 3$

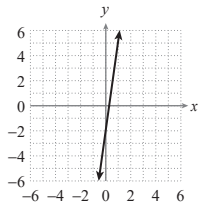
17. $\sqrt{x + h}$

19. $\sqrt[3]{(x + h)^2}$

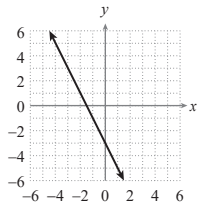
21. Dom $= \mathbb{N}$, Cod $= \mathbb{R}$,
Ran $= \left\{\dfrac{3}{4}, \dfrac{3}{2}, \dfrac{9}{4}, \ldots\right\}$

23. $\mathbb{R}$

25.



27.



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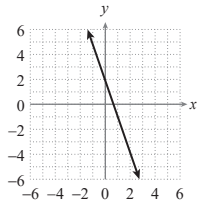
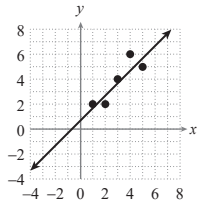


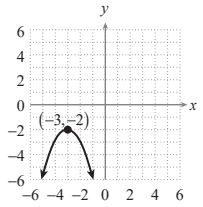
31. $f(x) = 2x - 1$

33. a. $y = x + 0.8$

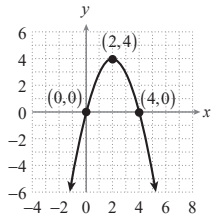


b. $r \approx 0.884$

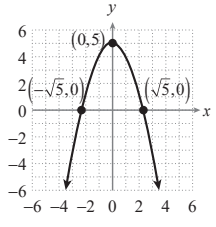
35.



37.



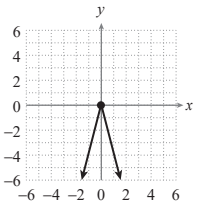
39.



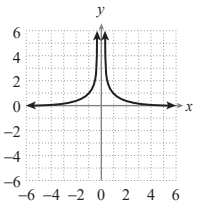
41. a. $f(x) = x^2 - 3x + 1$

b. $(1.5, -1.25)$

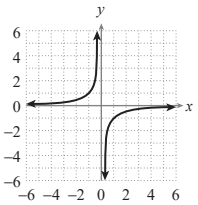
43.



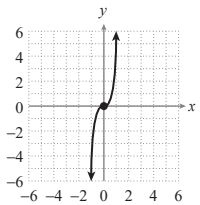
45.



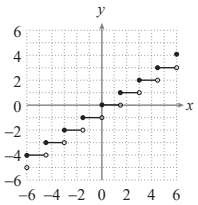
47.



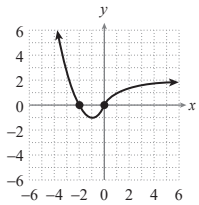
49.



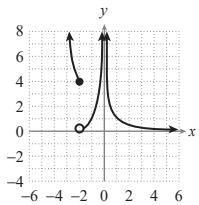
51.



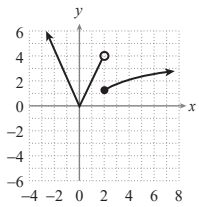
53.



55.



57.



59. $y = \dfrac{ka^3}{\sqrt{b}}$

61. $y = 72$

63. 7.44×10^7 m

65. a. $C(x) = \dfrac{12{,}000}{x} + 13x$, where x is the length of fence along the road.

b. Approximately 30.38 ft (along the road) by 39.50 ft (not along the road); approximately \$789.94

Chapter 5: Working with Functions

5.1 Exercises

1. $y = x^2$

3. $y = \sqrt[3]{x}$

5. $y = \sqrt{x}$

7. $y = \dfrac{1}{x^2}$

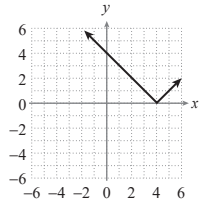
9. $y = x^3$

11. $y = |x|$

13. $y = \sqrt{x}$

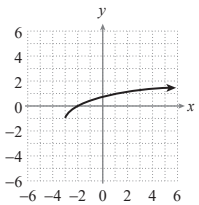
15. $y = x^3$

17.



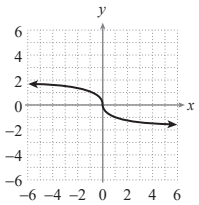
Dom $= \mathbb{R}$, Ran $= [0, \infty)$

19.



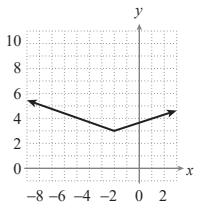
Dom $= [-3, \infty)$ Ran $= [-1, \infty)$

21.



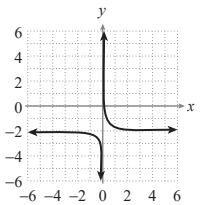
Dom $=$ Ran $= \mathbb{R}$

23.



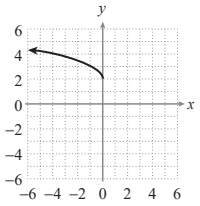
Dom $= \mathbb{R}$, Ran $= [3, \infty)$

25.



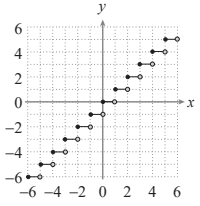
Dom $= (-\infty, 0) \cup (0, \infty)$,
Ran $= (-\infty, -2) \cup (-2, \infty)$

27.



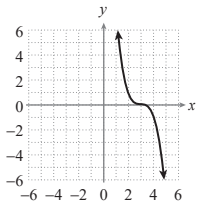
Dom $= (-\infty, 0]$, Ran $= [2, \infty)$

29.



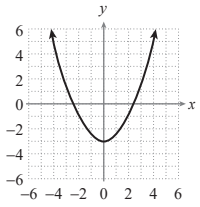
Dom $= \mathbb{R}$, Ran $= \mathbb{Z}$

31.



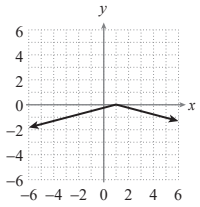
Dom = Ran = $\mathbb{R}$

33.



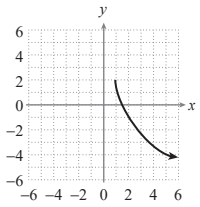
Dom = $\mathbb{R}$, Ran = $[-3,\infty)$

35.



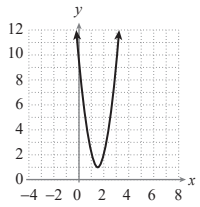
Dom = $\mathbb{R}$, Ran = $(-\infty,0]$

37.



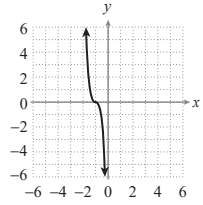
Dom = $[1,\infty)$, Ran = $(-\infty,2]$

39.



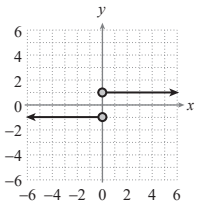
Dom = $\mathbb{R}$, Ran = $[1,\infty)$

41.



Dom = $\mathbb{R}$, Ran = $\mathbb{R}$

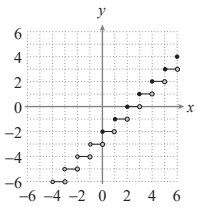
43.



Dom = $(-\infty,0)\cup(0,\infty)$,

Ran = $\{-1,1\}$

45.



Dom = $\mathbb{R}$, Ran = $\mathbb{Z}$

47. $f(x)=(x-4)^2+2$

49. $f(x)=(-x-2)^2=(x+2)^2$

51. $f(x)=(-x+1)^3$

53. $f(x)=-\sqrt{x+5}$

55. $f(x)=\sqrt{-\frac{x}{2}}+3$

57. $f(x)=-|x-8|-2$

59. $f(x)=-\sqrt{-(x+1)}$

61. $f(x)=\sqrt{2x}$

63. $f(x)=-\sqrt{x+4}$

65. $f(x)=1-(x-3)^3$

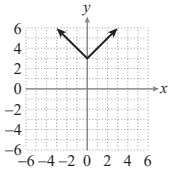
73. $f(x)=|x+4|-1$

75. $f(x)=-\sqrt{6-x}+2$

77. $f(x)=6-(x-3)^2$

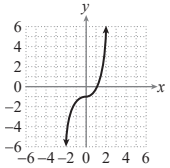
5.2 Exercises

1.



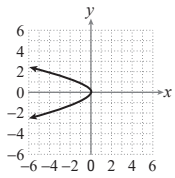
Even function; y-axis symmetry

3.



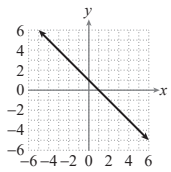
Function; neither; none of the symmetries

5.



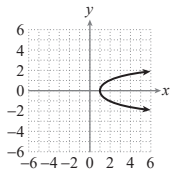
Not a function; x-axis symmetry

7.



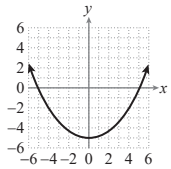
Function; neither; none of the symmetries

9.



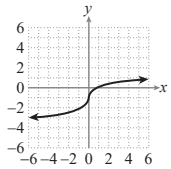
Not a function; x-axis symmetry

11.



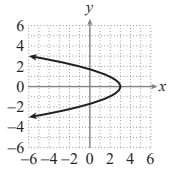
Even function; y-axis symmetry

13.



Function; neither; none of the symmetries

15.



Not a function; x-axis symmetry

17. Inc. on $(-\infty,2)$,
Dec. on $(2,\infty)$

19. Inc. on $(-\infty,-3)$,
Dec. on $(-3,\infty)$

21. Constant on $\mathbb{R}$

23. Dec. on $(-\infty,4)$,
Inc. on $(4,\infty)$

25. Dec. on $(-\infty,-3)$,
Inc. on $(-3,-1)$,
Constant on $(-1,\infty)$

27. a. local min at -1, local max at 2

b. value at -1 is 0, value at 2 is 3

29. a. local min at -1, local max at 2

b. value at -1 is -12, value at 2 is 15

31. a. Local min at -2, local max at 0, local min at 3

b. Value at -2 is $\frac{5}{3}$, value at 0 is 7, value at 3 is $-\frac{35}{4}$

33. a. Local min at 5

b. value at 5 is 2

35. a. local max at -2

b. value at -2 is 1

37. a. local min at 3

b. value at 3 is -2

39. 11

41. 2

43. $\frac{2-\sqrt{2}}{2}$

45. $2c+h$

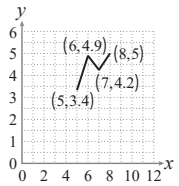
47. $-\frac{1}{12}$

49. a. $[a,b]$

b. $[b,c],[b,d]$

c. $[a,c],[a,d],[c,d]$

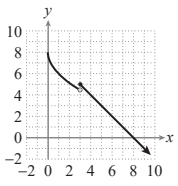
51.



Inc. on $(5,6)$ and $(7,8)$

Dec. on $(6,7)$

53.



Dec. on $(0,3)\cup(3,\infty)$

55. The average rate of change is m for any interval.

57. $2cp+hp+q$; The difference quotient approaches $2cp+q$ as h gets very small.

59. Nothing; Exercise 49a illustrates this conclusion.

5.3 Exercises

1. a. 2 **b.** -8
c. -15 **d.** $-\frac{3}{5}$

3. a. -3 **b.** -1
c. 2 **d.** 2

5. a. 12 **b.** 18
c. -45 **d.** -5

7. a. 3 **b.** 1
c. 2 **d.** 2

9. a. 6 **b.** 0
c. 9 **d.** 1

11. a. 5 **b.** -1
c. 6 **d.** $\frac{2}{3}$

13. a. 3 **b.** 5
c. -4 **d.** -4

15. a. $|x|+\sqrt{x}$, Dom $=[0,\infty)$

b. $\frac{|x|}{\sqrt{x}}$, Dom $=(0,\infty)$

17. a. x^2+x-2, Dom $=\mathbb{R}$

b. $\frac{1}{x+1}$, Dom $=(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

19. a. x^3+3x-8, Dom $=\mathbb{R}$

b. $\frac{3x}{x^3-8}$, Dom $=(-\infty,2)\cup(2,\infty)$

21. a. $-2x^2+|x+4|$, Dom $=\mathbb{R}$

b. $\frac{-2x^2}{|x+4|}$, Dom $=(-\infty,-4)\cup(-4,\infty)$

23. 2

25. 0

27. 8

29. 3

31. 1

33. $\frac{1}{3}$

35. a. $\frac{4-2x}{3x}$, Dom $=(-\infty,0)\cup(0,\infty)$

b. $\frac{3}{4x-2}$, Dom $=\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

37. a. $|x^3-2|$, Dom $=\mathbb{R}$

b. $|x-3|^3+1$, Dom $=\mathbb{R}$

39. a. $\sqrt{\frac{x-1}{2}}$, Dom $=[1,\infty)$

b. $\frac{\sqrt{x-1}+1}{2}$, Dom $=[1,\infty)$

41. a. $-3x^2-4$, Dom $=\mathbb{R}$

b. $9x^2-12x+6$, Dom $=\mathbb{R}$

43. a. $\sqrt{x^2-1}$, Dom $=(-\infty,-1]\cup[1,\infty)$

b. $x-1$, Dom $=\mathbb{R}$

45. $g(x)=\frac{2}{x}$, $h(x)=5x-1$, $f(x)=g(h(x))$

47. $g(x)=x+\sqrt{x}-5$, $h(x)=x+2$, $f(x)=g(h(x))$

49. $g(x)=\frac{\sqrt{x}}{x^2}$, $h(x)=x-3$, $f(x)=g(h(x))$

51. $g(x)=x-3$, $h(x)=|x^2+3x|$, $f(x)=g(h(x))$

53. $g(x)=\sqrt{x+5}$

55. $g(x)=-x^3-7$

57. $V=3\pi r^3$

59. $V=\frac{1}{12}\pi r^2t^2$

61. $(f\circ g)(x)=\sqrt[3]{\frac{-x^3}{3x^2-9}}$,

$(f\circ g)(-x)=\sqrt[3]{\frac{x^3}{3x^2-9}}$

$=-(f\circ g)(x)$

63. Yes

65. Yes

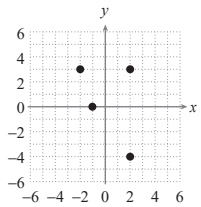
67. No

69. No

71. No

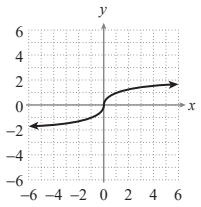
5.4 Exercises

1.



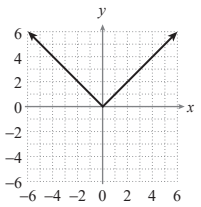
Dom $= \{2,-1,-2\}$,
Ran $= \{-4,3,0\}$

3.



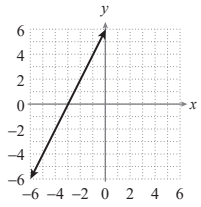
Dom = Ran = $\mathbb{R}$

5.



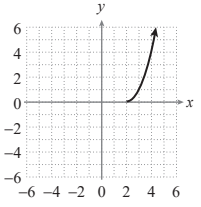
Dom = $\mathbb{R}$, Ran = $[0,\infty)$

7.



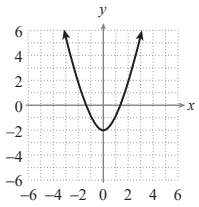
Dom = $\mathbb{R}$, Ran = $\mathbb{R}$

9.



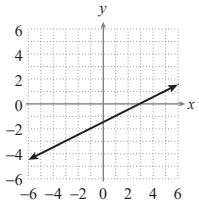
Dom = $[2,\infty)$, Ran = $[0,\infty)$

11.



Dom = $\mathbb{R}$, Ran = $[-2,\infty)$

13.



Dom = $\mathbb{R}$, Ran = $\mathbb{R}$

15. Not a one-to-one function
$f(-1) = f(1) = 1$

17. Restrict to $[0,\infty)$

19. Inverse exists

21. Inverse exists

23. Inverse exists

25. Restrict to $[2,\infty)$

27. Restrict to $[12,\infty)$

29. $f^{-1}(x) = (x+2)^3$

31. $r^{-1}(x) = \dfrac{-2x-1}{3x-1}$

33. $F^{-1}(x) = (x-2)^{\frac{1}{3}} + 5$

35. $V^{-1}(x) = 2x-5$

37. $h^{-1}(x) = (x+2)^{\frac{5}{3}}$

39. $J^{-1}(x) = \dfrac{x-2}{3x}$

41. $h^{-1}(x) = (x-6)^{\frac{1}{7}}$

43. $r^{-1}(x) = \dfrac{x^5}{2}$

45. $f^{-1}(x) = \dfrac{x^3}{54}$

47. $f^{-1}(x) = \sqrt{x-2}+3$
or $f^{-1}(x) = -\sqrt{x-2}+3$

49. $f^{-1}(x) = \sqrt[4]{x+2}-1$
or $f^{-1}(x) = -\sqrt[4]{x+2}-1$

61. f

63. c

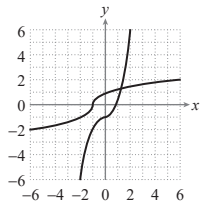
65. d

67. 184 72 96 96 72 160 144 8 72 112 160 120 32 8 200

69. REMEMBER
YOUR
SUNBLOCK

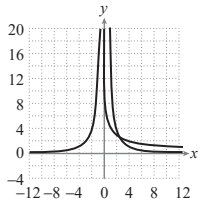
71. BEACH FUN IN THE SUN

73.



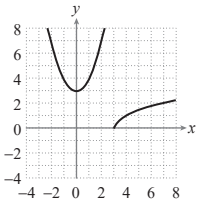
Dom $= (-\infty,\infty)$, Ran $= (-\infty,\infty)$

75.



Dom $= (0,\infty)$, Ran $= (0,\infty)$

77.



Dom $= [3,\infty)$, Ran $= [0,\infty)$

Chapter 5 Project

1. $A(r) = \pi r^2$

3. $A(t) = 6.76\pi t^2$

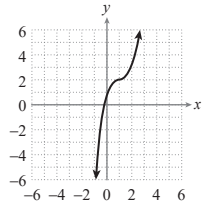
5. $r(5.5) = 14.3$ km

7. $A(5.5) \approx 642.4$ km^2

9. Approximately 286.7 km^2/hr

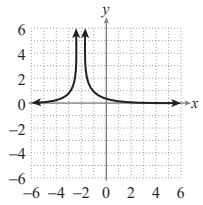
Chapter 5 Review Exercises

1.



Dom = Ran = $\mathbb{R}$

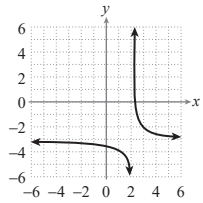
3.



Dom $= (-\infty,-2)\cup(-2,\infty)$,

Ran $= (0,\infty)$

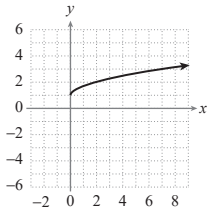
5.



Dom $= (-\infty,2)\cup(2,\infty)$,

Ran $= (-\infty,-3)\cup(-3,\infty)$

7.



Dom $= [0,\infty)$, Ran $= [1,\infty)$

9. $f(x)=(x-1)^2-2$

11. $f(x)=-\sqrt{x}+4$

13. Even function;
y-axis symmetry

15. Dec. on $(-\infty,2)$,

Inc. on $(2,\infty)$

17. a. Local min at 1,
local max at 2, local min at 3

b. Value at 1 is 1,
value at 2 is 4, value at 3 is 1

19. $-\frac{1}{3}$

21. -2

23. a. 3 **b.** 5

c. -4 **d.** -4

25. a. -2 **b.** 18

c. -80 **d.** $-\frac{4}{5}$

27. a. $\frac{1}{x-2}+\sqrt[3]{x}$,

Dom $= (-\infty,2)\cup(2,\infty)$

b. $\frac{1}{\sqrt[3]{x}(x-2)}$,

Dom $= (-\infty,0)\cup(0,2)\cup(2,\infty)$

29. a. $x^2+\sqrt[3]{x}-5$, Dom $=\mathbb{R}$

b. $\frac{x^2-4}{\sqrt[3]{x}-1}$,

Dom $= (-\infty,1)\cup(1,\infty)$

31. $-\frac{9}{2}$

33. $-\frac{2}{3}$

35. a. $\frac{1}{\sqrt{x-2}}$, Dom $=(2,\infty)$

b. $\frac{1}{\sqrt{x-4}}+2$, Dom $=(4,\infty)$

37. a. $3\sqrt{x-3}$, Dom $=[3,\infty)$

b. $\sqrt{3x-3}$, Dom $=[1,\infty)$

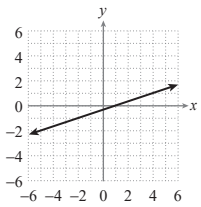
39. $g(x)=\frac{\sqrt{x}}{x^2}$,

$h(x)=x+2$,

$f(x)=g(h(x))$

41. $g(x)=\frac{2}{x}+1$

43.



Dom $=$ Ran $=\mathbb{R}$

45. $r^{-1}(x)=\frac{x+2}{7x}$

47. $f^{-1}(x)=(x+6)^5$

49. $f^{-1}(x)=\frac{-x-3}{x-2}$

51. $f^{-1}(x)=\frac{x-3}{8}$

Chapter 6: Polynomial and Rational Functions

6.1 Exercises

19. Yes

21. Yes

23. Yes

25. $1\pm 2i$

27. $-3, \frac{1}{2}$

29. $\pm\sqrt{3}, \pm\sqrt{5}$

31. $-\frac{5}{2}$

33. $0, 4\pm 3i$

35. $\pm 1, \pm 2i\sqrt{2}$

37. 7th-degree; lead coef. $= 4$;

$j(x)\to-\infty$ as $x\to-\infty$

$j(x)\to\infty$ as $x\to\infty$

39. 5th-degree; lead coef. $=-6$;

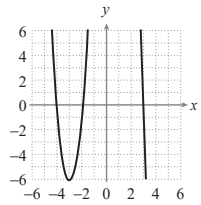
$h(x)\to\infty$ as $x\to-\infty$

$h(x)\to-\infty$ as $x\to\infty$

41. 4th-degree; lead coef. $=-2$;

$f(x)\to-\infty$ as
$x\to-\infty$ and $x\to\infty$

43.

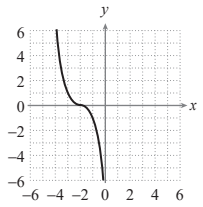


$g(x)\to\infty$ as $x\to-\infty$;

$g(x)\to-\infty$ as $x\to\infty$

x-int : $(-4,0),(-2,0),(3,0)$;

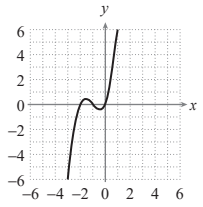
y-int : $(0,24)$

45. 

$h(x) \to \infty$ as $x \to -\infty$;
$h(x) \to -\infty$ as $x \to \infty$

x-int: $(-2,0)$

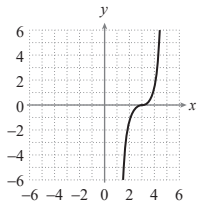
y-int: $(0,-8)$

47. 

$s(x) \to -\infty$ as $x \to -\infty$;
$s(x) \to \infty$ as $x \to \infty$

x-int: $(-2,0),(-1,0),(0,0)$

y-int: $(0,0)$

49. 

$g(x) \to -\infty$ as $x \to -\infty$;
$g(x) \to \infty$ as $x \to \infty$

x-int: $(3,0)$;

y-int: $(0,-243)$

51. $p(x) = \left(\frac{1}{2}\right)(x+2)$
$\times (x+1)(x-1)(x-3)$

53. $p(x) = -(x-1)(x-2)(x-3)$

55. e

57. a

59. f

61. d

63. f

65. b

67. $(-\infty,-2)\cup(3,\infty)$

69. $(-\infty,-2)\cup(-1,0)$

71. $[-2,1]\cup[3,\infty)$

73. $[-5,-1]\cup[1,4]$

75. $\left(-\frac{1}{2},2\right)$

77. $(-\infty,-4)\cup(2,3)$

79. All integers between 5 and 27, inclusive

81. All integers between 11 and 23, inclusive

83. Between 3490 and 17,740 phones, inclusive.

85. About 141.4 weeks

6.2 Exercises

1. $3x^2 - x + 1 + \frac{5x-1}{2x^2+2}$

3. $x - 2 + \frac{-2}{x^2-4x+4}$

5. $4x^2 - 14x + 29 + \frac{-65}{x+2}$

7. $x^3 + 6x^2 - 2x + 5 + \frac{2x+5}{3x^2-1}$

9. $2x^3 - 3x^2 + 2x - 5$

11. $x^3 + 3x^2 + 10x + 10 + \frac{22}{x-3}$

13. $3x^2 + 5x + 9 + \frac{45}{3x-5}$

15. $2x - 5 + \frac{7}{x+3}$

17. $x^2 - ix + 6 + \frac{1+i}{2x-i}$

19. $x^2 + 3$

21. $p(1) = 4$

23. c is a zero

25. c is a zero

27. $p(1) = 12$

29. c is a zero

31. c is a zero

33. c is a zero

35. c is a zero

37. $p(5) = -2$

39. c is a zero

41. $x^2 - 4x + 2 + \frac{-1}{x+5}$

43. $x^7 - 3x^2 + \frac{3}{x+1}$

45. $4x^2 - 4x + 2$

47. $x^4 - x^3 - x^2 - 7x - 14 - \frac{10}{x-2}$

49. $x^3 - x^2 + x$

51. $2x^2 - 4ix + 17 + \frac{8+48i}{x-3i}$

53. $f(x) = -x^2 - x + 12$

55. $f(x) = -x^2 + 4x - 13$

57. $f(x) = x^4 - 12x^3 + 54x^2 - 108x + 81$

59. $f(x) = 3x^4 + 9x^3 - 9x^2 - 21x + 18$

61. $SA = (x+5)(x+2)$
$= x^2 + 7x + 10$

6.3 Exercises

1. $\pm\left\{\frac{1}{3},\frac{2}{3},1,\frac{4}{3},2,\frac{8}{3},4,8\right\}$,
$\left\{-4,\frac{1}{3},2\right\}$

3. $\pm\{1,2,3,4,6,8,12,24\},\{\pm 2i,2,3\}$

5. $\pm\{1,2,7,14\},\{1,2,7\}$

7. $\pm\left\{\frac{1}{2},1,\frac{3}{2},\frac{5}{2},3,5,\frac{15}{2},15\right\}$,
$\left\{-1,\frac{5}{2},3\right\}$

9. $\pm\left\{\frac{1}{3},1,3\right\},\{-1,1,-i,i\}$

11. $\pm\{1,11\},\{-11,-1,1\}$

13. $\{-1,1,-i,i\}$

15. $\{-1,2-3i,2+3i\}$

17. $\{-2i,2i,2,3\}$

19. $\{4,1-2i,1+2i\}$

21. $\{-5i,5i,-2,1\}$

23. $\{-11,-1,1\}$

25. 0 pos., 3 or 1 neg.

27. 2 or 0 pos., 1 neg.

29. 3 or 1 pos., 1 neg.

31. 1 pos., 1 neg.

33. 3 or 1 pos., 0 neg.

35. 0 pos., 0 neg.

37. $[-5,1]$

39. $[-1,6]$

41. $[-3,6]$

43. $[-3,3]$

45. $[-3,6]$

47. $\{-4,-1,1\}$

49. $\{-1,2,5\}$

51. $\{2,3,\pm 2\sqrt{2}\}$

53. $\pm\{\sqrt{5},i\sqrt{5}\}$

55. $\left\{-2,-\frac{1}{2},6\right\}$

57. $f(-3)=-84,$
$f(-1)=16$

59. $f(2)=-15,$
$f(3)=24$

61. $f(2)=15,$
$f(3)=-24$

63. $\{-1,2,5\}$

65. $\left\{-\frac{7}{3},\pm 1\right\}$

67. $\left\{-1,-\frac{2}{5},2,\frac{7}{3}\right\}$

69. $\left\{\pm 3,-2,-\frac{1}{3}\right\}$

71. $\left\{-5,\pm 1,\frac{5}{2},4\right\}$

73. $\{-3,-2,-1\}$

75. $\left\{-9,\pm\sqrt{2}\right\}$

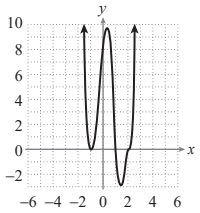
77. $\left\{-3,-\frac{1}{2},8\right\}$

79. $\left\{\pm 2,-\frac{3}{4},1\right\}$

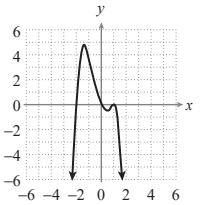
81. $\left\{\sqrt[3]{-2},\pm\frac{\sqrt{6}}{2},\frac{5}{3}\right\}$

6.4 Exercises

1.



3.



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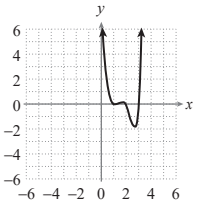
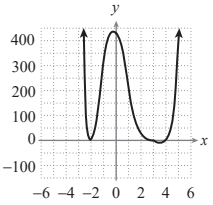
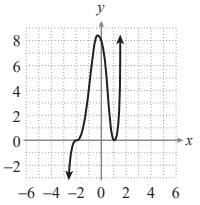


7.

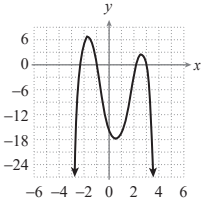


9.



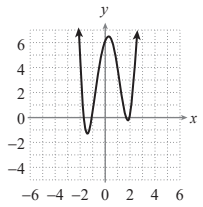
$f(x)=(x+2)^3(x-1)^2$

11.



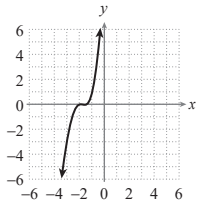
$s(x)=-(x-3)(x+1)$
$(x-\sqrt{5})(x+\sqrt{5})$

13.



$H(x)=(x-2)(x+1)(x-\sqrt{3})(x+\sqrt{3})$

15.



$f(x)=(x+2)^2(2x+3)$

17. $\{-2,1\}$

19. $\{-2,1,\pm i\sqrt{5}\}$

21. $\{-3,1,\pm 2i\}$

23. $\{\pm 1,\pm 4i\}$

25. $(x-3+2i)(x-3-2i)$
$\times(x-4)(x+1)$

27. $(x-1-3i)(x-2)(x+1)$

29. $(x+\sqrt{7})(x-\sqrt{7})(x-2+3i)$
$\times(x-2-3i)$

31. $(x-2)(x+1)(x-1+2i)$
$\times(x-1-2i)$

33. $x(x-3)(x-1)(x+11)$

35. $(x-2)(x-2+4i)(x-2-4i)$

37. $f(x)=-2x^3+18x^2-32x-52$

39. $f(x)=2x^5+2x^4-10x^3-2x^2$
$+16x-8$

41. $f(x)=3x^4-18x^3+12x^2-72x$

43. $f(x)=-x^3+2x^2+14x-40$

45. $f(x)=-x^3+4x^2+15x-68$

47. **a.** $V(x)=4x(5-x)(9-x)$
b. $x=0, x=5, x=9$
c. $x=0$ and $x=5$

49. **a.** $V(x)=x(17-2x)(9-2x)$
b. $x=0, x=\frac{9}{2}, x=\frac{17}{2}$
c. $x=0$ and $x=\frac{9}{2}$

6.5 Exercises

1. $x = 1$

3. No vertical asymptote

5. $x = 2$

7. $x = 0$

9. $x = -\frac{1}{2}$

11. No vertical asymptote

13. $x = 7$

15. $x = -2$

17. $x = -2, x = 2$

19. $y = 0$

21. No horizontal or oblique asymptote

23. $y = 0$

25. $y = 2$

27. $y = 3x + 6$

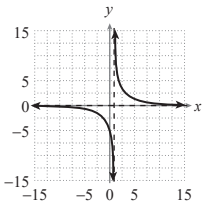
29. $y = 0$

31. $y = 0$

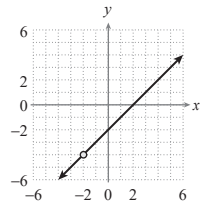
33. $y = x - 11$

35. $y = 5x + 4$

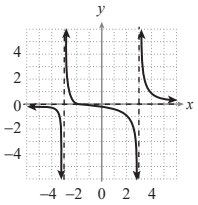
37.



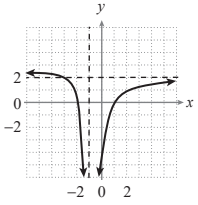
39.



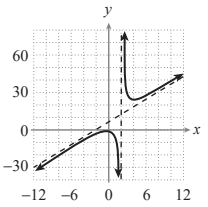
41.



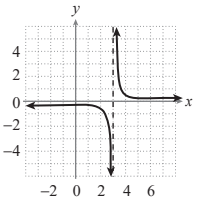
43.



45.



47.



49. **a.** $x = -2$
b. $y = 0$
c. None
d. None
e. $(0, 5)$

51. **a.** $x = 9$
b. $y = 0$
c. None
d. None
e. $\left(0, -\frac{1}{3}\right)$

53. **a.** $x = -1, x = 1$
b. None
c. $y = x$
d. $\left(\sqrt[3]{3}, 0\right)$
e. $(0, 3)$

55. **a.** $x = 1$
b. None
c. $y = 3x$
d. None
e. $(0, -3)$

57. $(-\infty, -2) \cup (-1, 1)$

59. $(-8, -2) \cup (2, \infty)$

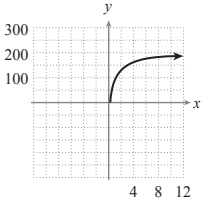
61. $(-\infty, -2) \cup (-2, 3)$

63. $(0, 3)$

65. $(-2, -1) \cup (1, \infty)$

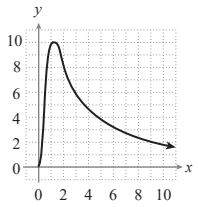
67. $(-\infty, -1) \cup \left[-\frac{1}{2}, 0\right)$

69. **a.**



b. April's fish population approaches a maximum of 200 fish.

71. **a.**



b. The concentration of the drug disappears in the long run.

Chapter 6 Project

1. $P(x) = -3x^2 + 5500x - 729{,}000$

3. \$4933 for $x = 1689$ to \$9568 for $x = 144$

5. $[212, 1621]$

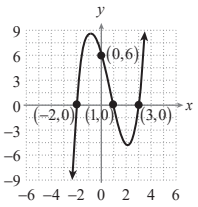
Chapter 6 Review Exercises

5. $\pm\sqrt{2}, \pm\sqrt{5}$

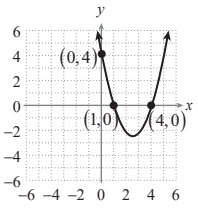
7. $\pm\sqrt{2}$

9. $0, \frac{-1 \pm \sqrt{5}}{2}$

11. x-int: -2, 1, 3
y-int: 6
$f(x) \to -\infty$ as $x \to -\infty$;
$f(x) \to \infty$ as $x \to \infty$



13. x-int: 1, 4
y-int: 4
$g(x) \to \infty$ as $x \to \pm\infty$



15. $\left[\frac{5}{2}, 3\right]$

17. $[-2, -1] \cup [1, 4]$

19. $(-\infty, 0) \cup (0, 1) \cup (2, \infty)$

21. All integers between 4 and 30, inclusive

23. $11x+35+\dfrac{100}{x-3}$

25. $2x^3+2x^2-2x-3+\dfrac{-2x-2}{x^2-x}$

27. $p(1)=90$

29. $p\left(\dfrac{2}{3}\right)=-\dfrac{7}{3}$

31. $-x^3+2x^2-7x+23$

33. $-x^3+7x^2+x-3+\dfrac{-1}{x-1}$

35. $f(x)=x^2-4x-12$

37. $f(x)=2(x^2-4)(x-3)$

39. $\pm\left\{\dfrac{1}{2},1,\dfrac{3}{2},3,\dfrac{9}{2},9\right\},\left\{1,\dfrac{3}{2},3\right\}$

41. $\pm\{1,3,9\},\{-3,-1\}$

43. $\left\{1,\dfrac{3}{2},3\right\}$

45. $\{-3,-1\}$

47. 4, 2 or 0 pos., 2 or 0 neg.

49. $[-3,7]$

51. $\left\{-\dfrac{5}{2},-\dfrac{1}{2},7\right\}$

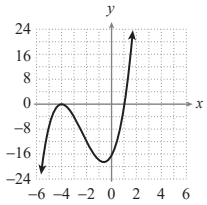
53. $f(2)=3;f(4)=-15$

55. $\{\pm 3i,4\}$

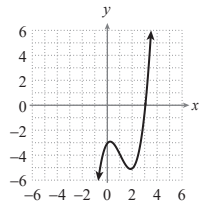
57. $\left\{3,2\pm\sqrt{3}\right\}$

59. $\left\{-\dfrac{1}{2},2\right\}$

61.



63. $(x^2+1)(x-3)$



65. $\left\{-1,\dfrac{5}{3},\pm 2i\right\}$

67. $\{\pm 2i,-3,\pm 1\}$

69. $(x-5i)(x+5i)(x-6)(x+1)$

71. $(x+3)\left(x-\dfrac{1+i\sqrt{19}}{4}\right)\times\left(x-\dfrac{1-i\sqrt{19}}{4}\right)$

73. $f(x)=x^5+3x^4-3x^3-17x^2-18x-6$

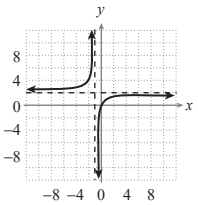
75. $x=\dfrac{5}{2}$

77. $x=0$

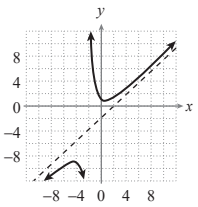
79. $y=2x+9$

81. No horizontal or oblique asymptote

83.



85.



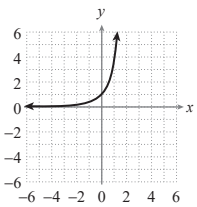
87. $\left(-3,\dfrac{7}{2}\right]$

89. $\left(-3,\dfrac{8}{9}\right)\cup(2,\infty)$

Chapter 7: Exponential and Logarithmic Functions

7.1 Exercises

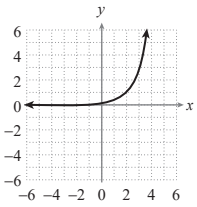
1.



Dom $=(-\infty,\infty)$,

Ran $=(0,\infty)$

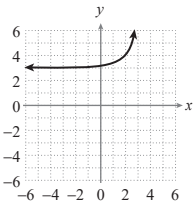
3.



Dom $=(-\infty,\infty)$,

Ran $=(0,\infty)$

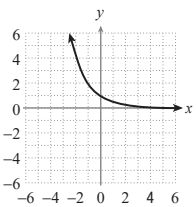
5.



Dom $=(-\infty,\infty)$,

Ran $=(3,\infty)$

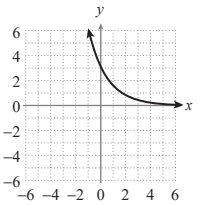
7.



Dom $=(-\infty,\infty)$,

Ran $=(0,\infty)$

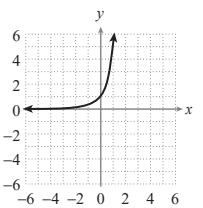
9.



Dom $=(-\infty,\infty)$,

Ran $=(0,\infty)$

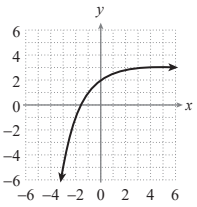
11.



Dom $=(-\infty,\infty)$,

Ran $=(0,\infty)$

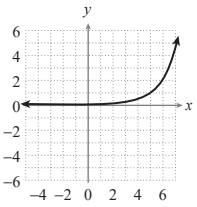
13.



Dom $= (-\infty, \infty)$,

Ran $= (-\infty, 3)$

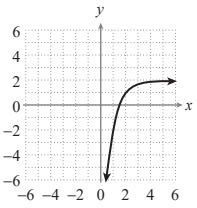
15.



Dom $= (-\infty, \infty)$,

Ran $= (0, \infty)$

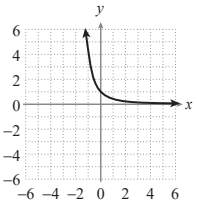
17.



Dom $= (-\infty, \infty)$,

Ran $= (-\infty, 2)$

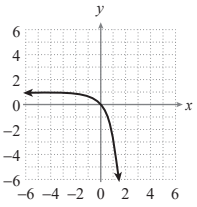
19.



Dom $= (-\infty, \infty)$,

Ran $= (0, \infty)$

21.



Dom $= (-\infty, \infty)$,

Ran $= (-\infty, 1)$

23. $\{2\}$

25. $\{-2\}$

27. $\{-13\}$

29. $\{3\}$

31. $\{-2\}$

33. $\{-2, -1\}$

35. $\{7\}$

37. $\{3\}$

39. $\{9\}$

41. $\{-3\}$

43. $\{2\}$

45. $\{-1\}$

47. a

49. i

51. d

53. e

55. h

7.2 Exercises

1. $V \approx 178$ people

3. $C \approx \$8526.20$

5. $W \approx 93$ computers

7. a. $a \approx 0.999567$

b. $A \approx 0.958$ grams

c. $A \approx 0.648$ grams

9. a. 3 years

b. 9 years

11. 1118 rabbits

13. The bank offering 2.75% and monthly compounding.

15. Approximately 3.18%

17. \$134,392

19. a. 10

b. 7490 people

c. The function approaches 10,000 as time goes on.

21. a. $a \approx 0.965936$

b. $A \approx 0.707$ kg

c. $A \approx 7.628$ mg

23. a. \$1521.74

b. \$271.74

25. \$9459.48;

\$9942.41

27. \$2835.71

29. \$20,000

31. \$7318.71

33. a. \$7647.95

b. \$7647.57

c. Yes; daily compounding is a frequency close enough to continuous compounding to make little difference at the hundredths place.

35. a. Linear:

$y = 1.06098 \times 10^6 x - 1.94223 \times 10^9$

Quadratic:

$y = 8501.22x^2 - 3.08186 \times 10^7 x + 2.79424 \times 10^{10}$

Exponential:

$y = 2.172414 \times 10^{-12} (1.023986)^x$

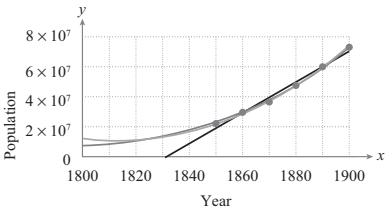
b. Linear: –32,466,000

Quadratic: 12,872,800

Exponential: 7,348,129

Actual: 5,308,483

None of these regression models appear to be very accurate this far from known data, though the population estimates given by the exponential model are closer to the actual population.



7.3 Exercises

1. $4 = \log_5 625$

3. $3 = \log_x 27$

5. $3 = \log_{4.2} C$

7. $x = \log_4 31$

9. $\sqrt{3} = \log_{4x} 13$

11. $e^x = \log_2 11$

13. $81 = 3^4$

15. $4 = b^{\frac{1}{2}}$

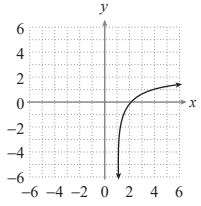
17. $15 = 2^b$

19. $W = 5^{12}$

21. $2x = \pi^4$

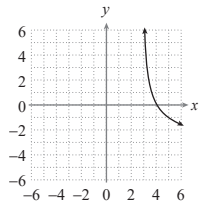
23. $e^x = 2$

25.



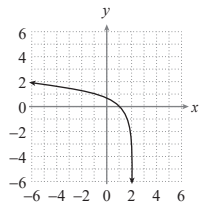
Dom: $(1, \infty)$, Ran: $(-\infty, \infty)$

27.



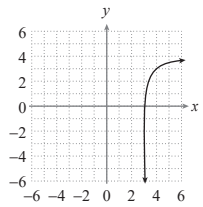
Dom: $(3,\infty)$, Ran: $(-\infty,\infty)$

29.



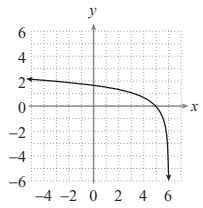
Dom: $(-\infty,2)$, Ran: $(-\infty,\infty)$

31.



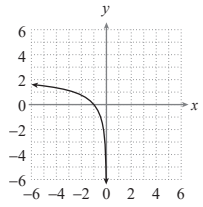
Dom: $(3,\infty)$, Ran: $(-\infty,\infty)$

33.



Dom: $(-\infty,6)$, Ran: $(-\infty,\infty)$

35.



Dom: $(-\infty,0)$, Ran: $(-\infty,\infty)$

37. e **39.** b **41.** h

43. d **45.** i **47.** -2

49. 3 **51.** $-\frac{1}{2}$ **53.** $\frac{3}{4}$

55. 2.89 **57.** $\frac{5}{3}$ **59.** 1

61. $\frac{1}{5}$ **63.** 2 **65.** $\{64\}$

67. $\{9\}$ **69.** $\left\{-\frac{1}{2}\right\}$ **71.** $\left\{\frac{1}{21}\right\}$

73. $\{10\}$ **75.** $\{36\}$

77. $\left\{\pm\frac{1}{10}\right\}$ **79.** $\{0.18\}$

81. $\left\{\pm\sqrt{e}\right\}$ or $\{\pm1.65\}$

83. $\{12.89\}$

85. $\{10,000,000,002\}$

7.4 Exercises

1. $3+3\log_5 x$

3. $2+\ln p-3\ln q$

5. $1+\log_9 x-3\log_9 y$

7. $\frac{3}{2}\ln x+\ln p+5\ln q-7$

9. $\log(2+3\log x)$

11. $1-\frac{1}{2}\log(x+y)$

13. $\log_2(y^2+z)-4\log_2 x-4$

15. $2\log_b x+\frac{1}{2}\log_b y-\log_b z$

17. $2+\log_b a+b\log_b c$

19. $\log\left(\frac{x}{y}\right)$

21. $\log_5(x+5)$

23. $\log_2\left(x^{\frac{4}{3}}+3x^{\frac{1}{3}}\right)$

25. $\ln\left(\frac{3p}{q^2}\right)$

27. $\log\left(\frac{x-10}{x}\right)$

29. $\ln\left(\frac{z^2}{x^3y^3}\right)$

31. $\log_5 4$ **33.** $\ln 45$

35. $\log_3 1=0$ **37.** $\ln 12$

39. $\log 11$ **41.** $\log_8(x^2-y)$

43. x^2 **45.** $\frac{e^2p}{x}$

47. $\frac{x^3}{y^4}$ **49.** x^2

51. 4 **53.** $12x^2$ **55.** 2.04

57. 0.95 **59.** 0.95 **61.** 2.45

63. 3.30 **65.** 0.74 **67.** 1.20

69. 1.86 **71.** -1 **73.** 3.85

75. 0.77 **77.** -1.76 **79.** 2

81. 7 **83.** 1

85. $4\sqrt{2}\approx 5.66$

87. 9.05 **89.** 2.08 **91.** 12

93. 1,048,576

95. 10.25

97. 5,011,872 times stronger

99. 133 decibels

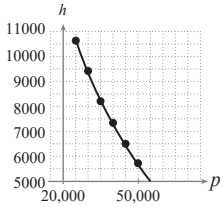
101. 7.62; yes

103. **a.** 15.05 minutes

b. 7:00 p.m.

c. 112 °F; no

105. $h(p)=81,751.7-7027.82\ln p$



7.5 Exercises

1. $x\approx 0.26$

3. $x\approx 3.12$

5. $x\approx 3.89$

7. $x\approx -2.28$

9. $x\approx 8.09$

11. $x\approx 2.68$

13. $x\approx -1.12$

15. $x\approx 52.77$

17. $x\approx \pm 0.71$

19. $x=-12$

21. $x\approx 1.32$

23. $x\approx 3.27$

25. $x=125$

27. $x=5$

29. $x\approx 40.17$

31. $x=35$

33. $x\approx 9.38$

35. $x=4$

37. $x=\frac{1}{162}$

39. $x=1$

41. $x\approx 100.04$

43. No solution

45. $x=5$

47. $x=8$

49. $x=\frac{37}{8}$

51. $x=5$

53. No solution

55. $x=6$

57. $x=\sqrt{2}-1$

59. $x=2,3$

61. $x=1,2$

63. $f(x)=\log(2x^2)$

65. $f(x) = \ln(9x^2)$

67. $f(x) = 256x$

69. $f(x) = \ln 1 = 0$

71. $f(x) = \ln(5^x)$

73. $f(x) = \ln 5$

75. **a.** 17.36 years

b. 9.90 years

77. 4.98 hours

79. 4.99 years

81. 0.271 years (about 99 days)

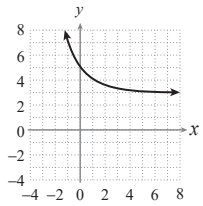
Chapter 7 Project

1. 242

3. 20,219

Chapter 7 Review Exercises

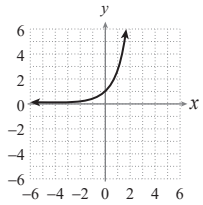
1.



Dom $= (-\infty, \infty)$,

Ran $= (3, \infty)$

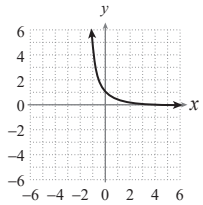
3.



Dom $= (-\infty, \infty)$,

Ran $= (0, \infty)$

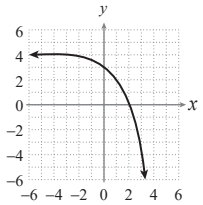
5.



Dom $= (-\infty, \infty)$,

Ran $= (0, \infty)$

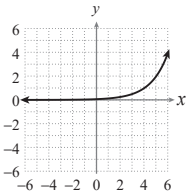
7.



Dom $= (-\infty, \infty)$,

Ran $= (-\infty, 4)$

9.



Dom $= (-\infty, \infty)$,

Ran $= (0, \infty)$

11. $x = 5$

13. $x = 2$

15. $x = -1$

17. $x = \dfrac{8}{5}$

19. $x = 2$

21. 8 days

23. **a.** 173 flies

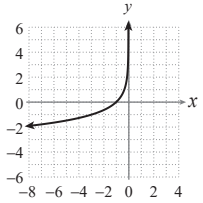
b. 20 flies

25. $x = \log_3 8$

27. $\log_4 4096 = 3a$

29. $3^{-3} = \dfrac{1}{27}$

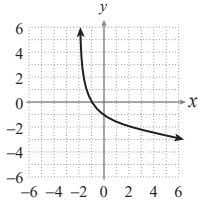
31.



Dom $= (-\infty, 0)$,

Ran $= (-\infty, \infty)$

33.



Dom $= (-2, \infty)$,

Ran $= (-\infty, \infty)$

35. -2

37. -3 **39.** $\dfrac{2}{3}$

41. $x = 27$

43. $x = \dfrac{17}{2}$

45. $x \approx 155.41$

47. $\dfrac{3}{2}\log x - \dfrac{5}{2}\log \pi - \log 2$

49. $3 + 3\log_3 a$

51. $\log_2 \left(\dfrac{a^{\frac{5}{3}}}{b^{\frac{1}{3}} c} \right)$

53. $\log_2 (x - 3)$

55. $\log_3 \left(\dfrac{x^2 - 2x}{x^2 + 4} \right)$

57. $\dfrac{x}{y^2}$ **59.** 6.18

61. 0

63. **a.** 251,188,643

b. 7,079,458

c. 1,258,925,412

65. $\dfrac{6}{\log 321} \approx 2.39$

67. $\dfrac{\ln 5}{4 - 3\ln 5} \approx -1.94$

69. $\dfrac{\log 12}{\log 18} \approx 0.86$

71. -2

73. 73.5

75. $f(x) = \ln(x^3)$

77. $f(x) = \log(x^{10})$

79. $f(x) = \log 7$

81. 20.4 months

(1.7 years)

Chapter 8: Trigonometric Functions

8.1 Exercises

1. 225°

3. −67.5°

5. 120°

7. 150°

9. −405°

11. $\frac{47\pi}{180}$

13. $\frac{11\pi}{15}$

15. $\frac{37\pi}{45}$

17. $\frac{8\pi}{3}$

19. $\frac{25\pi}{36}$

21. 270°

23. 540°

25. −72°

27. $\frac{\pi}{9}$

29. $-\frac{4\pi}{5}$

31. $\frac{\pi}{6}$

33. $\frac{83\pi}{90}$

35. 115°

37. 50°

39. 345°

41. 295°

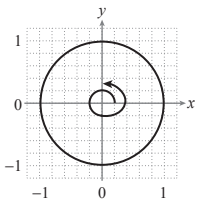
43. $\frac{5\pi}{4}$

45. $\frac{\pi}{4}$

47. $\frac{4\pi}{3}$

49. $\frac{13\pi}{4}$

51.



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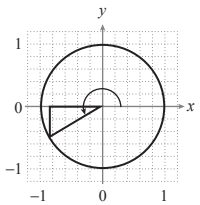
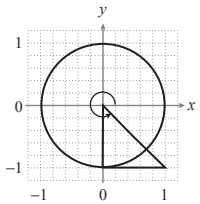


55.



57. 4 in.

59. 11.78 ft

61. 8.64 m

63. $\frac{9}{2}$

65. $\frac{21}{47}$

67. $\frac{45}{4}$

69. 1.48 inches

71. 3.56 feet

73. 1.83 feet

75. 1.45°

77. 445.79 miles

79. 1335.24 km

81. 7853.58 km

83. 1039.61 mph

85. 17.14 mph

87. a. 10π rad / s

b. 122.84 in./s

89. $\frac{5\pi}{12} \approx 1.31$

91. 29.93 cm^2

93. 15.08 m^2

95. 314.16 ft^2

97. 9.82 in.^{2}

99. 26.18 cm

101. 11.7 in.

8.2 Exercises

1. $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$, $\tan\theta = \cot\theta = 1$, $\csc\theta = \sec\theta = \sqrt{2}$

3. $\sin\theta = \frac{1}{3}$, $\cos\theta = \frac{2\sqrt{2}}{3}$, $\tan\theta = \frac{\sqrt{2}}{4}$, $\csc\theta = 3$, $\sec\theta = \frac{3\sqrt{2}}{4}$, $\cot\theta = 2\sqrt{2}$

5. $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$, $\tan\theta = \cot\theta = 1$, $\csc\theta = \sec\theta = \sqrt{2}$

7. $\sin\theta = \frac{5}{7}$, $\cos\theta = \frac{2\sqrt{6}}{7}$, $\tan\theta = \frac{5\sqrt{6}}{12}$, $\csc\theta = \frac{7}{5}$, $\sec\theta = \frac{7\sqrt{6}}{12}$, $\cot\theta = \frac{2\sqrt{6}}{5}$

9. $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = \frac{12}{5}$, $\csc\theta = \frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = \frac{5}{12}$

11. $\sin\theta = \frac{33}{65}$, $\cos\theta = \frac{56}{65}$, $\tan\theta = \frac{33}{56}$, $\csc\theta = \frac{65}{33}$, $\sec\theta = \frac{65}{56}$, $\cot\theta = \frac{56}{33}$

13. $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$, $\tan\theta = \frac{\sqrt{3}}{3}$, $\csc\theta = 2$, $\sec\theta = \frac{2\sqrt{3}}{3}$, $\cot\theta = \sqrt{3}$

15. $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, $\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$

17. $\sec 60° = 2$

19. $\csc\left(\frac{\pi}{6}\right) = 2$

21. $\sec 5° \approx 1.0038$, $\tan 5° \approx 0.0875$

23. $\cot\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$

25. $\tan(87.2°) \approx 20.4465$

27. 0.9945

29. 1.0355

31. 28.6537

33. 3.0777

35. 0.3827

37. 2

39. 38.9053°

41. 25.325°

43. 21.6656°

45. 0.1149

47. 0.7746

49. 5

51. True

53. True

55. False

57. 751.19 feet

59. 17.47 feet

61. 48.54 yards

63. 12.04 m

65. 314.57 feet

67. 13.86 feet

69. 6.86 feet

71. 20 m

73. 3196.80 feet

75. The two triangles are similar, meaning that the lengths of the sides of Belinda's triangle are scaled by some common factor k relative to the corresponding sides of Adam's triangle. So, for instance, Adam would find that $\sin\theta = \frac{a}{c}$ and Belinda would find that $\sin\theta = \frac{ka}{kc} = \frac{a}{c}$.

8.3 Exercises

1. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

3. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

5. $(-1, 0)$

7. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

9. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

11. $\frac{5\pi}{3} + 2n\pi, n \in \mathbb{Z}$

13. $\frac{7\pi}{6} + 2n\pi, n \in \mathbb{Z}$

15. $\sin\left(\frac{\pi}{2}\right) = \csc\left(\frac{\pi}{2}\right) = 1,$
$\cos\left(\frac{\pi}{2}\right) = \cot\left(\frac{\pi}{2}\right) = 0,$
$\tan\left(\frac{\pi}{2}\right) = \sec\left(\frac{\pi}{2}\right) = \text{undefined}$

17. $\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2},$
$\tan\left(\frac{3\pi}{4}\right) = \cot\left(\frac{3\pi}{4}\right) = -1,$
$\csc\left(\frac{3\pi}{4}\right) = \sqrt{2}, \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

19. $\sin(-520°) \approx -0.3420,$
$\cos(-520°) \approx -0.9397,$
$\tan(-520°) \approx -0.3640,$
$\csc(-520°) \approx -2.9238,$
$\sec(-520°) \approx -1.0642,$
$\cot(-520°) \approx -2.7475$

21. $\sin(-1105°) \approx -0.4226,$
$\cos(-1105°) \approx 0.9063,$
$\tan(-1105°) \approx -0.4663,$
$\csc(-1105°) \approx -2.3662,$
$\sec(-1105°) \approx 1.1034,$
$\cot(-1105°) \approx -2.1445$

23. $\sin 670° \approx -0.7660,$
$\cos 670° \approx 0.6428,$
$\tan 670° \approx -1.1918,$
$\csc 670° \approx -1.3054,$
$\sec 670° \approx 1.5557,$
$\cot 670° \approx -0.8391$

25. $\sin(-215°) \approx 0.5736,$
$\cos(-215°) \approx -0.8192,$
$\tan(-215°) \approx -0.7002,$
$\csc(-215°) \approx 1.7434,$
$\sec(-215°) \approx -1.2208,$
$\cot(-215°) \approx -1.4281$

27. $\sin 780° = \frac{\sqrt{3}}{2}, \cos 780° = \frac{1}{2},$
$\tan 780° = \sqrt{3}, \csc 780° = \frac{2\sqrt{3}}{3},$
$\sec 780° = 2, \cot 780° = \frac{\sqrt{3}}{3}$

29. $\theta' = 82°$

31. $\theta' = 60°$

33. $\theta' = \frac{\pi}{2}$

35. $\theta' = \frac{\pi}{6}$

37. $\theta' = \frac{\pi}{5}$

39. $\theta' = \frac{\pi}{2}$

41. $\theta' = \frac{\pi}{4}$

43. $\theta' = 75°$

45. IV

47. IV

49. I

51. II

53. a

55. a

57. b

59. c

61. c

63. $\sin\left(\frac{9\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

65. $\tan\left(\frac{5\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

67. $\sin 313° = -\sin 47° \approx -0.7314$

69. $\tan(-168°) = \tan 12° \approx 0.2126$

71. $\sin 216° = -\sin 36° \approx -0.5878$

73. $\cos(-330°) = \cos 30° = \frac{\sqrt{3}}{2}$

75. $\tan 718° = -\tan 2° \approx -0.0349$

77. b

79. e

81. $\cot 135° = \cos(-45°) = -1$

83. $\sin(-60°) = \cos 150° = -\frac{\sqrt{3}}{2}$

85. $\csc\left(\frac{5\pi}{6}\right) = \sec\left(-\frac{\pi}{3}\right) = 2$

87. $\cos\left(-\frac{3\pi}{6}\right) = \sin\pi = 0$

89. $\sin\left(-\frac{4\pi}{5}\right) = \cos\left(\frac{13\pi}{10}\right) \approx -0.5878$

91. $\csc\left(\frac{3\pi}{2}\right) = \sec(-\pi) = -1$

93. $\cot\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

95. $\sec 105° = \csc(-15°) \approx -3.8637$

97. $\tan\theta \approx -0.727, \cot\theta \approx -1.376$

99. $\tan\theta = 1.732, \cot\theta \approx 0.577$

101. $\tan\theta \approx 9.476, \cot\theta \approx 0.106$

103. $\csc\theta = \frac{125}{100} = \frac{5}{4}, \cot\theta = -\frac{3}{4}$

105. No such angle exists, as this would require the opposite side to be longer than the hypotenuse.

107. $\theta = -\dfrac{2\pi}{3}$, $\tan\theta = -\sqrt{3}$

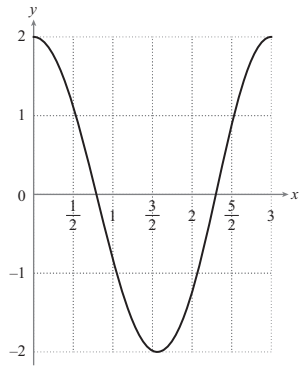
109. $\cos\left(-\dfrac{5\pi}{4}\right)$, -0.7071

111. $\tan\left(-\dfrac{\pi}{4}\right)$, -1

113. cot 75°, 0.2679

8.4 Exercises

1. Amplitude = 2, frequency = $\dfrac{1}{\pi}$



3. Amplitude = $\dfrac{1}{2}$,
frequency = $\dfrac{3}{2\pi}$

5. A = 2, P = 2π, no phase shift

7. A = 4, P = 2π, no phase shift

9. A = 1, P = 2π, no phase shift

11. A = 3, P = 2π,
shifted left 7 units

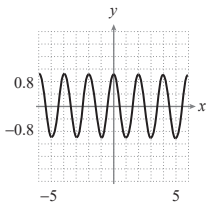
13. A = 2, P = π, no phase shift

15. A = $\dfrac{3}{2}$, P = 2, no phase shift

17. A = 0.5, P = $\dfrac{\pi}{4}$,
shifted left $\dfrac{1}{8}$ units

19. A = $\dfrac{8}{5}$, P = 1,
shifted left $\dfrac{2}{\pi}$ units

21.



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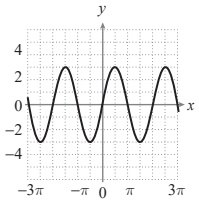

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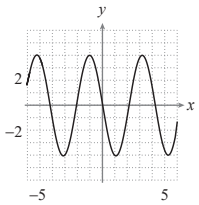

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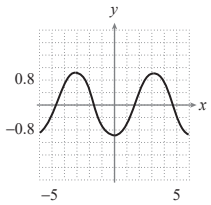

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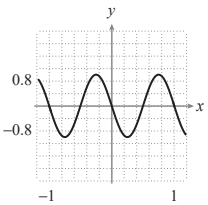

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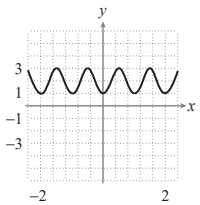

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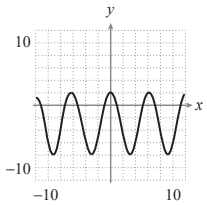

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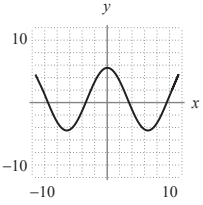

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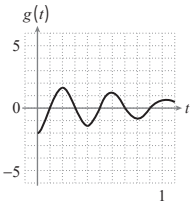

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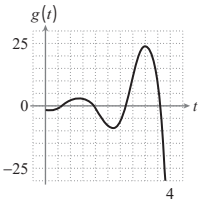

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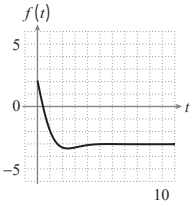

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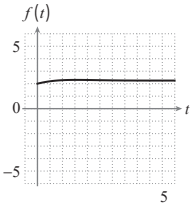


45. Frequency = 0.5 Hz

47. A = 3 inches,
P = 2 seconds,
$g(t) = 3\cos(\pi t)$

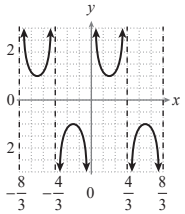
49. A = 1.5 ft,
P = $\dfrac{3}{5}$ seconds,
$g(t) = 1.5\cos\left(\dfrac{10\pi}{3}t\right)$

51. a. $\cos\left(x + \dfrac{3\pi}{2}\right)$

b. $\cos\left(x - \dfrac{(4n+1)\pi}{2}\right)$

8.5 Exercises

1.



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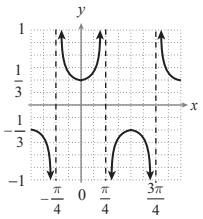

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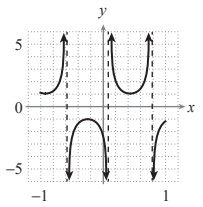

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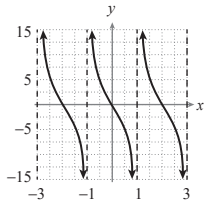

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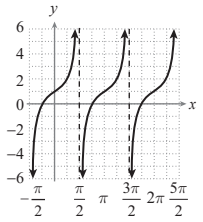

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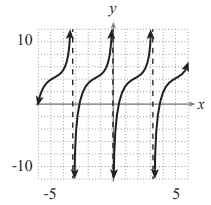

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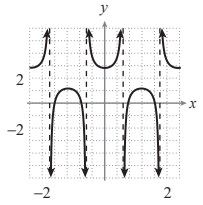


8.6 Exercises

1. $-\frac{\pi}{2}$

3. $\frac{\pi}{4}$

5. $\frac{\pi}{6}$

7. 0

9. $-\frac{\pi}{3}$

11. $\frac{\pi}{3}$

13. $-\frac{\pi}{4}$

15. $\frac{2\pi}{3}$

17. $-\frac{\pi}{6}$

19. $\frac{\pi}{2}$

21. $\frac{\pi}{4}$

23. $\frac{2\pi}{3}$

25. −0.2014

27. −1.1198

29. 1.9823

31. 0.2014

33. 0.0067

35. −0.1126

37. $\frac{\pi}{2}$

39. $\frac{1}{2}$

41. −0.8

43. 0.3714

45. 0.8944

47. 0.75

49. $\frac{\sqrt{3}}{2}$

51. 1

53. −1

55. $\frac{\sqrt{2}}{2}$

57. 1

59. $\frac{\sqrt{2}}{2}$

61. $-\sqrt{2}$

63. $\frac{\sqrt{2}}{2}$

65. −1

67. $\frac{\sqrt{1-x^2}}{x}$

69. $\sqrt{9x^2+1}$

71. $\left|\frac{1}{x}\right|\sqrt{x^2-1}$

73. $\theta = 47.88848865$

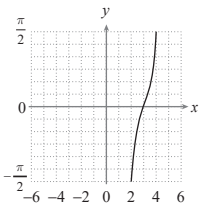
75. $\theta = 32.76975166$

77. $\theta = 60.10239082$

79. $\theta = 0.5736213262$

81. $\theta = -0.8480623625$

83.



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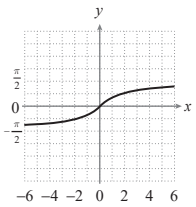


87. **a.** 0.2450

b. 0.7854

c. 0.9505

Chapter 8 Project

1. United Nations, NY: approx. 235.0°;
California Academy of Sciences: approx. 220.5°;
Smithsonian, Washington, DC: approx. 226.0°;
St. Isaac's Cathedral, Russia: approx. 311.4°;
Paris Observatory, France: approx. 270.9°

3. Approx. 14.92 ft

5. Approx. 12.6°

Chapter 8 Review Exercises

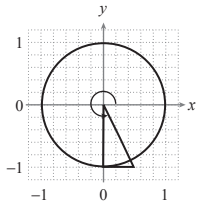
1. 4°

3. –315°

5. $\frac{7\pi}{30}$

7. $-\frac{79\pi}{180}$

9.



11. 15.71 ft

13. 30.72 meters

15. 1296.35 km

17. 3 rad

19. $108\pi\ \text{ft}^2$ or $339.29\ \text{ft}^2$

21. $\sin\theta = \frac{2\sqrt{5}}{5}, \cos\theta = \frac{\sqrt{5}}{5}, \tan\theta = 2, \csc\theta = \frac{\sqrt{5}}{2}, \sec\theta = \sqrt{5}, \cot\theta = \frac{1}{2}$

23. 4.0108

25. 0.2225

27. 15.2203°

29. False

31. 129.13 feet

33. $\sin(-460°) \approx -0.9848$,
$\cos(-460°) \approx -0.1736$,
$\tan(-460°) \approx 5.6713$,
$\csc(-460°) \approx -1.0154$,
$\sec(-460°) \approx -5.7588$,
$\cot(-460°) \approx 0.1763$

35. $\sin\left(\frac{7\pi}{3}\right)=\frac{\sqrt{3}}{2}, \cos\left(\frac{7\pi}{3}\right)=\frac{1}{2}$,
$\tan\left(\frac{7\pi}{3}\right)=\sqrt{3}, \csc\left(\frac{7\pi}{3}\right)=\frac{2\sqrt{3}}{3}$,
$\sec\left(\frac{7\pi}{3}\right)=2, \cot\left(\frac{7\pi}{3}\right)=\frac{\sqrt{3}}{3}$

37. $\theta' = 37°$

39. $\theta' = \frac{\pi}{4}$

41. III

43. $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

45. $\sin(-42°) = \cos 132°$
≈ -0.6691

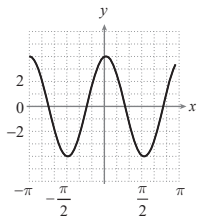
47. $\cos\left(\frac{5\pi}{4}\right) = \sin\left(-\frac{3\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

49. $\sin\theta = \frac{13}{5}$

51. A = 6, P = 2π, no phase shift

53. A = 9, P = π, shifted left π units

55.



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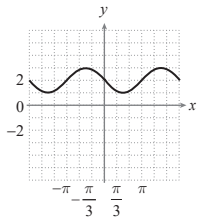

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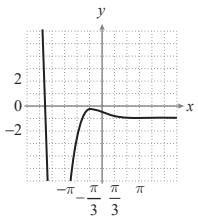

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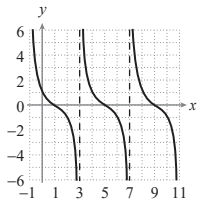

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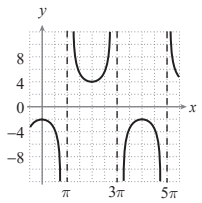


65. $\frac{\pi}{2}$

67. $\frac{\pi}{3}$

69. 0.4636 rad or 26.5651 deg

71. 0.9

73. $-\frac{\pi}{4}$

75. $\frac{\sqrt{3}}{3}$

77. 2

79. −1.336110366

81. 55.32339906

Chapter 9: Trigonometric Identities and Equations

9.1 Exercises

1. $\sec x$

3. $\sin t$

5. $\cos x - \sec x$

7. $\tan\alpha$

9. $\sec y$

11. $\tan^2 x$

13. $\sec\theta$

15. $3\sin\theta$

17. $8\cos\theta$

19. $5\sec\theta$

9.2 Exercises

1. $\frac{-\sqrt{6}+\sqrt{2}}{4}$

3. $-\sqrt{3}-2$

5. $-\sqrt{3}-2$

7. $\frac{\sqrt{6}+\sqrt{2}}{4}$

9. $\frac{\sqrt{2}+\sqrt{6}}{4}$

11. $\sqrt{3}+2$

13. $\frac{\sqrt{6}-\sqrt{2}}{4}$

15. $\sqrt{3}+2$

17. $\frac{-\sqrt{6}-\sqrt{2}}{4}$

19. $\sqrt{3}+2$

21. $\frac{\sqrt{6}+\sqrt{2}}{4}$

23. $\frac{\sqrt{6}-\sqrt{2}}{4}$

25. $\frac{\sqrt{6}+\sqrt{2}}{4}$

27. $\frac{56}{65}$

29. $\frac{36}{85}$

31. $\frac{4\sqrt{6}-\sqrt{21}}{25}$

33.
$\sin 45° = \frac{\sqrt{2}}{2}$

35.
$\tan 135° = -1$

37. $\tan\left(\frac{\pi}{4}\right) = 1$

39. $\sin 150° = \frac{1}{2}$

41. $\cos 135° = \frac{-\sqrt{2}}{2}$

43. $\tan 60° = \sqrt{3}$

45. $-\frac{56}{33}$

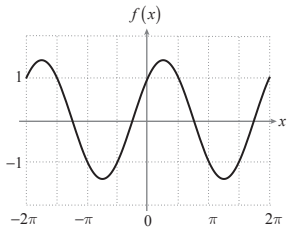
47. $\frac{4+3\sqrt{3}}{10}$

49. 1

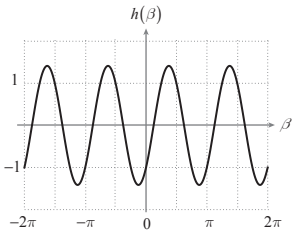
51. $\frac{2x^2+\sqrt{1-x^2}}{\sqrt{1+4x^2}}$

53. $x\left(\sqrt{1-4x^2} - 2\sqrt{1-x^2}\right)$

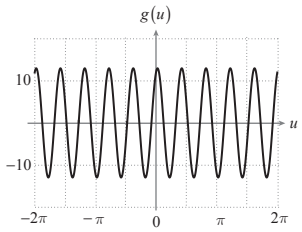
55. $\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$



57. $\sqrt{2}\sin\left(2\beta-\frac{\pi}{4}\right)$



59. $13\sin(5u+1.1760)$



73. True

75. False

79. True

81. True

83. False

9.3 Exercises

1. $\cos(2x)=\frac{7}{25}$;
$\sin(2x)=\frac{24}{25}$;
$\tan(2x)=\frac{24}{7}$

3. $\cos(2x)=\frac{1}{3}$;
$\sin(2x)=\frac{-2\sqrt{2}}{3}$;
$\tan(2x)=-2\sqrt{2}$

5. $\cos(2x)=\frac{1}{2}$;
$\sin(2x)=\frac{\sqrt{3}}{2}$;
$\tan(2x)=\sqrt{3}$

7. $\frac{\sin x-\sin x\cos(2x)}{2}$

9. $\frac{1-\cos(2x)-\cos(4x)}{16}+\frac{\cos(2x)\cos(4x)}{16}$

11. $\frac{3\sin x-4\sin x\cos(2x)}{3+4\cos(2x)+\cos(4x)}+\frac{\sin x\cos(4x)}{3+4\cos(2x)+\cos(4x)}$

13. $\frac{\sqrt{2+\sqrt{2}}}{2}$

15. $\frac{\sqrt{2+\sqrt{3}}}{2}$

17. $\frac{\sqrt{2+\sqrt{3}}}{2}$

19. $\frac{\sin(6x)+\sin 0}{2}$

21. $\frac{5(\sin 120°-\sin 90°)}{2}$

23. $\frac{\cos(2y)-\cos(2x)}{2}$

25. $\frac{\cos\left(\frac{7\pi}{12}\right)-\cos\left(\frac{23\pi}{12}\right)}{2}$

27. $\sin\left(\frac{\pi}{2}\right)-\sin\left(\frac{\pi}{6}\right)$

29. $2\sin(4x)\cos(2x)$

31. $-2\sin(2\beta)\sin\beta$

33. $2\cos 75°\sin 60°$

35. $-2\sin\left(\frac{17\pi}{24}\right)\sin\left(\frac{11\pi}{24}\right)$

47. $\cos(2x)=2\cos^2 x-1$
$\cos(3x)=4\cos^3 x-3\cos x$
$\cos(4x)=8\cos^4 x-8\cos^2 x+1$
$\sin(2x)=2\sin x\cos x$
$\sin(3x)=\sin x(4\cos^2 x-1)$
$\sin(4x)=\sin x(8\cos^3 x-4\cos x)$

9.4 Exercises

1. $x=\frac{7\pi}{6}+2n\pi, x=\frac{11\pi}{6}+2n\pi$

3. $x=\frac{\pi}{4}+2n\pi, x=\frac{7\pi}{4}+2n\pi$

5. $x=\frac{\pi}{6}+2n\pi, x=\frac{11\pi}{6}+2n\pi$

7. No solution

9. $x=\frac{2\pi}{3}+n\pi$

11. $x=n\pi$

13. $x=2n\pi$

15. No solution

17. $x=\frac{2\pi}{3}+2n\pi, x=\frac{4\pi}{3}+2n\pi$

19. $x=\frac{5\pi}{12}+\frac{n\pi}{2}$

21. $x=\frac{\pi}{6},\frac{5\pi}{6}$

23. $x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

25. $x=0,\pi,2\pi$

27. $x=\frac{2\pi}{3},\frac{4\pi}{3}$

29. $x=\tan^{-1}(-4+\sqrt{59})$,
$\tan^{-1}(-4+\sqrt{59})+\pi$,
$\tan^{-1}(-4-\sqrt{59})+\pi$,
$\tan^{-1}(-4-\sqrt{59})+2\pi$

41. True

43. True

45. True

47. $x=\frac{2\pi}{3}+n\pi$

49. True

51. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

53. $x = \frac{\pi}{2}, \pi$

55. $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

57. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

59. $x = 45°, 135°, 225°, 315°$

61. $x = 90°, 210°, 330°$

63. No solution

65. $x = 30°, 210°, 240°, 300°$

67. $s = -4, 3$

t has no solution.

69. $s = \frac{\left(1 \pm \sqrt{2}\right)}{2}$

$t = 1.7794, 4.5038$

71. 17.3°

73. 2.6779 and 5.8195

75. 0.6662, 2.4754, 3.8078, and 5.6169

77. 1.9948 and 5.8592

79. 0.6993, 2.6078, 3.6754, and 5.5839

81. $x = 10°, 50°, 130°, 170°, 250°, 290°$

83. They can have at most two real solutions.

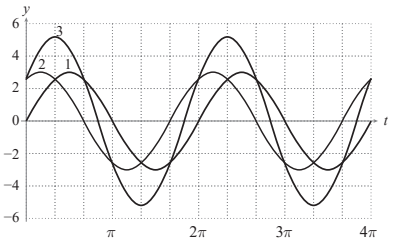
85. They may have as many as $2a$ solutions from 0° to 360°.

Chapter 9 Project

1. $y_1 + y_2 = 2\sqrt{2}\sin\left(kx - \omega t + \frac{\pi}{4}\right)$,

amplitude $= 2\sqrt{2}$

3.



We can suggest that the addition of two waves with equal amplitudes averages their displacements.

(Answers will vary.)

Chapter 9 Review Exercises

1. $\csc x$

3. $-\tan^2 y$

5. $-1 - \sin\theta$

13. $4\sec\theta$

15. $10\cot\theta$

17. $\frac{\sqrt{3}}{2}$

19. $\frac{\sqrt{2} - \sqrt{6}}{4}$

21. $\sqrt{3}$

23. $\frac{13}{84}$

25. $\sin 210°; -\frac{1}{2}$

29. $\frac{\sqrt{1-x^2} - x^2}{\sqrt{1+x^2}}$

31. $2\sin\left(x + \frac{7\pi}{4}\right)$

33. $\cos(2x) = \frac{-7}{25}, \sin(2x) = \frac{24}{25}$,

$\tan(2x) = \frac{-24}{7}$

37. $\frac{\sin x - \sin x\cos(4x)}{8}$

39. $2 + \sqrt{3}$

41. $2 - \sqrt{3}$

43. $\frac{\sin(2x) - \sin(2y)}{2}$

45. $\frac{1}{2}(\sin 180° + \sin 150°)$

47. $2\cos(4\alpha)\sin\alpha$

49. $-2\sin\left(\frac{17\pi}{24}\right)\sin\left(\frac{\pi}{24}\right)$

51. $x = \frac{\pi}{6} + 2n\pi, x = -\frac{\pi}{6} + 2n\pi$

53. $x = \frac{\pi}{2} + 2n\pi, x = \frac{-\pi}{2} + 2n\pi$

55. $x = \frac{\pi}{3} + 2n\pi, x = \frac{5\pi}{3} + 2n\pi$,

$x = \pi + 2n\pi$

57. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

59. True

61. $x = \frac{\pi}{3} + 2n\pi, x = \frac{2\pi}{3} + 2n\pi$

63. $x = 0°, 180°$

Chapter 10: Additional Topics in Trigonometry

10.1 Exercises

1. $C = 105°, b \approx 4.2426, c \approx 5.7956$

3. $C = 60°, a \approx 4.9067, c \approx 4.5221$

5. $A = 80°, a \approx 3.9392, b \approx 3.7588$

7. $C = 150°, b \approx 1.0154, c \approx 2.9238$

9. No triangle

11. $A \approx 20.7048°, B \approx 114.2952°, b \approx 5.1559$

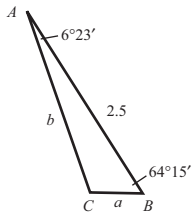
13. $A \approx 35.9044°, B \approx 4.0956°, a \approx 8.2110$

15. $A \approx 19.1638°, C \approx 80.8362°, c \approx 6.0148$

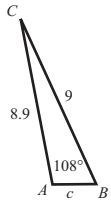
17. $A \approx 6.7456°, C \approx 63.2544°, c \approx 7.6026$

19. $h \approx 6.02$, $c < h$, so a triangle cannot be created.

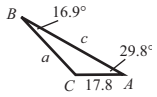
21. $C = 109°22'$, $a \approx 0.29$, $b \approx 2.39$



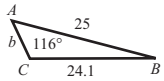
23. $c \approx 0.31$, $B \approx 70.13°$, $C \approx 1.87°$



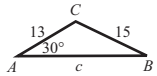
25. $C = 133.4°$, $a \approx 30.34$, $c \approx 44.49$



27. $b \approx 1.92$, $A \approx 60.05°$, $B \approx 3.95°$



29. $c \approx 24.78$, $C \approx 124.32°$, $B \approx 25.68°$



31. 94.34

33. 9.74

35. 18.47

37. 335.18

39. 21 feet

41. 442.3 feet

43. 9.4 feet

45. 26.3 feet

47. 32.6 feet, 26.5 feet

49. 105.5 miles

51. 3.5 feet

53. 178.3882 ft^2

10.2 Exercises

1. $a = \sqrt{37}$, $B \approx 25.2850°$, $C \approx 94.7150°$

3. $b \approx 4.5985$, $A \approx 41.7854°$, $C \approx 88.2146°$

5. $c \approx 4.1063$, $A \approx 103.0643°$, $B \approx 46.9357°$

7. $c \approx 7.0752$, $A \approx 41.6113°$, $B \approx 68.3887°$

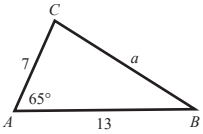
9. $A \approx 46.5675°$, $B \approx 104.4775°$, $C \approx 28.9550°$

11. $A \approx 121.8554°$, $B \approx 39.5712°$, $C \approx 18.5734°$

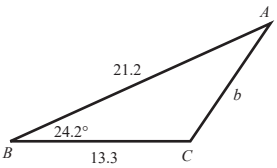
13. $A = B = C = 60°$

15. $A = 90°$, $B \approx 36.8699°$, $C \approx 53.1301°$

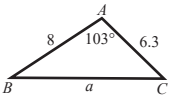
17. $a \approx 11.88$, $B \approx 32.28°$, $C \approx 82.72°$



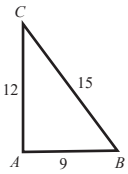
19. $b \approx 10.58$, $A \approx 31.01°$, $C \approx 124.79°$



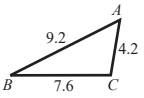
21. $a \approx 11.24$, $B \approx 33.1°$, $C \approx 43.9°$



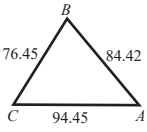
23. $A \approx 90°$, $B \approx 53.13°$, $C \approx 36.87°$



25. $A \approx 54.82°$, $B \approx 26.85°$, $C \approx 98.33°$



27. $A \approx 50.22°$, $B \approx 71.71°$, $C \approx 58.07°$



29. 89.29

31. 11.15

33. 65.5744 feet

35. 2.4×10^7 miles

37. 21.4413 feet

39. 22.6199°, 67.3801°

41. 160.1188°

43. 337,940.9589 square nautical miles

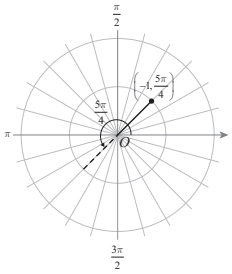
45. **a.** 61.9372 in.^{2}

b. 584.2397 in.^{2}

c. 136.1041 in.^{2}

10.3 Exercises

1.



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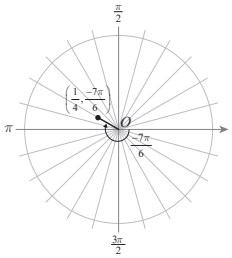

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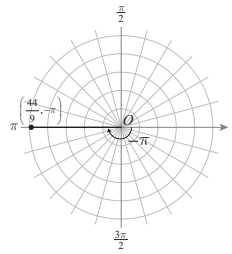


7. $(3.54, -3.54)$

9. $(-4.42, -4.42)$

11. $(-2.60, -1.50)$

13. $(-3, 0)$ and $(3, \pi)$

15. $\left(\sqrt{145}, -0.08\right)$ and $\left(-\sqrt{145}, 3.06\right)$

17. $\left(2\sqrt{21}, 1.76\right)$ and $\left(-2\sqrt{21}, -1.38\right)$

19. $r^2 = 25$

21. $r\cos\theta = 12$

23. $\sin\theta = \cos\theta$

25. $r\cos\theta = 16a$

27. $r^2 - 4ar\cos\theta = 0$

29. $r^2\sin^2\theta - 4r\cos\theta - 4 = 0$

31. $x^2 + y^2 = 5x$

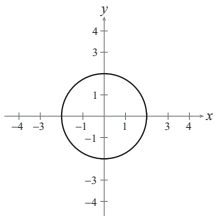
33. $x^2 + y^2 = 49$

35. $y = \dfrac{1}{2}$

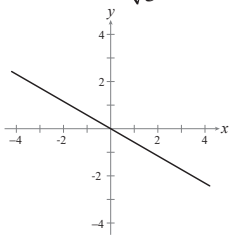
37. $x^4 + y^4 + 2x^2y^2 = 2xy$

39. $4y + 7x = 12$

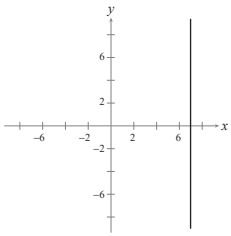
41. $x^2 + y^2 = 4$



43. $y = -\dfrac{x}{\sqrt{3}}$



45. $x = 7$



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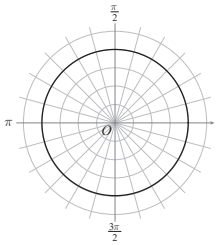

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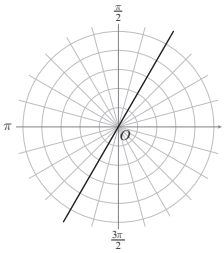

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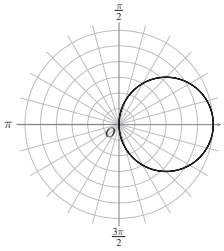

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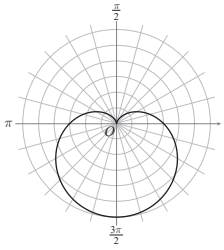

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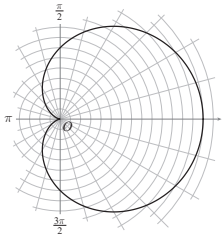

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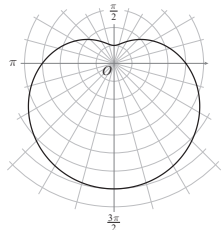
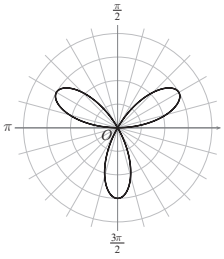


59.



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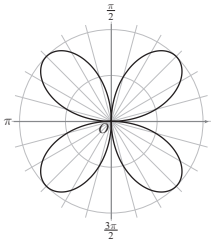

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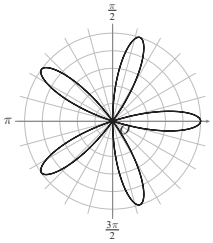
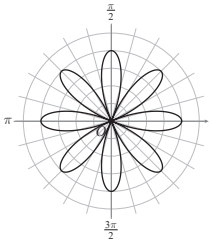
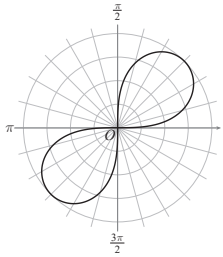


65.



67.



69. $(0,0), \left(\dfrac{\sqrt{2}}{2}, \dfrac{\pi}{4}\right)$

71. $(0,0), \left(1+\dfrac{\sqrt{2}}{2}, \dfrac{3\pi}{4}\right),$

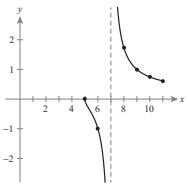
$\left(1-\dfrac{\sqrt{2}}{2}, -\dfrac{\pi}{4}\right)$

73. The graph of $f(\theta - a)$ is that of $f(\theta)$ rotated about the origin by a radians.

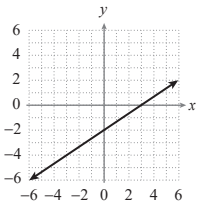
10.4 Exercises

1.

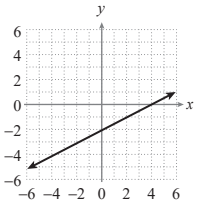
t	x	y
0	5	0
1	6	1
2	7	undefined
3	8	$\sqrt{3}$
4	9	1
5	10	$\frac{\sqrt{5}}{3}$
6	11	$\frac{\sqrt{6}}{4}$



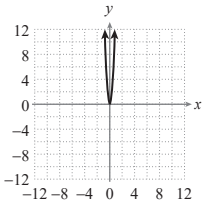
3. $2x = 3y + 6$



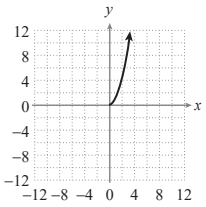
5. $x = 2y + 4$



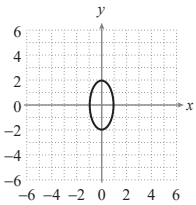
7. $y = 16x^2$



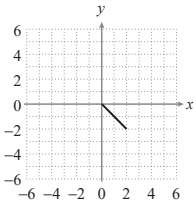
9. $x = \sqrt{y}$



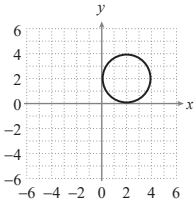
11. $y = \pm 2\sqrt{1 - x^2}$



13. $y = -x,\ 0 \le x \le 2$



15. $y = \pm\sqrt{4x - x^2} + 2$



17. $x = t,\ y = t^2 + 2t + 1$

19. $x = t,\ y = -t^2 - 5$

21. $x = t,\ y = \pm\sqrt{t - 4}$

23. $x = t,\ y = \dfrac{1}{t}$

25. $x = t,\ y = |t - 1|$

27. $x = t,\ y = \pm\sqrt{1 - t^2}$

29. $x = t,\ y = t^2 - t - 6$

31. $x = t,\ y = \dfrac{1}{4}t + \dfrac{19}{2}$

33. $x = t,\ y = \dfrac{4}{7}t$

35. $x = t,\ y = \dfrac{1}{2}t - 3$

37. $x = t - 5,\ y = 2 - |t - 5|$

39. $x = 4t,\ y = -2t + \dfrac{5}{2}$

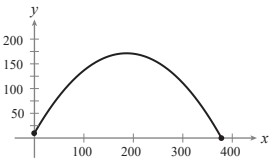
41. $x = 5t + 1,$
$y = 25t^2 - 10t + 1$

43. $x = -4 + 3\cos\theta,$
$y = 2 + 3\sin\theta$

45. $x = 6\cos\theta,$
$y = -2 + 6\sin\theta$

47. a. $x \approx 58.67t,$
$y \approx -16t^2 + 101.61t + 10$

b.



c. Approx. 126.42 ft

d. Approx. 378.42 ft

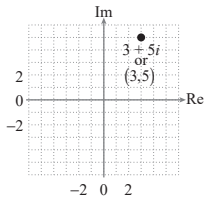
e. $t \approx 6.45$ s

f. Yes

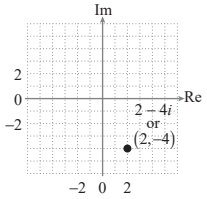
49. $x = 12(\theta - \sin\theta),$
$y = 12(1 - \cos\theta)$

10.5 Exercises

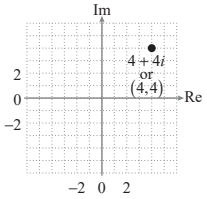
1. $\sqrt{34}$



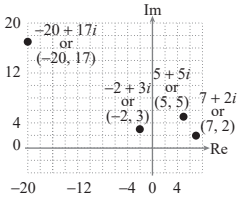
3. $\sqrt{20} = 2\sqrt{5}$



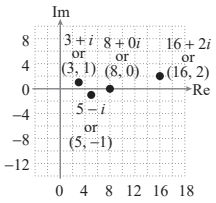
5. $\sqrt{32} = 4\sqrt{2}$



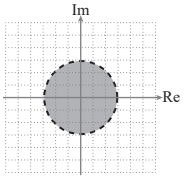
7.



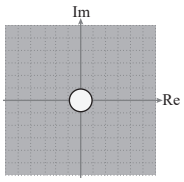
9.



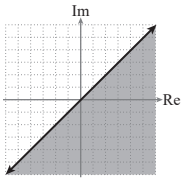
11.



13.



15.



17. $\sqrt{10}\left(\cos\left(3.46\right)+i\sin\left(3.46\right)\right)$

19. $\sqrt{5}\left(\cos\left(1.11\right)+i\sin\left(1.11\right)\right)$

21. $2\sqrt{5}\left(\cos\left(0.46\right)+i\sin\left(0.46\right)\right)$

23. $2\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$

25. $5\left(\cos\left(0.93\right)+i\sin\left(0.93\right)\right)$

27. $8\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)$

29. $\frac{-3\sqrt{3}}{2}+\frac{3i}{2}$

31. $-1-i\sqrt{3}$

33. $-\frac{5}{\sqrt{2}}+\frac{5i}{\sqrt{2}}$

35. $\frac{-3\sqrt{3}}{4}+\frac{3i}{4}$

37. $1.01+4.9i$

39. $16\left(\cos 30°+i\sin 30°\right)=8\sqrt{3}+8i$

41. $3\sqrt{6}\left(\cos\left(\frac{17\pi}{12}\right)+i\sin\left(\frac{17\pi}{12}\right)\right)$
$=-1.9-7.1i$

43. $2\sqrt{10}\left(\cos\left(2.42\right)+i\sin\left(2.42\right)\right)$
$=\left(-3-\sqrt{3}\right)+\left(3\sqrt{3}-1\right)i$

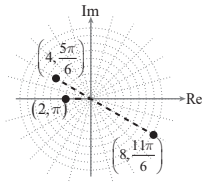
45. $2\left(\cos 180°+i\sin 180°\right)=-2$

47. $\frac{10}{3}\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)=\frac{10i}{3}$

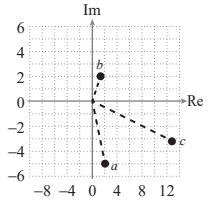
49. $\frac{1}{\sqrt{2}}\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)$
$=-\frac{1}{2}-\frac{i}{2}$

51. $2\left(\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right)$
$=0.52+1.93i$

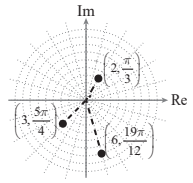
53. $a=4\left(\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right)$,
$b=2\left(\cos\pi+i\sin\pi\right)$,
$c=8\left(\cos\left(\frac{11\pi}{6}\right)+i\sin\left(\frac{11\pi}{6}\right)\right)$



55. $a=2-5i, b=\sqrt{2}+2i,$
$c=10+2\sqrt{2}+i\left(4-5\sqrt{2}\right)$



57. $a=2e^{\frac{\pi}{3}i}, b=3e^{\frac{5\pi}{4}i}, c=6e^{\frac{19\pi}{12}i}$

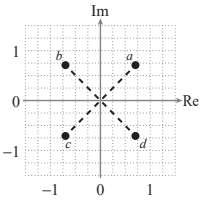


59. $32e^{\frac{\pi}{3}i}$

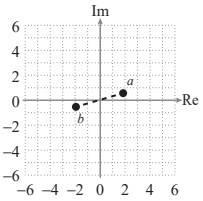
61. $1.04\times10^{13}e^{2.9i}$

63. $e^{2\pi i}$

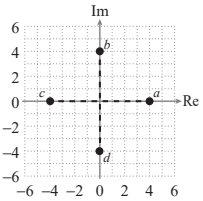
65. $a=e^{\frac{\pi i}{4}}, b=e^{\frac{3\pi i}{4}}, c=e^{\frac{5\pi i}{4}}, d=e^{\frac{7\pi i}{4}}$



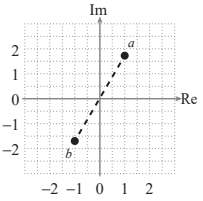
67. $a=2e^{\frac{\pi i}{12}}, b=2e^{\frac{13\pi i}{12}}$



69. $a=4, b=4e^{\frac{\pi i}{2}}, c=4e^{\pi i}, d=4e^{\frac{3\pi i}{2}}$



71. $a=2e^{60°i}, b=2e^{240°i}$



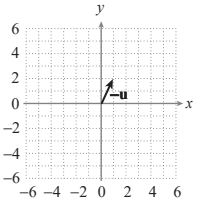
73. $2\sqrt{2}e^{\frac{\pi i}{12}}, 2\sqrt{2}e^{\frac{13\pi i}{12}}$

75. $2e^{\frac{\pi i}{5}}, 2e^{\frac{3\pi i}{5}}, 2e^{\pi i}, 2e^{\frac{7\pi i}{5}}, 2e^{\frac{9\pi i}{5}}$

77. $5e^{\frac{\pi i}{4}}, 5e^{\frac{3\pi i}{4}}$

10.6 Exercises

1.



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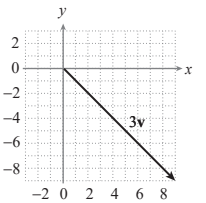

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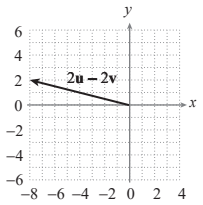


7. $\mathbf{v} = \langle 3, -3 \rangle, \|\mathbf{v}\| = 3\sqrt{2}$

9. $\mathbf{v} = \langle 5, 3 \rangle, \|\mathbf{v}\| = \sqrt{34}$

11. $\mathbf{v} = \langle 5, -1 \rangle, \|\mathbf{v}\| = \sqrt{26}$

13. $\mathbf{v} = \langle -7, 7 \rangle, \|\mathbf{v}\| = 7\sqrt{2}$

15. $\mathbf{v} = \langle -4, -6 \rangle, \|\mathbf{v}\| = 2\sqrt{13}$

17. a. $\langle -2, 8 \rangle$ **b.** $\langle 8, -4 \rangle$
c. $\langle -4, 0 \rangle$

19. a. $\langle 1, 4 \rangle$ **b.** $\langle -11, 12 \rangle$
c. $\langle 6, -8 \rangle$

21. a. $\langle -5, -10 \rangle$
b. $\langle -8, -2 \rangle$
c. $\langle 6, 4 \rangle$

23. $-\mathbf{u} = \langle -1, -1 \rangle$,
$2\mathbf{u} - \mathbf{v} = \langle -1, 5 \rangle$,
$\mathbf{u} + \mathbf{v} = \langle 4, -2 \rangle$,
$\|\mathbf{u}\| = \sqrt{2}$,
$\|\mathbf{v}\| = 3\sqrt{2}$

25. $-\mathbf{u} = \langle 4, -4 \rangle$,
$2\mathbf{u} - \mathbf{v} = \langle -12, 12 \rangle$,
$\mathbf{u} + \mathbf{v} = \langle 0, 0 \rangle$,
$\|\mathbf{u}\| = 4\sqrt{2}$,
$\|\mathbf{v}\| = 4\sqrt{2}$

27. a. $\left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$
b. $\mathbf{u} = 6\mathbf{i} - 3\mathbf{j}$

29. a. $\left\langle \frac{-5}{\sqrt{26}}, \frac{1}{\sqrt{26}} \right\rangle$
b. $\mathbf{u} = 5\mathbf{i} - \mathbf{j}$

31. a. $\left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$
b. $\mathbf{u} = 3\mathbf{i} + 3\mathbf{j}$

33. $\|\mathbf{v}\| = 5, \theta = 30°$

35. $\|\mathbf{v}\| = 5, \theta = 36.9°$

37. $\langle 3\sqrt{3}, 3 \rangle$

39. $\langle -9\sqrt{2}, 9\sqrt{2} \rangle$

41. $\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

43. $\left\langle \frac{8}{\sqrt{13}}, \frac{12}{\sqrt{13}} \right\rangle$

45. $\langle 2\sqrt{3}, 2 \rangle$

47. 38.67 mph, N 77.76° W

49. $\mathbf{F} = \langle 150, -1235 \rangle$,
$|\mathbf{F}| = 1244.08$ pounds

10.7 Exercises

1. 17
3. 6
5. 8
7. 1
9. −7
11. 32
13. $\langle -26, 39 \rangle$
15. $\sqrt{37}$
17. $\sqrt{53}$
19. 123.7°
21. 14.0°
23. 161.6°
25. $\frac{\pi}{4}$
27. 8°, 69°, 103°
29. 57.1°, 60.8°, 62.1°
31. −62.5
33. $-32\sqrt{2}$
35. $\langle 1, 1 \rangle, \langle 5, 5 \rangle$
37. $\langle 3, 1 \rangle, \langle -6, -2 \rangle$
39. Neither
41. Orthogonal

43. $\text{proj}_{\mathbf{v}}\mathbf{u} = \langle 2, 1 \rangle$,
$\text{perp}_{\mathbf{v}}\mathbf{u} = \langle -1, 2 \rangle$

45. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{6}{5}, \frac{2}{5} \right\rangle$,
$\text{perp}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{9}{5}, -\frac{27}{5} \right\rangle$

47. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle -\frac{60}{17}, -\frac{15}{17} \right\rangle$,
$\text{perp}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{9}{17}, -\frac{36}{17} \right\rangle$

49. 14
51. 3
53. 3479.3 pounds
55. 109.6 pounds
57. 1638.3 ft-lb
59. 11,431,535.3 ft-lb

10.8 Exercises

1. 0
3. 1
5. $\frac{3}{5}$
7. −1.04
9. 74.21
11. Even
13. Odd
15. Odd
37. 0
45. −0.33
47. 0.69
49. 0.35

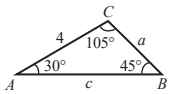
Chapter 10 Project

1. Approx. 921,479 ft^2

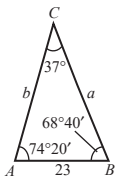
3. a. Approx. 8414 pounds
b. Approx. 1298.4 pounds
c. Approx. 2045.6 feet
d. 2,822,928 foot-pounds

Chapter 10 Review Exercises

1. $C = 105°$, $c \approx 5.46$, $a \approx 2.83$

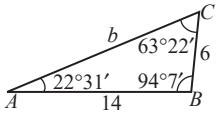


3. $B = 68°40'$, $a \approx 36.80$, $b \approx 35.60$

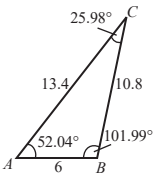


5. 10.14

7. $A \approx 22°31'$, $C \approx 63°22'$, $b \approx 15.62$



9. $A \approx 52.04°$, $B \approx 101.99°$, $C \approx 25.98°$



11. 22.19

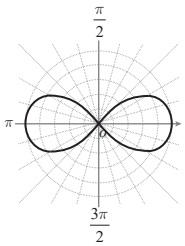
13. $(-6.06, -3.5)$

15. $(15.62, 0.88)$ and $(-15.62, -2.26)$

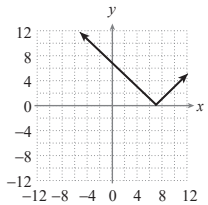
17. $r^2 - 9ar\cos\theta = 0$

19. $x + y = 4$

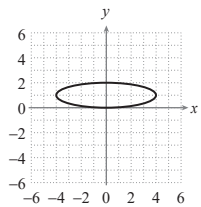
21.



23. $y = |x - 7|$

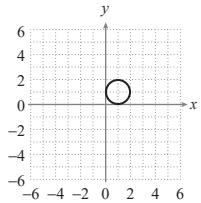


25. $\frac{x^2}{16} + (y-1)^2 = 1$

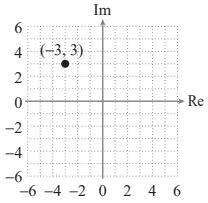


27. $x = t, y = 2 - 6t$

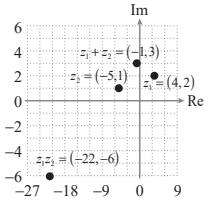
29. $x = 1 + \cos\theta, y = 1 + \sin\theta$



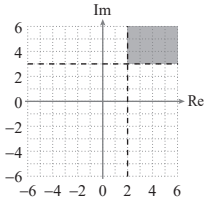
31. $3\sqrt{2}$



33.



35.



37. $\sqrt{17}\left(\cos(1.33) + i\sin(1.33)\right)$

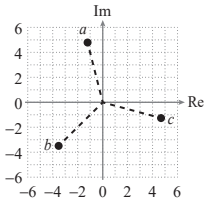
39. $\frac{3}{2} + \frac{3i\sqrt{3}}{2}$

41. $5(\cos 120° + i\sin 120°)$, $-\frac{5}{2} + \frac{5i\sqrt{3}}{2}$

43. $24(\cos 315° + i\sin 315°)$, $12\sqrt{2} - 12i\sqrt{2}$

45. $177{,}147e^{120°i}$

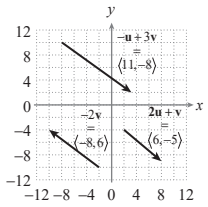
47. $a = 5e^{i\left(\frac{7\pi}{12}\right)}, b = 5e^{i\left(\frac{5\pi}{4}\right)}, c = 5e^{i\left(\frac{23\pi}{12}\right)}$



49. $2e^{i\left(\frac{\pi}{4}\right)}, 2e^{i\left(\frac{11\pi}{12}\right)}, 2e^{i\left(\frac{19\pi}{12}\right)}$

51. $\mathbf{v} = \langle -10, -6\rangle, \|\mathbf{v}\| = 2\sqrt{34}$

53.



55. $-\mathbf{u} = \langle -5, -1\rangle, 2\mathbf{u} - \mathbf{v} = \langle 7, 1\rangle$, $\mathbf{u} + \mathbf{v} = \langle 8, 2\rangle, \|\mathbf{u}\| = \sqrt{26}$, $\|\mathbf{v}\| = \sqrt{10}$

57. a. $\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$

b. $6\mathbf{i} + 3\mathbf{j}$

59. $\|\mathbf{v}\| = \sqrt{26}, \theta = -11.3°$

61. $\langle 3.6, -4.8\rangle$

63. 40.01 mph, N 40.23° W

65. $\langle -108, -270\rangle$

67. $\sqrt{10}$

69. $\frac{5\pi}{12}$

71. −36

73. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{3}{2}, \frac{3}{2} \right\rangle$, $\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{5}{2}, \frac{-5}{2} \right\rangle$

75. 31

77. 417,558.5 ft-lb

79. $\frac{15}{8}$

81. 1.31

Chapter 11: Conic Sections

11.1 Exercises

1. Center: $(5,2)$
Foci: $\left(5,2\pm\sqrt{21}\right)$
Vertices: $(5,7),(5,-3)$

3. Center: $(-2,-5)$
Foci: $\left(-2\pm\sqrt{6},-5\right)$
Vertices: $(1,-5),(-5,-5)$

5. Center: $(-3,2)$
Foci: $\left(-3\pm\sqrt{2},2\right)$
Vertices: $(-1,2),(-5,2)$

7. Center: $(-5,1)$
Foci: $\left(-5,1\pm2\sqrt{3}\right)$
Vertices: $(-5,5),(-5,-3)$

9. Center: $(-4,2)$
Foci: $\left(-4\pm3\sqrt{2},2\right)$
Vertices: $\left(-4\pm3\sqrt{3},2\right)$

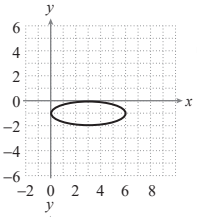
11. Center: $(2,0)$
Foci: $(4,0),(0,0)$
Vertices: $\left(2\pm\sqrt{5},0\right)$

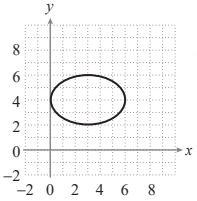
13. e

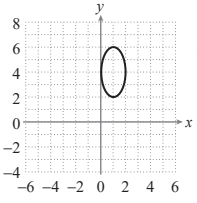
15. f

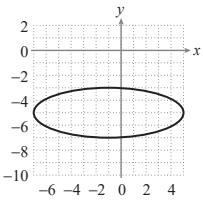
17. c

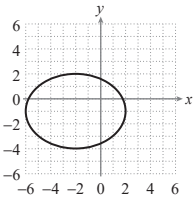
19. b

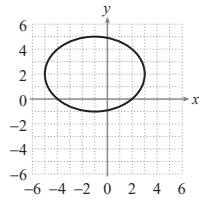
21.  $\left(3\pm2\sqrt{2},-1\right)$

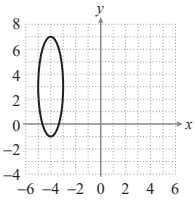
23.  $\left(3\pm\sqrt{5},4\right)$

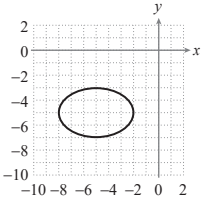
25.  $\left(1,4\pm\sqrt{3}\right)$

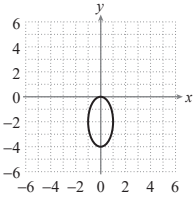
27.  $\left(-1\pm\sqrt{21},-5\right)$

29.  $\left(-2\pm\sqrt{7},-1\right)$

31.  $\left(-1\pm\sqrt{7},2\right)$

33.  $\left(-5,3\pm\sqrt{15}\right)$

35.  $\left(-5\pm\sqrt{5},-5\right)$

37.  $\left(0,-2\pm\sqrt{3}\right)$

39. $\dfrac{x^2}{16}+\dfrac{y^2}{25}=1$

41. $(x-1)^2+\dfrac{(y-1)^2}{9}=1$

43. $\dfrac{(x-3)^2}{36}+\dfrac{y^2}{27}=1$

45. $(x+2)^2+\dfrac{(y+3)^2}{4}=1$

47. $\dfrac{(x-5)^2}{16}+\dfrac{(y-3)^2}{15}=1$

49. $\dfrac{(x-2)^2}{4}+\dfrac{(y+2)^2}{9}=1$

51. $\dfrac{(x-1)^2}{9}+\dfrac{y^2}{16}=1$

53. $e=\dfrac{\sqrt{11}}{6}$
major $=24$
minor $=20$

55. $e=\dfrac{2\sqrt{2}}{3}$
major $=12$
minor $=4$

57. $e=\dfrac{\sqrt{3}}{2}$
major $=4$
minor $=2$

59. $e=\dfrac{\sqrt{2}}{2}$
major $=4$
minor $=2\sqrt{2}$

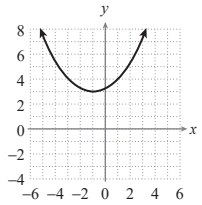
61. $e=\dfrac{\sqrt{42}}{7}$
major $=14$
minor $=2\sqrt{7}$

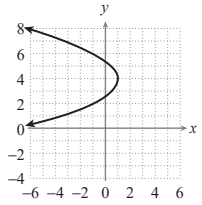
63. $e\approx0.249$

65. Yes, just barely, if the boat is centered on the river.

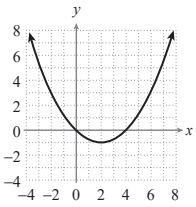
67. The string should be 5 cm long, and the tacks should be 4 cm apart.

11.2 Exercises

1. 
$(-1,4),y=2$

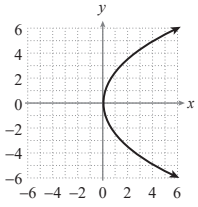
3. 
$\left(\dfrac{1}{2},4\right),x=\dfrac{3}{2}$

5.



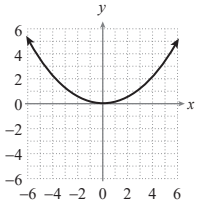
$(2,0), y=-2$

7.



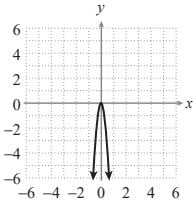
$\left(\frac{3}{2},0\right), x=-\frac{3}{2}$

9.



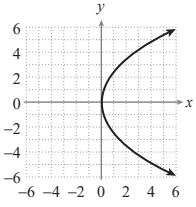
$\left(0,\frac{7}{4}\right), y=-\frac{7}{4}$

11.



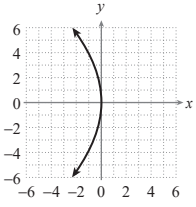
$\left(0,-\frac{1}{48}\right), y=\frac{1}{48}$

13.



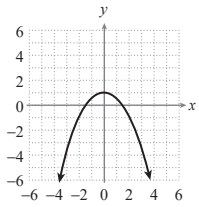
$\left(\frac{3}{2},0\right), x=-\frac{3}{2}$

15.



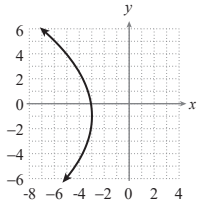
$(-4,0), x=4$

17.



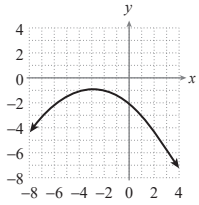
$\left(0,\frac{1}{2}\right), y=\frac{3}{2}$

19.



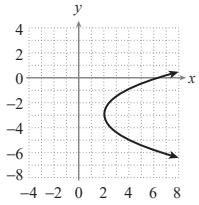
$(-6,-1), x=0$

21.



$(-3,-3), y=1$

23.



$\left(\frac{5}{2},-3\right), x=\frac{3}{2}$

25. g **29.** e

27. b **31.** d

33. $(y-1)^2=-4(x+1)$

35. $(x-3)^2=8(y+1)$

37. $(y+2)^2=24(x-3)$

39. $(x+3)^2=-2(y+1)$

41. $(y-3)^2=10(x+4)$

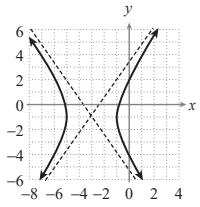
43. $(y+1)^2=-8(x-2)$

45. 2 feet

47. 1.5 inches

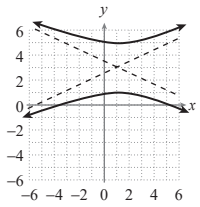
11.3 Exercises

1.



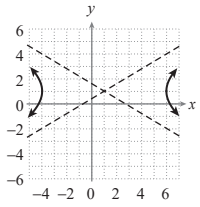
$\left(-3\pm\sqrt{13},-1\right)$

3.



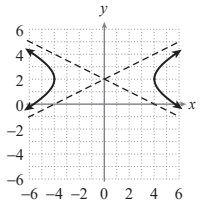
$\left(1,3\pm2\sqrt{5}\right)$

5.



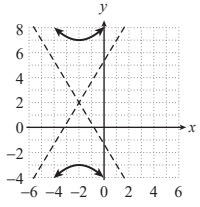
$\left(1\pm\sqrt{34},1\right)$

7.



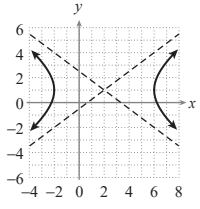
$\left(\pm2\sqrt{5},2\right)$

9.



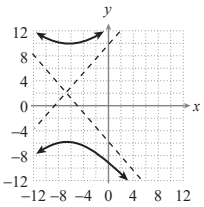
$\left(-2,2\pm\sqrt{34}\right)$

11.



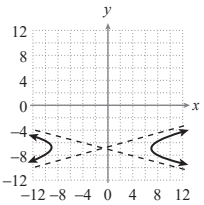
$(7,1),(-3,1)$

13.



$\left(-7, 2 \pm \sqrt{113}\right)$

15.



$\left(-1 \pm 2\sqrt{17}, -7\right)$

17. Center: $(-3, 2)$

Foci: $\left(-3 \pm \sqrt{13}, 2\right)$

Vertices: $(-1, 2), (-5, 2)$

19. Center: $(1, -4)$

Foci: $\left(1 \pm 2\sqrt{3}, -4\right)$

Vertices: $\left(1 \pm \sqrt{3}, -4\right)$

21. Center: $(-2, 1)$

Foci: $\left(-2 \pm \sqrt{30}, 1\right)$

Vertices: $(3, 1), (-7, 1)$

23. Center: $(-3, -1)$

Foci: $\left(-3 \pm 2\sqrt{3}, -1\right)$

Vertices: $(-1, -1), (-5, -1)$

25. Center: $(1, 0)$

Foci: $\left(1 \pm \dfrac{\sqrt{5}}{2}, 0\right)$

Vertices: $(2, 0), (0, 0)$

27. Center: $(8, 5)$

Foci: $\left(8 \pm 4\sqrt{5}, 5\right)$

Vertices: $(12, 5), (4, 5)$

29. a **33.** g

31. b **35.** h

37. $\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$

39. $y^2 - 4x^2 = 1$

41. $\dfrac{5x^2}{2} - \dfrac{5(y-4)^2}{18} = 1$

43. $\dfrac{(x-6)^2}{9} - \dfrac{(y-5)^2}{7} = 1$

45. $\dfrac{(x+4)^2}{4} - \dfrac{(y-3)^2}{16} = 1$

47. $\dfrac{(x-1)^2}{9} - \dfrac{(y+1)^2}{4} = 1$

49. $\dfrac{(y+4)^2}{16} - \dfrac{(x-3)^2}{25} = 1$

51. $\dfrac{x^2}{\left(6 \times 10^7\right)^2} - \dfrac{y^2}{\left(7.2 \times 10^7\right)^2} = 1$

11.4 Exercises

1. $\left(4\sqrt{3} + 3, -4 + 3\sqrt{3}\right)$

3. $\left(\dfrac{-5\sqrt{2}}{16}, \dfrac{3\sqrt{2}}{16}\right)$

5. $(-1.2097, -13.5476)$

7. $(3.3485, -1.5002)$

9. Ellipse

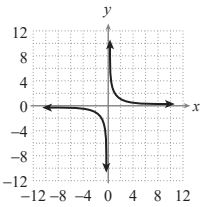
11. Hyperbola

13. Hyperbola

15. Ellipse

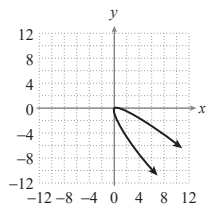
17. Hyperbola

$\theta = \dfrac{\pi}{4}, x'^2 - y'^2 = 4$



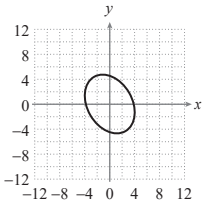
19. Parabola

$\theta = \dfrac{\pi}{4}, y' = -\sqrt{2}x'^2$



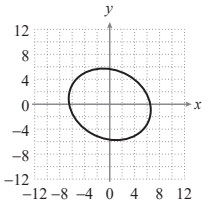
21. Ellipse

$\theta = \dfrac{\pi}{6}, \dfrac{x'^2}{13} + \dfrac{y'^2}{25} = 1$

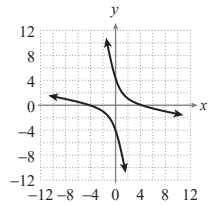


23. Ellipse

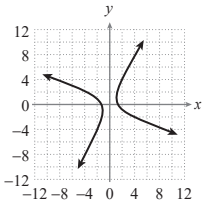
$\theta = \dfrac{\pi}{3}, \dfrac{x'^2}{30} + \dfrac{y'^2}{46} = 1$



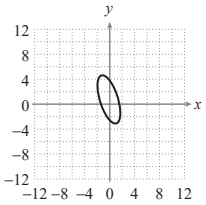
25.



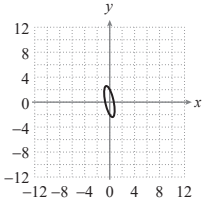
27.



29.



31.



33. c **37.** g

35. a **39.** e

41. The objective of the rotation of axes is to eliminate the $x'y'$-term. If your final equation contains an $x'y'$-term, you know that a mistake has occurred.

43. a. Use the rotation of axes procedure to obtain the equation $4x'^2 + 16y'^2 - 16 = 0$. Now we know $F = -16$ and $F' = -16$. We can plug these values in $F = F'$ and obtain $-16 = -16$, which is true.

b. Use the rotation of axes procedure to obtain the equation $4x'^2 + 16y'^2 - 16 = 0$. Now we know $A = 7$, $C = 13$, $A' = 4$, and $C' = 16$. We can plug these values in $A + C = A' + C'$ and obtain $7 + 13 = 4 + 16$, or $20 = 20$, which is true.

c. Use the rotation of axes procedure to obtain the equation $4x'^2 + 16y'^2 - 16 = 0$. Now we know

$A = 7,\ B = -6\sqrt{3},\ C = 13,$
$A' = 4,\ B' = 0,$ and $C' = 16.$

We can plug these values in $B^2 - 4AC = B'^2 - 4A'C'$ and obtain

$$\left(-6\sqrt{3}\right)^2 - 4(7)(13)$$
$$= (0)^2 - 4(4)(16)$$

or $-256 = -256$, which is true.

11.5 Exercises

1. c **3.** f **5.** b

7. Hyperbola, $y = \frac{7}{6}$

9. Ellipse, $x = -3$

11. Hyperbola, $x = \frac{1}{3}$

13. Ellipse, $x = 5$

15. Hyperbola, $x = -\frac{6}{5}$

17. Parabola, $y = \frac{3}{2}$

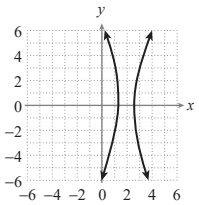
19. Hyperbola, $x = -\frac{4}{7}$

21. $r = \frac{2}{1 - \cos\theta}$

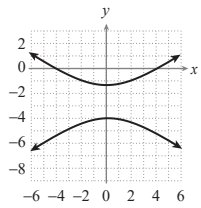
23. $r = \frac{3}{1 - 4\sin\theta}$

25. $r = \frac{3}{1 + \frac{1}{4}\cos\theta}$

27.



29.



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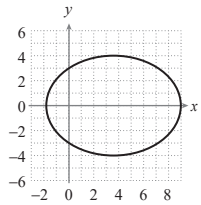
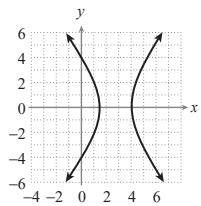


33.



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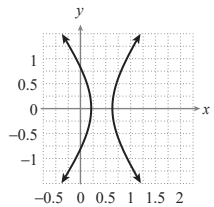
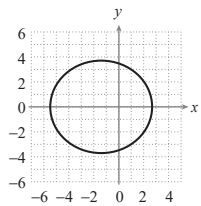
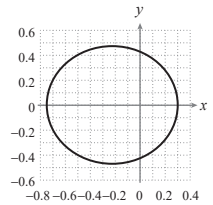


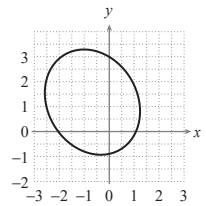
37.



39.



41.



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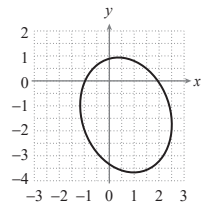
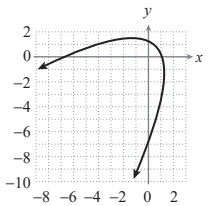


45.



Chapter 11 Project

1. $x^2 = -\frac{1125}{4}(y - 80)$

3. $\frac{x^2}{22{,}500} + \frac{y^2}{6400} = 1, y \geq 0$

5. Answers may vary. The semiellipse design, since it gives more space for ships to pass through.

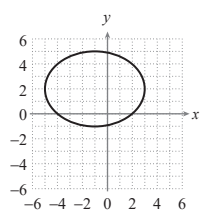
Chapter 11 Review Exercises

1. Center: $(3, -1)$

Vertices: $(7, -1), (-1, -1)$

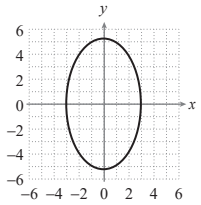
Foci: $\left(3 \pm 2\sqrt{3}, -1\right)$

3.



$\left(-1 \pm \sqrt{7}, 2\right)$

5.



$(0, \pm 3\sqrt{2})$

7. $\dfrac{(x+1)^2}{9} + \dfrac{(y-4)^2}{16} = 1$

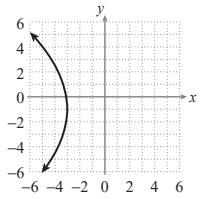
9. $\dfrac{4(x-2)^2}{9} + \dfrac{4(y+1)^2}{9} = 1$

11. $\dfrac{(x-2)^2}{16} + \dfrac{y^2}{12} = 1$

13. $\dfrac{(x+1)^2}{4} + \dfrac{(y-1)^2}{25} = 1$

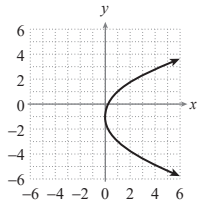
15. $30\pi a - \pi a^2$

17.



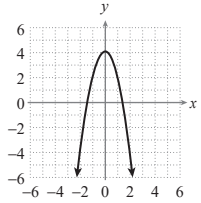
$(-6, -1), x = 0$

19.



$(1, -1),\ x = -1$

21.



$\left(0, \dfrac{31}{8}\right),\ y = \dfrac{33}{8}$

23. $(x+2)^2 = 4(y-3)$

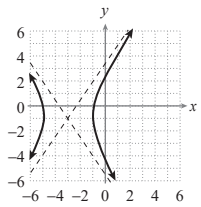
25. $(y+1)^2 = 2\left(x - \dfrac{5}{2}\right)$

27. $(y+1)^2 = 16(x-2)$

29. $(y+1)^2 = -16(x-5)$

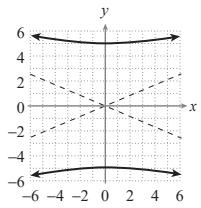
31. $\dfrac{5}{4}$ inches

33.



$\left(-3 \pm \sqrt{13}, -1\right)$

35.



$(0, \pm 13)$

37. Center: $(0, 2)$

Foci: $\left(\pm 2\sqrt{10}, 2\right)$

Vertices: $(\pm 6, 2)$

39. Center: $(3, 3)$

Foci: $\left(3, 3 \pm \sqrt{53}\right)$

Vertices: $(3, 1), (3, 5)$

41. $\dfrac{(x+1)^2}{4} - \dfrac{(y+2)^2}{25} = 1$

43. $\dfrac{(y-2)^2}{9} - \dfrac{(x-2)^2}{1} = 1$

45. $\dfrac{(y-7)^2}{4} - \dfrac{(x+1)^2}{9} = 1$

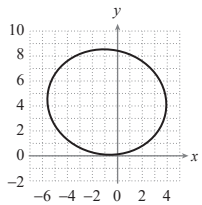
47. $\dfrac{(y+2)^2}{9} - \dfrac{x^2}{4} = 1$

49. $\left(11 + 43\sqrt{3}, -11\sqrt{3} + 43\right)$

51. $\left(3 + 3\sqrt{3}, 9 - \sqrt{3}\right)$

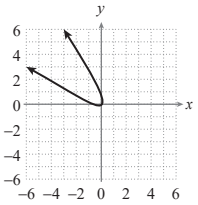
53. Ellipse,

$$\theta = \frac{\pi}{3}, \frac{\left(x' - \dfrac{25\sqrt{3}}{13}\right)^2}{\dfrac{25,000}{1521}} + \frac{\left(y' - \dfrac{25}{9}\right)^2}{\dfrac{25,000}{1053}} = 1$$



55. Parabola,

$$\theta = \frac{\pi}{4}, y' = \sqrt{2}x'^2$$



57. Parabola, $y = \dfrac{7}{4}$

59. Ellipse, $x = \dfrac{7}{2}$

61. $r = \dfrac{4}{1 + \dfrac{1}{4}\cos\theta}$

63. $r = \dfrac{3}{1 + 9\cos\theta}$

Chapter 12: Systems of Equations and Inequalities

12.1 Exercises

1. $(-5, 2)$

3. $(5, 3)$

5. $\varnothing$

7. $\left\{\left(\dfrac{y-3}{2}, y\right) \middle| y \in \mathbb{R}\right\}$

9. $(-1, 7)$

11. $(3, 11)$

13. $\{(x, 4x+1) | x \in \mathbb{R}\}$

15. $(2,19)$

17. $(-5,1)$

19. $(5,6)$

21. $\{(-y-2,\ y)\,|\,y\in\mathbb{R}\}$

23. $(-5,4)$

25. $(-1,1)$

27. $(3,-5)$

29. $\varnothing$

31. $(-1,3,0)$

33. $(2,2,-1)$

35. $\left\{\left(\dfrac{y-z+2}{3},y,z\right)\middle|\,y\in\mathbb{R},z\in\mathbb{R}\right\}$

37. $\varnothing$

39. $(1,1,0)$

41. $(9,1,1)$

43. $(3,1,-2)$

45. $(4,5,5)$

47. $\left(\dfrac{49}{3},\dfrac{-16}{3},\dfrac{5}{4}\right)$

49. $(0,3,2)$

51. 22 pennies, 23 nickels

53. 25 people

55. Eliza is 15 years old.

57. 7 shirts and 4 pairs of shorts

59. 3 quarters, 11 dimes, and 28 pennies

61. Jim is 28 years old.

63. 3 thumb screws

65. Apples: \$0.78, Oranges: \$0.93, Mangos: \$1.05

67. $(0.43,1.28,3.64)$

69. $(-3.42,2.98,2.76)$

71. $(6,8,7)$

12.2 Exercises

1. **a.** 3×2
b. -1
c. None

3. **a.** 5×2
b. None
c. 10

5. **a.** 3×4
b. None
c. 286

7. **a.** 3×2
b. 1
c. None

9. **a.** 2×5
b. 5
c. 2

11. $\left[\begin{array}{ccc|c} -3 & 1 & -2 & -4 \\ \frac{1}{2} & -4 & -1 & 1 \\ 0 & -3 & 3 & 1 \end{array}\right]$

13. $\left[\begin{array}{ccc|c} -\frac{3}{2} & -1 & 0 & -1 \\ 2 & 2 & 3 & 0 \\ 0 & -1 & 6 & 0 \end{array}\right]$

15. $\left[\begin{array}{ccc|c} \frac{12}{5} & \frac{1}{2} & -\frac{3}{2} & \frac{1}{5} \\ 1 & 0 & 3 & 1 \\ 5 & 2 & 1 & -2 \end{array}\right]$

17. $\left[\begin{array}{ccc|c} \frac{2}{3} & -\frac{4}{3} & -2 & 0 \\ 8 & -2 & 6 & 7 \\ 3 & -2 & 0 & 0 \end{array}\right]$

19. $\left[\begin{array}{ccc|c} \frac{1}{2} & -14 & -\frac{1}{4} & -8 \\ \frac{1}{5} & -\frac{7}{6} & \frac{1}{4} & -3 \\ 5 & -5 & \frac{8}{3} & -5 \end{array}\right]$

21. $\begin{cases} x=8 \\ y=3 \end{cases}$

23. $\begin{cases} x+3y+6z=16 \\ y+2z=9 \\ z=4 \end{cases}$

25. $\begin{cases} 9y+13z=27 \\ 2x+21z=19 \\ 7x+18y=32 \end{cases}$

27. $\left[\begin{array}{cc|c} 2 & -5 & 3 \\ 0 & -7 & 5 \end{array}\right]$

29. $\left[\begin{array}{cc|c} 1 & 3 & -2 \\ 9 & -2 & 7 \end{array}\right]$

31. $\left[\begin{array}{cc|c} 8 & -2 & -4 \\ -6 & 2 & -14 \end{array}\right]$

33. $\left[\begin{array}{cc|c} 4 & 12 & -6 \\ 9 & 9 & 6 \end{array}\right]$

35. $\left[\begin{array}{cc|c} 4 & -1 & 5 \\ -6 & 2 & 0 \end{array}\right]$

37. $\left[\begin{array}{ccc|c} 18 & -6 & 15 & 42 \\ -7 & 19 & 2 & 3 \\ -4.5 & 5.5 & -2 & 3.5 \end{array}\right]$

39. $\left[\begin{array}{ccc|c} 5 & 18 & 22 & 5 \\ 32 & -9 & -27 & -23 \\ -9 & 21 & 12 & 9 \end{array}\right]$

41. $\left[\begin{array}{ccc|c} 0 & 1 & -9 & -3 \\ 1 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

43. $\left[\begin{array}{cc|c} -1 & 4 & -3 \\ 1 & -6 & \frac{5}{2} \end{array}\right]$

45. $\left[\begin{array}{ccc|c} 1 & 5 & -9 & 11 \\ 0 & -1 & 8 & -7 \\ 0 & -17 & 41 & 1 \end{array}\right]$

47. Neither

49. Neither

51. Neither

53. $(3,-1)$

55. $(1,3)$

57. $(-7,3)$

59. $\varnothing$

61. $(3,2)$

63. $\{(-2y-4,\ y)\,|\,y\in\mathbb{R}\}$

65. $\varnothing$

67. $(4,0,3)$

69. $(15,-21,8)$

71. $(3,-5)$

73. $\{(x,-3x-2)\,|\,x\in\mathbb{R}\}$

75. $(-4,1)$

77. $(6,4)$

79. $(-11,-5)$

81. $(7,3,3)$

83. $(2,2,-1)$

85. $\{(1,y,0)\,|\,y\in\mathbb{R}\}$

87. $(3,-2,3)$

89. $(2,3,4)$

91. $(9,-19,7)$

93. $(1,-2,-1,3)$

95. 42, 26, 87

97. Small: 10, Medium: 24, Large: 48

12.3 Exercises

1. 11

3. 15

5. $ab-x^2$

7. 8

9. -10

11. -39

13. $\{-2,3\}$

15. $\{-5,1\}$

17. $\{-5,-4\}$

19. $\{-6,4\}$

21. $\{2,5\}$

23. 3

25. -9

27. 2

29. -2

31. 159

33. 78

35. -254

37. 404

39. 4

41. 120

43. 10

45. x^4

47. x^8

49. $(76,-53)$

51. $\{(-y-2,y)\,|\,y\in\mathbb{R}\}$

53. $\varnothing$

55. $\{(-3z-5,-6z-10,z)\,|\,z\in\mathbb{R}\}$

57. $\left\{\left(\dfrac{-5y-z-5}{2},\ \dfrac{-5y+3z-19}{2},\ y,\ z\right)\,\middle|\,y\in\mathbb{R},\ z\in\mathbb{R}\right\}$

59. $\{(-z+8,\ -5z+31,\ -2z+37,\ z)\,|\,z\in\mathbb{R}\}$

61. $(1647,2071)$

63. $\varnothing$

65. $(-3,-1,0,-4)$

67. Candy bars: 5, Ice cream: 6

69. 0.012

71. 0.564

73. 1194

75. $(1,-1,2)$

77. $(2,1,0,3)$

12.4 Exercises

1. $\begin{bmatrix}5&-1\\0&0\\2&13\end{bmatrix}$

3. $\begin{bmatrix}6&-3\\18&30\\-9&21\end{bmatrix}$

5. Not possible

7. $\begin{bmatrix}14&-14\\8&0\\-4&14\end{bmatrix}$

9. $\begin{bmatrix}-7&5\\3&10\\-3&-8\end{bmatrix}$

11. Not possible

13. $a=3,\ b=-1,\ c=10$

15. $a=2,\ b=-2,\ c=-1$

17. Not possible

19. $x=10,\ y=5$

21. $x=3,\ y=1$

23. Not possible

25. $a=8,\ b=5$

27. $\begin{bmatrix}24&-5\end{bmatrix}$

29. $\begin{bmatrix}35&18\end{bmatrix}$

31. Not possible

33. $\begin{bmatrix}-30&-3\end{bmatrix}$

35. $\begin{bmatrix}15&-3&-24\\25&-5&-40\\30&-6&-48\end{bmatrix}$

37. $\begin{bmatrix}-34&-7\end{bmatrix}$

39. $\begin{bmatrix}11&0\\0&11\end{bmatrix}$

41. $\begin{bmatrix}32&-20\\56&-35\\-16&10\end{bmatrix}$

43. Not possible

45. $\begin{bmatrix}14&-13\\-13&5\end{bmatrix}$

47. $\begin{bmatrix}179&76\end{bmatrix}$

49. $\frac{2}{3}$ for store A; $\frac{1}{3}$ for store B

51. Solution is incorrect. Explanations may vary.

53. $\begin{bmatrix}23.94&-7.56&28.98\\21.66&-6.84&26.22\end{bmatrix}$

55. $\begin{bmatrix}-23.94&-26.72\end{bmatrix}$

57. $\begin{bmatrix}-79.59\\39.21\\10.08\end{bmatrix}$

12.5 Exercises

1. $\begin{bmatrix}14&-5\\1&9\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}7\\2\end{bmatrix}$

3. $\begin{bmatrix}1&2\\9&-3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-14\end{bmatrix}$

5. $\begin{bmatrix} 3 & -7 & 1 \\ 1 & -1 & 0 \\ 0 & 8 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ -3 \end{bmatrix}$

7. $\begin{bmatrix} \frac{3}{5} & -\frac{8}{5} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

9. $\begin{bmatrix} 4 & -3 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9 \\ 13 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -1 & 3 \\ -1 & 1 & 0 \\ 4 & -5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 17 \\ -2 \end{bmatrix}$

13. $\begin{bmatrix} -\frac{1}{20} & -\frac{1}{5} \\ \frac{1}{4} & 0 \end{bmatrix}$

15. $\begin{bmatrix} -5 & -4 \\ 4 & 3 \end{bmatrix}$

17. $\begin{bmatrix} -5 & 0 \\ 2 & 2 \end{bmatrix}$

19. Not invertible

21. $\begin{bmatrix} 2 & 1 & -4 \\ -4 & -2 & -3 \\ -1 & -1 & -4 \end{bmatrix}$

23. $\begin{bmatrix} -1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & -4 & 3 \end{bmatrix}$

25. $\begin{bmatrix} -1 & -2 & 1 \\ -2 & 1 & -3 \\ 1 & 2 & 0 \end{bmatrix}$

27. $\begin{bmatrix} -2 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

29. $\begin{bmatrix} 2 & -1 & 2 \\ 0 & 1 & -1 \\ -3 & -2 & -4 \end{bmatrix}$

31. No

33. Yes

35. No

37. $\left(-2, -\frac{5}{2}\right)$

39. $\left\{\left(\frac{3y-1}{2}, y\right) \middle| y \in \mathbb{R}\right\}$

41. $(-2, 0)$

43. $(8, -19)$

45. $(0, 5)$

47. $(-4, 5, -1)$

49. $(-13, 19, 23)$; $(0, 0, -1)$; $(1, -1, -1)$

51. $(1, -8, 7)$; $(3, 1, 1)$; $(4, 2, 0)$

53. $\begin{bmatrix} \frac{-1}{4} & \frac{1}{10} \\ \frac{-1}{4} & \frac{3}{10} \end{bmatrix}$

55. $\begin{bmatrix} 0.053 & -0.258 \\ 0.113 & 0.076 \end{bmatrix}$

57. $\begin{bmatrix} 0.004 & -0.003 & 0.009 \\ 0 & 0.020 & 0.029 \\ 0.012 & 0.014 & 0.013 \end{bmatrix}$

12.6 Exercises

1. $\frac{A_1}{x-3} + \frac{A_2}{x+3}$

3. $\frac{A_1}{x+3} + \frac{A_2}{x+4} + \frac{A_3}{(x+4)^2}$

5. $\frac{A_1}{x+3} + \frac{A_2}{x-2} + \frac{A_3}{x+2}$

7. d

9. h

11. a

13. c

15. $-\frac{1}{x} + \frac{2}{x-2} + \frac{2}{x+2}$

17. $\frac{5}{72(x-2)} - \frac{1}{32x} - \frac{7}{288(x+4)} - \frac{x+17}{72(x^2+8)}$

19. $\frac{10}{x-4} - \frac{5}{x-2}$

21. $\frac{3}{2(x+3)} + \frac{3}{14(x-1)} - \frac{12}{7(x+6)}$

23. $\frac{1}{2(x-1)} - \frac{1}{2(x+1)}$

25. $\frac{1}{48(x-2)} - \frac{x}{80(x^2+4)} - \frac{3}{40(x^2+4)} - \frac{1}{120(x+4)}$

27. $\frac{5}{4(x-2)} - \frac{1}{4(x+2)}$

29. $\frac{1}{16(x+2)} + \frac{1}{16(x-2)} - \frac{x}{8(x^2+4)}$

31. $\frac{1}{24(x+6)} + \frac{1}{30(x+3)} - \frac{7}{40(x-2)} + \frac{1}{6(x-3)}$

33. $\frac{15}{16(x+3)} + \frac{1}{16(x-1)} - \frac{9}{4(x+3)^2}$

35. $\frac{1}{x+3} + \frac{1}{x-3}$

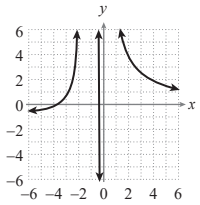
37. $\frac{1}{4(x-2)} + \frac{3}{4(x+6)}$

39. $\frac{1}{a}\left(\frac{1}{x} - \frac{1}{x+a}\right)$

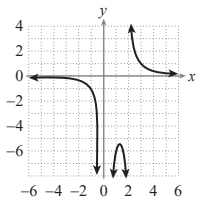
41. $\frac{1}{2a}\left(\frac{1}{a+x} + \frac{1}{a-x}\right)$

43. $\frac{1}{a-1}\left(\frac{1}{x+1} + \frac{1}{x+a}\right)$

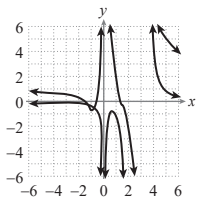
45. True



47. True

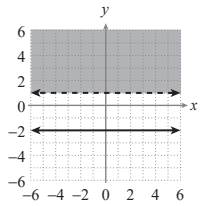


49. False

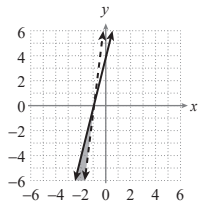


12.7 Exercises

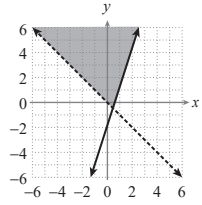
1.



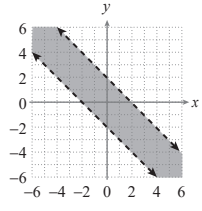
3.



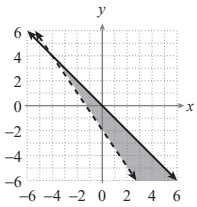
5.



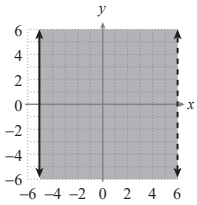
7.



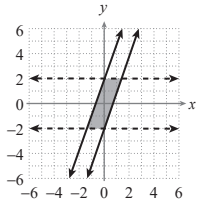
9.



11.



13.



15. Min = 0 at $(0,0)$;
Max = 12 at $(3,0)$

17. Min = 0 at $(0,0)$;
Max = 12 at $(0,3)$

19. Min = 100 at $\left(\frac{10}{3},10\right)$;
Max = 250 at $(20,10)$

21. Min = $\frac{150}{7}$ at $\left(\frac{50}{7},0\right)$;
Max = 66 at $(8,6)$

23. Min = 0 at $(0,0)$;
Max = $\frac{165}{8}$ at $\left(\frac{111}{8},\frac{27}{8}\right)$

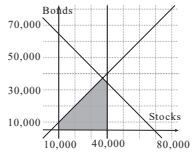
25. Min = 210 at $(14,0)$;
Max = 600 at $(0,20)$

27. Min = $\frac{680}{7}$ at $\left(\frac{88}{7},\frac{-36}{7}\right)$;
Max = $\frac{1980}{7}$ at $\left(\frac{108}{7},\frac{-6}{7}\right)$

29.

$40x + 60y \le 200$;
$8x + 5y \le 25$;
$x \ge 0$; $y \ge 0$

31.



$x + y \le 75{,}000$;
$x - y \ge 0$;
$x \le 40{,}000$;
$x \ge 10{,}000$;
$y \ge 5000$

33. Type X: 75 units;
Type Y: 100 units;
Maximum profit: \$712.50

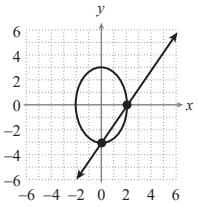
35. The volunteer could choose any of the following points: $(9,9),(11,6),(13,3),(15,0)$. In each of these points, the first coordinate represents the number of packages from Company A and the second coordinate represents the number of packages from Company B. The minimum cost is \$22.50.

37. a. 120 flip phones;
80 smart phones;
Minimum cost: \$1160

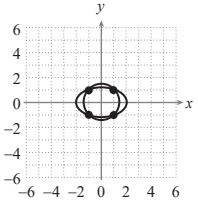
b. 100 flip phones;
170 smart phones;
Maximum profit: \$650

12.8 Exercises

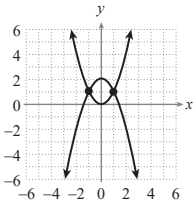
1. $\{(0,-3),(2,0)\}$



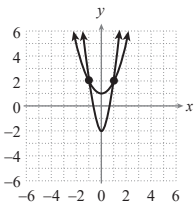
3. $\{(-1,-1),(1,1),(-1,1),(1,-1)\}$



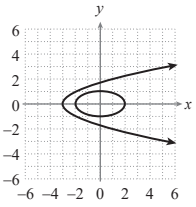
5. $\{(-1,1),(1,1)\}$



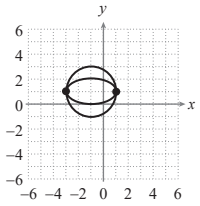
7. $\{(-1,2),(1,2)\}$



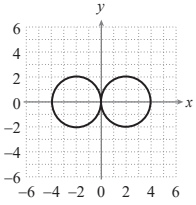
9. No solution



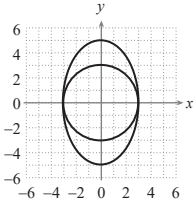
11. $\{(-3,1),(1,1)\}$



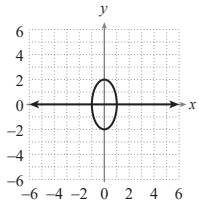
13. $\{(0,0)\}$



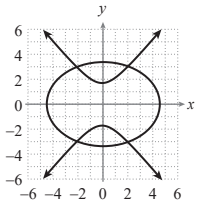
15. $\{(\pm 3,0)\}$



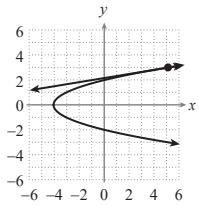
17. $\{(\pm 1,0)\}$



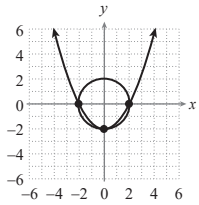
19. $\{(2,\pm 3),(-2,\pm 3)\}$



21. $\{(5,3)\}$



23. $\{(\pm 2,0),(0,-2)\}$



25. $\left\{\begin{array}{l}(-i\sqrt{6},-6),(i\sqrt{6},-6),\\(-\sqrt{5},5),(\sqrt{5},5)\end{array}\right\}$

27. $\{(-2,3)\}$

29. $\left\{\begin{array}{l}(-1,-4),(1,4),\\(-2\sqrt{2},-\sqrt{2}),\\(2\sqrt{2},\sqrt{2})\end{array}\right\}$

31. $\left\{\begin{array}{l}(-3i,-5),\\(3i,-5),(0,4)\end{array}\right\}$

33. $\{(-\sqrt{3},0),(\sqrt{3},0)\}$

35. $\left\{\begin{array}{l}(-i,-6),(i,-6),\\(-\sqrt{2},3),(\sqrt{2},3)\end{array}\right\}$

37. $\left\{\begin{array}{l}(1,1),\left(-\frac{3}{2},\frac{\sqrt{14}}{2}\right),\\(1,-1),\left(-\frac{3}{2},-\frac{\sqrt{14}}{2}\right)\end{array}\right\}$

39. $\left\{\left(\frac{7}{2},\frac{17}{4}\right)\right\}$

41. $\left\{\begin{array}{l}(-6,-i\sqrt{15}),(2,1),\\(-6,i\sqrt{15}),(2,-1)\end{array}\right\}$

43. $\left\{(0,1),\left(-\frac{100}{101},-\frac{99}{101}\right)\right\}$

45. $\left\{\begin{array}{l}\left(-\frac{\sqrt{42}}{6},-\frac{\sqrt{66}}{6}\right),\\\left(-\frac{\sqrt{42}}{6},\frac{\sqrt{66}}{6}\right),\\\left(\frac{\sqrt{42}}{6},-\frac{\sqrt{66}}{6}\right),\\\left(\frac{\sqrt{42}}{6},\frac{\sqrt{66}}{6}\right)\end{array}\right\}$

47. $\{(-5,0),(-2,0),(-1,0)\}$

49. $\left\{\begin{array}{l}(3,2),(2,3),\\\left(\frac{-1\pm i\sqrt{23}}{2},\frac{-1\pm i\sqrt{23}}{2}\right)\end{array}\right\}$

51. $\left\{(0,-1),\left(\frac{\sqrt{6}}{3},\frac{2\sqrt{6}}{9}-1\right)\right\}$

53. $\left\{\begin{array}{l}\left(\sqrt{2}+1,2\sqrt{2}\right),\left(-\sqrt{2}+1,-2\sqrt{2}\right),\\\left(2\sqrt{2}+1,\sqrt{2}\right),\left(-2\sqrt{2}+1,-\sqrt{2}\right)\end{array}\right\}$

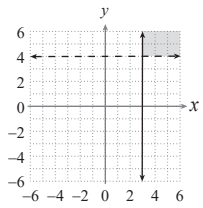
55. $\left\{(4,1),\left(1,1+i\sqrt{3}\right)\right\}$

57. $\left\{\begin{array}{l}(0,4),(0,-3),\\(3,-2),(3,3)\end{array}\right\}$

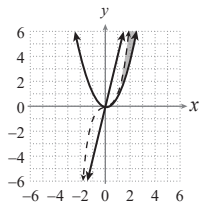
59. $\{(4,0)\}$

61. $\left\{\left(\pm\frac{1}{2},\pm\frac{1}{4}\right)\right\}$

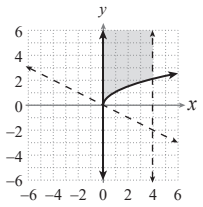
63. b



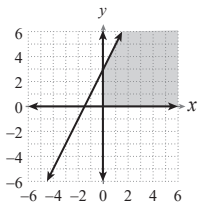
65. b, d



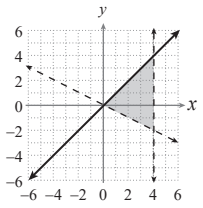
67. a, d



69.



71.



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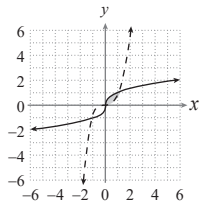
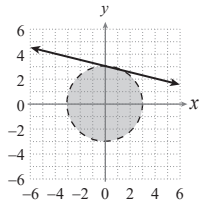


75.



77. 9 inches by 5 inches

79. 60 mph and 70 mph

81. -12 and 7

83. $h = 6$ cm, $r = 3$ cm

Chapter 12 Project

1.

Shop Name	The number of months passed					
	1	2	3	4	5	6
Joe's Java	26.85%	28.47%	29.88%	31.12%	32.21%	33.16%
Buck's Café	43.45%	42.10%	40.92%	39.89%	39.00%	38.21%
Tweak's Coffee	29.70%	29.43%	29.20%	28.98%	28.80%	28.63%

3. As our market share increases, the shares of both Buck's Café and Tweak's Coffee decrease. As such, they will have to adjust their marketing strategies to stop the decrease and potentially increase their shares.

Chapter 12 Review Exercises

1. $\varnothing$

3. $(3,0,2)$

5. $\varnothing$

7. $\{(3-3y, y) \mid y \in \mathbb{R}\}$

9. $(3,2)$

11. $(8,12,10)$

13. $y = \frac{4}{15}x^2 - x + \frac{11}{15}$

15. a. 1×4

b. 8

c. None

17. $\left[\begin{array}{ccc|c} 4 & 5 & -1 & 0 \\ 1 & 3 & 2 & 3 \\ 10 & -1 & -6 & 0 \end{array}\right]$

19. $\begin{cases} 8x + 7z = 5 \\ -3y + 4z = 16 \\ 16x - 2y + z = 2 \end{cases}$

21. $\left[\begin{array}{cc|c} 0 & -5 & -11 \\ 1 & 2 & 3 \end{array}\right]$

23. $\left[\begin{array}{cc|c} 1 & -4 & -4 \\ 1 & 7 & 11 \end{array}\right]$

25. $(2,-1)$

27. $(3,-5)$

29. $2x^4$

31. 7

33. 9, -9

35. $(-4,1)$

37. $\varnothing$

39. $\begin{bmatrix} 4 & -16 & 4 \\ -5 & 8 & 12 \end{bmatrix}$

41. Not possible

43. $\begin{bmatrix} 9 & -23 & 3 \\ 5 & -3 & 8 \end{bmatrix}$

45. $w = -2$, $x = 1$, $y = 3$, $z = -4$

47. $x = 2$, $y = -3$

49. $\begin{bmatrix} 12 & 46 \end{bmatrix}$

51. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & -1 \\ -3 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ 5 \end{bmatrix}$

53. $\begin{bmatrix} \frac{3}{16} & \frac{1}{8} \\ -\frac{1}{8} & \frac{1}{4} \end{bmatrix}$

55. Not possible

57. No

59. Yes

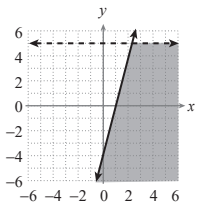
61. $\left(1,-\frac{1}{3}\right)$

63. $(-15,20,-33),(-2,3,-4),$ $(-9,11,-17)$

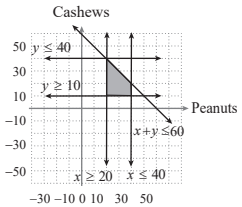
65. $\frac{A_1}{x+4}+\frac{A_2}{x-1}$

67. $-\frac{3}{2(2x-5)^2}+\frac{1}{2(2x-5)}$

69.



71. $x \geq 20,\ y \geq 10,\ x \leq 40,$ $y \leq 40,\ x+y \leq 60$

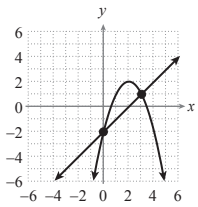


73. Min = 0 at $(0,0)$
Max = 30 at $(5,0)$

75. Min = 8 at $(0,2)$
Max = 24 at $\left(\frac{24}{7},\frac{12}{7}\right)$

77. 12 vases should be produced, 12 pitchers should be produced, Max profit: $660

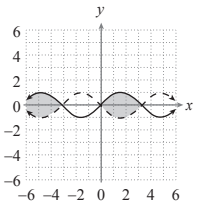
79. $\{(0,-2),(3,1)\}$



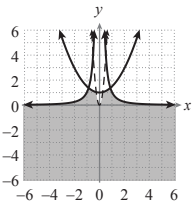
81. $\left\{(i,-1),(-i,-1),\left(\frac{\sqrt{2}}{2},\frac{1}{2}\right),\left(-\frac{\sqrt{2}}{2},\frac{1}{2}\right)\right\}$

83. $\{(2,1)\}$

85.



87.



89. 36 mph and 24 mph

Chapter 13: Sequences, Series, Combinatorics, and Probability

13.1 Exercises

1. Infinite

3. Finite

5. Finite

7. Infinite

9. Infinite

11. $2,-1,-4,-7,-10$

13. $1,\frac{3}{2},\frac{9}{5},2,\frac{15}{7}$

15. $\frac{2}{3},-\frac{4}{9},\frac{8}{27},-\frac{16}{81},\frac{32}{243}$

17. $\frac{1}{2},\frac{4}{3},\frac{9}{4},\frac{16}{5},\frac{25}{6}$

19. $-1,3,-6,10,-15$

21. $26,7,0,-1,-2$

23. $-1,\sqrt{2},-\sqrt{3},2,-\sqrt{5}$

25. $1,5,9,13,17$

27. $\frac{1}{2},1,2,4,8$

29. $3,36,729,20,736,759,375$

31. $\frac{5}{4},2,\frac{5}{2},\frac{20}{7},\frac{25}{8}$

33. $1,3,6,10,15$

35. Undefined, 9, 4, $\frac{25}{9},\frac{9}{4}$

37. $\frac{1}{3},\frac{1}{2},\frac{5}{9},\frac{7}{12},\frac{3}{5}$

39. $0,-1,-4,-9,-16$

41. $2,4,16,256,65,536$

43. $1,2,6,24,120$

45. $2,\sqrt{5},\sqrt{6},\sqrt{7},2\sqrt{2}$

47. 1, 0, −27, 0, 125

49. $a_n=7n-2$

51. $a_1=-1, a_n=-na_{n-1}, n\geq 2$

53. $a_n=\left(\frac{1}{n}\right)^2$

55. $a_n=9n-43$

57. $a_n=2^{n-3}$

59. $a_n=\frac{n}{2^n}$

61. $-2+1+4+7+10$ $+13+16=49$

63. $\sum_{i=1}^{6} i^3=441$

65. $45+80+125+180$ $+245+320+405$ $+500=1900$

67. $-6-12-24-48$ $-96-192=-378$

69. $\sum_{i=2}^{9} 3^i=29,520$

71. $S_n=\frac{n}{4(n+4)}, S=\frac{1}{4}$

73. $S_n=2^n-1, S_{15}=32,767$

75. $S_n = \dfrac{n}{2(n+1)}, S = \dfrac{1}{2}$

77. $S_n = -\ln(n+1)$, series diverges

79. $S_n = \dfrac{3n}{4(3n+4)}, S = \dfrac{1}{4}$

81. 4, 7, 11, 18, 29

83. 10, 20, 30, 50, 80

85. 13, −17, −4, −21, −25

87. 1, −3, −3, 9, −27

89. 987

91. 1, 2, 4, 8, 16, 32, 64, 128

13.2 Exercises

1. $a_n = 3n - 5$

3. $a_n = -2n + 9$

5. $a_n = 9n - 4$

7. $a_n = -6n + 9$

9. $a_n = 19n + 5$

11. $a_n = n + \dfrac{5}{2}$

13. $a_n = -\dfrac{19}{2}n + \dfrac{43}{2}$

15. $a_n = -2n + 1$

17. $a_n = -4n + 33$

19. $d = 2$

21. $d = 1$

23. No

25. No

27. 13

29. 2

31. 2

33. $d = 2.5$; 5, 7.5, 10, 12.5, 15

35. $d = 7$; 7, 14, 21, 28, 35

37. $d = 9$; −62, −53, −44, −35, −26

39. $d = 5$

41. $d = 1$

43. $d = -2$

45. 195

47. a_{73}

49. 117

51. 26

53. −8

55. 13

57. 55

59. 14,350

61. 17,114

63. −1475

65. $-\dfrac{3219}{5}$

67. 902

69. −1316

71. 6 years

73. 1620 pounds

75. \$625; \$8100

77. 266.4

79. −152

81. 10,382.05

13.3 Exercises

1. $a_n = -3(2)^{n-1}$

3. $a_n = 2\left(-\dfrac{1}{3}\right)^{n-1}$

5. $a_n = \left(-\dfrac{1}{4}\right)^{n-1}$

7. $a_n = \left(\dfrac{1}{7}\right)^{n-1}$

9. $a_n = (-3)^{n-1}$

11. $a_n = 3\left(\dfrac{2}{3}\right)^{n-1}$

13. $a_n = 7(-2)^{n-1}$

15. $a_n = \dfrac{1}{16}(2)^{n-1}$

17. $a_n = \dfrac{39}{68}\left(\dfrac{4}{3}\right)^{n-1}$

19. No

21. $r = \dfrac{1}{2}$

23. $r = 2$

25. $r = 7$

27. $r = 3$; 8, 24, 72, 216, 648

29. $r = 2$; $\dfrac{1}{4}, \dfrac{1}{2}, 1, 2, 4$

31. $r = \dfrac{1}{5}$;
62,500, 12,500, 2500, 500, 100

33. $\dfrac{5}{16,384}$

35. −2,147,483,648

37. $r = \pm 2$

39. $r = \pm\dfrac{1}{5}$

41. $\dfrac{52,222,139,775}{1,048,576}$
$\approx 49,802.9$

43. $\dfrac{10,923}{16,384} \approx 0.666687$

45. $\dfrac{73,810}{19,683} \approx 3.749936$

47. −109,200

49. $-\dfrac{3}{2}$

51. Series diverges

53. $\dfrac{2,476,099}{160,000} \approx 15.475619$

55. Series diverges

57. $\dfrac{28,561}{152,064} \approx 0.187822$

59. $\dfrac{123}{999}$

61. $-\dfrac{35}{9}$

63. $\dfrac{989}{99}$

65. \$14,802.44

67. 1.845×10^{19}

69. Approximately 13,778 students

71. $S_{30} = 1.1 \times 10^{30}$; $r = 10$

73. Yes; explanations will vary (any example such that $r = 1$ and $d = 0$).

75. 9.98

77. 2.137×10^{-18}

79. 6.54×10^{51}

13.4 Exercises

1. $S_{k+1}=\dfrac{1}{3k+9}$

3. $S_{k+1}=\dfrac{(k+1)(k+2)(2k+3)}{4}$

5. Basic Step:

$n=1, 1=1$ and $\dfrac{1(1+1)}{2}=1$;

Induction Step:

If $1+2+3+\cdots+k=\dfrac{k(k+1)}{2}$,

then $(1+2+3+\cdots+k)+(k+1)$

$=\dfrac{k(k+1)}{2}+(k+1)$

$=\dfrac{k^2+k+2k+2}{2}$

$=\dfrac{(k+1)(k+2)}{2}$

7. Basic Step:

$n=1, 2(1)=2$ and $1(1+1)=2$;

Induction Step:

If $2+4+6+\cdots+2k=k(k+1)$,

then

$(2+4+6+\cdots+2k)+2(k+1)$

$=k^2+k+2k+2$

$=(k+1)(k+2)$

9. Basic Step:

$n=1, 4^{1-1}=1$ and $\dfrac{4^1-1}{3}=1$;

Induction Step:

If $4^0+4^1+4^2+\cdots+4^{k-1}=\dfrac{4^k-1}{3}$,

then $4^0+4^1+4^2+\cdots+4^{k-1}+4^{k+1-1}$

$=\dfrac{4^k-1}{3}+4^k=\dfrac{4^k-1+3\cdot 4^k}{3}$

$=\dfrac{4\cdot 4^k-1}{3}=\dfrac{4^{k+1}-1}{3}$

11. Basic Step:

$n=1, \dfrac{1}{(3(1)-2)(3(1)+1)}=\dfrac{1}{4}$

and $\dfrac{1}{3(1)+1}=\dfrac{1}{4}$;

Induction Step:

If $\dfrac{1}{1\cdot 4}+\dfrac{1}{4\cdot 7}+\dfrac{1}{7\cdot 10}$

$+\cdots+\dfrac{1}{(3k-2)(3k+1)}=\dfrac{k}{3k+1}$,

then $\dfrac{1}{1\cdot 4}+\dfrac{1}{4\cdot 7}+\dfrac{1}{7\cdot 10}$

$+\cdots+\dfrac{1}{(3k-2)(3k+1)}$

$+\dfrac{1}{(3(k+1)-2)(3(k+1)+1)}$

$=\left[\begin{array}{l}\dfrac{1}{1\cdot 4}+\dfrac{1}{4\cdot 7}+\dfrac{1}{7\cdot 10}\\ +\cdots+\dfrac{1}{(3k-2)(3k+1)}\end{array}\right]$

$+\dfrac{1}{(3k+1)(3k+4)}$

$=\dfrac{k}{3k+1}+\dfrac{1}{(3k+1)(3k+4)}$

$=\dfrac{3k^2+4k+1}{(3k+1)(3k+4)}$

$=\dfrac{(3k+1)(k+1)}{(3k+1)(3k+4)}=\dfrac{(k+1)}{(3(k+1)+1)}$

13.

Basic Step:

$n=1, 5(1)=5$ and $\dfrac{5(1)(1+1)}{2}=5$;

Induction Step:

If $5+10+15+\cdots+5k=\dfrac{5k(k+1)}{2}$,

then $5+10+15+\cdots+5k+5(k+1)$

$=(5+10+15+\cdots+5k)+5k+5$

$=\dfrac{5k(k+1)}{2}+5k+5$

$=\dfrac{5k^2+15k+10}{2}$

$=\dfrac{5(k+1)(k+2)}{2}$

$=\dfrac{5(k+1)[(k+1)+1]}{2}$

15.

Basic Step:

$n=1, 1+\dfrac{1}{1}=2$ and $1+1=2$

Induction Step:

If $\left(1+\dfrac{1}{1}\right)\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)\cdots\left(1+\dfrac{1}{k}\right)=k+1$,

then

$\left(1+\dfrac{1}{1}\right)\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)\cdots\left(1+\dfrac{1}{k}\right)\left(1+\dfrac{1}{k+1}\right)$

$=(k+1)\left(1+\dfrac{1}{k+1}\right)$

$=k+1+\dfrac{k+1}{k+1}=(k+1)+1$

17.

Basic Step:

$n=1, 3(1)-2=1$ and $\dfrac{1}{2}(3(1)-1)=1$;

Induction Step:

If $1+4+7+10+\cdots+(3k-2)=\dfrac{k}{2}(3k-1)$,

then

$[1+4+7+10+\cdots+(3k-2)]$

$+[3(k+1)-2]$

$=\dfrac{k}{2}(3k-1)+(3k+1)$

$=\dfrac{k(3k-1)+2(3k+1)}{2}$

$=\dfrac{3k^2+5k+2}{2}$

$=\dfrac{(k+1)(3k+2)}{2}$

$=\dfrac{k+1}{2}(3(k+1)-1)$

19. Basic Step:

$n=2, 3^2=9$ and $2(2)+1=5$,

so $3^2>2(2)+1$;

Induction Step:

If $3^k>2k+1$, then

$3^{k+1}=3^1\cdot 3^k>3(2k+1)$

$=6k+3>2k+3$

$=2k+2+1=2(k+1)+1$

21.

Basic Step:

$n=1, 1^3=1$ and $\dfrac{1^2(1+1)^2}{4}=1$;

Induction Step:

If $1^3+2^3+3^3+4^3+\cdots+k^3=\dfrac{k^2(k+1)^2}{4}$,

then

$\left(1^3+2^3+3^3+4^3+\cdots+k^3\right)+(k+1)^3$

$=\dfrac{k^2(k+1)^2}{4}+(k+1)^3$

$=\dfrac{k^2(k+1)^2+4(k+1)^3}{4}$

$=\dfrac{(k+1)^2(k+2)^2}{4}=\dfrac{(k+1)^2((k+1)+1)^2}{4}$

23. Basic Step:

$n=1, a^1=a$ so $a^1>1$, when $a>1$;

Induction Step:

If $a^k>1$, then

$a^{k+1}=a^k\cdot a^1>1\cdot a=a>1$

25.

Basic Step:

$n=1, 1^4=1$ and

$\dfrac{1(1+1)(2(1)+1)(3(1)^2+3(1)-1)}{30}=1$;

Induction Step:

If $1^4+2^4+3^4+\cdots+k^4$

$=\dfrac{k(k+1)(2k+1)(3k^2+3k-1)}{30}$,

then

$\left(1^4+2^4+3^4+\cdots+k^4\right)+(k+1)^4$

$=\dfrac{k(k+1)(2k+1)(3k^2+3k-1)}{30}+(k+1)^4$

$=\dfrac{6k^5+45k^4+130k^3+180k^2+119k+30}{30}$

$=\dfrac{(k+1)(k+2)(2k+3)(3k^2+9k+5)}{30}$

$=\dfrac{\left[\begin{array}{l}(k+1)(k+2)(2(k+1)+1)\\ \times(3(k+1)^2+3(k+1)-1)\end{array}\right]}{30}$

27.

Basic Step:

$n=2, \dfrac{1}{\sqrt{1}}+\dfrac{1}{\sqrt{2}}=1+\dfrac{\sqrt{2}}{2}$

and $1+\dfrac{\sqrt{2}}{2}>\sqrt{2}$;

Induction Step:

If $\dfrac{1}{\sqrt{1}}+\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}+\cdots+\dfrac{1}{\sqrt{k}}>\sqrt{k}$,

then

$\left[\dfrac{1}{\sqrt{1}}+\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}+\cdots+\dfrac{1}{\sqrt{k}}\right]+\dfrac{1}{\sqrt{k+1}}$

$>\sqrt{k}+\dfrac{1}{\sqrt{k+1}}=\dfrac{\sqrt{k}(\sqrt{k+1})+1}{\sqrt{k+1}}$

$=\dfrac{\sqrt{k^2+k}+1}{\sqrt{k+1}}>\dfrac{\sqrt{k^2}+1}{\sqrt{k+1}}$

$=\dfrac{k+1}{\sqrt{k+1}}=\sqrt{k+1}$

29. Basic Step:

$n=1, (ab)^1=ab$ and $a^1b^1=ab$;

Induction Step:

If $(ab)^k=a^kb^k$, then

$(ab)^{k+1}=(ab)^k\cdot(ab)$

$=a^kb^k\cdot ab=(a\cdot a^k)(b\cdot b^k)$

$=a^{k+1}b^{k+1}$

31.

Basic Step:

$n=1, \ln(x_1)=\ln x_1$;

Induction Step:

If $\ln(x_1\cdot x_2\cdot x_3\cdot\cdots\cdot x_k)$

$=\ln x_1+\ln x_2+\ln x_3+\cdots+\ln x_k$

when $x_1>0, x_2>0, \ldots, x_n>0$,

then $\ln(x_1\cdot x_2\cdot x_3\cdot\cdots\cdot x_k\cdot x_{k+1})$

$=\ln(x_1\cdot x_2\cdot x_3\cdot\cdots\cdot x_k)+\ln(x_{k+1})$

$=(\ln x_1+\ln x_2+\ln x_3+\cdots+\ln x_k)+\ln x_{k+1}$

33.

Basic Step: $n=2$,

$(9^2-8(2)-1)=64$ of which 64 is a factor;

Induction Step:

If $(9^k-8k-1)=64p$ for some integer p,

then $(9^{k+1}-8(k+1)-1)$

$=9\cdot 9^k-8k-9$

$=9\cdot 9^k-9\cdot 8k+8\cdot 8k-9$

$=9(9^k-8k-1)+64k$

$=9(64p)+64k=64(9p+k)$

35.

Basic Step: $n=1$,

$(1^3-1+3)=3$, which is divisible by 3;

Induction Step:

If $\dfrac{k^3-k+3}{3}=p$

or $k^3-k+3=3p$ for some integer p,

then $(k+1)^3-(k+1)+3$

$=k^3+3k^2+2k+3$

$=(k^3-k+3)+(3k^2+3k)$

$=3p+3(k^2+k)=3(p+k^2+k)$

37.

Basic Step: $n=1$,

$1(1+1)(1+2)=6$, which is divisible by 6;

Induction Step:

If $\dfrac{k(k+1)(k+2)}{6}=p$

or $k(k+1)(k+2)=6p$

for some integer p, then

$(k+1)(k+2)(k+3)$

$=k^3+6k^2+11k+6$

$=(k^3+3k^2+2k)+(3k^2+9k+6)$

$=k(k+1)(k+2)+3(k+1)(k+2)$

$=6p+3(k+1)(k+2)$.

$6p$ is clearly divisible by 6.

In order for $3(k+1)(k+2)$ to be divisible by 6, it must be divisible by 2 and 3. It is clearly divisible by 3.

If k is odd, then the term $(k+1)$ must be even, making it divisible by 2.

If k is even, then the term $(k+2)$ is even, making it divisible by 2.

Therefore, $3(k+1)(k+2)$ is divisible by 6.

39.

$0+1+2+3+\cdots+(n-1)=\dfrac{n(n-1)}{2}$;

Basic Step:

$n=1, (1-1)=0$ and $\dfrac{1(1-1)}{2}=0$;

Induction Step:

If $0+1+2+\cdots+(k-1)=\dfrac{k(k-1)}{2}$,

then

$$\left[0+1+2+\cdots+(k-1)\right]+(k+1-1)$$
$$=\frac{k(k-1)}{2}+k=\frac{k^2-k+2k}{2}$$
$$=\frac{k(k+1)}{2}=\frac{(k+1)\left((k+1)-1\right)}{2}$$

41. The induction step does not work for $n = 1$. In the case of $n = 1$, $n + 1 = 2$ and the groups formed by removing the first horse and then the last horse do not overlap.

13.5 Exercises

1. combination

3. combination

5. 12 **7.** 720

9. 15 **11.** 792

13. $\dfrac{5!}{2!}=60$

15. $\dfrac{7!}{2!2!}=1260$

17. $\dfrac{11!}{2!2!2!}=4,989,600$

19. $243x^5+405x^4y+270x^3y^2$
$+90x^2y^3+15xy^4+y^5$

21. $x^4-12x^3+54x^2-108x+81$

23. $7776x^{10}+6480x^8y+2160x^6y^2$
$+360x^4y^3+30x^2y^4+y^5$

25. $2401x^8+10,976x^6y^2$
$+18,816x^4y^4+14,336x^2y^6$
$+4096y^8$

27. $x^2+2xy+2xz+y^2+2yz+z^2$

29. $64x^6+960x^5+6000x^4$
$+20,000x^3+37,500x^2$
$+37,500x+15,625$

31. 32

33. $x^{16}+48x^{15}y+1080x^{14}y^2$
$+15,120x^{13}y^3$

35. $129,140,163x^{\frac{17}{4}}$
$+3,658,971,285x^4y$

37. $651,168x^5$

39. $10^3=1000$

41. $9^7=4,782,969$

43. $15!\approx 1.308\times 10^{12}$

45. $3!=6$

47. $5^{10}=9,765,625$

49. $36^6=2,176,782,336$

51. $26\cdot 25\cdot 24\cdot 10\cdot 9\cdot 8$
$=11,232,000$

53. ${}_{30}P_{12}\approx 4.143\times 10^{16}$

55. ${}_{36}P_{8}\approx 1.220\times 10^{12}$

57. ${}_7P_6=5040$
${}_7P_7=5040$ as well. (Having a child remain standing is numerically equivalent to putting a seventh chair in the room.)

59. ${}_{26}P_3=15,600$

61. ${}_7C_3=35$

63. ${}_9C_2=36$

65. ${}_{75}C_5=17,259,390$
${}_{52}C_5=2,598,960$

67. ${}_{10}C_4\cdot{}_8C_4\cdot{}_{13}C_4$
$=10,510,500$

69. 112 cones

71. 96 outfits

73. 288 schedules

75. 120 5-letter strings

77. 303,600 ways

79. 495 pizzas

81. 752,538,150 groups

83. 420 ways

85. $\dbinom{n}{n-k}=\dfrac{n!}{(n-k)!\left(n-(n-k)\right)!}$
$=\dfrac{n!}{(n-k)!(n-n+k)!}$
$=\dfrac{n!}{(n-k)!k!}=\dbinom{n}{k}$

87. $2^n=(1+1)^n$
$=\displaystyle\sum_{k=0}^{n}\binom{n}{k}(1)^k(1)^{n-k}=\sum_{k=0}^{n}\binom{n}{k}$
$=\dbinom{n}{0}+\dbinom{n}{1}+\cdots+\dbinom{n}{n}$

13.6 Exercises

1. $\dfrac{3}{5}$

3. $\dfrac{9}{13}$

5. $\dfrac{1}{3}$

7. a. 0 **b.** $\dfrac{5}{8}$

9. a. 0 **b.** $\dfrac{3}{5}$

11. a. 0 **b.** 1

13. a. $\dfrac{1}{8}$ **b.** $\dfrac{9}{16}$

15. The set of all ordered 4-tuples made up of H's and T's. There are 16 such 4-tuples.

17. The set of all ordered pairs that have either an H or a T in the first slot and one of the 13 hearts in the second slot. There are 26 such ordered pairs.

19. The set of all ordered triples with any of the 6 values in each slot. There are 216 such triples.

21. The set of the 38 pockets.

23. a. $\dfrac{2}{3}$ **b.** $\dfrac{1}{3}$

25. a. $\dfrac{3}{8}$ **b.** $\dfrac{1}{8}$ **c.** $\dfrac{1}{2}$

27. $\dfrac{3}{10}$

29. $\dfrac{387,420,489}{1,000,000,000}\approx 0.3874$

31. $\frac{3}{8}$

33. a. $\frac{11}{26}$ **b.** $\frac{9}{52}$ **c.** $\frac{2}{13}$

35. a. $\frac{1}{169}$ **b.** $\frac{1}{221}$

37. 18.75%

39. a. $\frac{1}{6}$ **b.** $\frac{5}{18}$ **c.** $\frac{1}{9}$

41. $\frac{1}{20}$

43. $\frac{2}{9}$

45. $\frac{3}{10}$

47. 27 tickets

Chapter 13 Project

1. a. $\frac{9}{19}$ **b.** $\frac{9}{19}$ **c.** $\frac{1}{38}$ **d.** $\frac{3}{38}$ **e.** $\frac{1}{38}$

3. Approximately –\$0.05 (or the person betting will lose about 5 cents per play on average)

Chapter 13 Review Exercises

1. –3, 9, –27, 81, –243

3. –3, –4, –5, –6, –7

5. $a_n = 6n - 13$

7. $a_n = n^2 - 1$

9. $a_1 = -2, a_n = n(a_{n-1})$ for $n \geq 2$

11. $-3-5-7-9-11-13 = -48$

13. $\sum_{i=2}^{7} i^3 = 783$

15. $-8 - 16 - 32 - 64 - 128 = -248$

17. $S_n = \frac{n}{2(n+2)}$, $S_{80} = \frac{20}{41}$

19. $S_n = 3 - 3^{n+1}$, series diverges

21. –10, –12, –22, –34, –56

23. $a_n = \frac{5}{2}n + 9$

25. $a_n = 3n - 1$

27. $a_n = 9n - 14$

29. 275

31. a_{36} **33.** 8827

35. 66

37. $a_n = 3\left(\frac{1}{5}\right)^{n-1}$

39. $a_n = 6(4)^{n-1}$

41. $a_n = 8\left(\frac{1}{4}\right)^{n-1}$

43. $r = \pm\frac{2}{3}$; $\pm\frac{45}{8}, \frac{15}{4}, \pm\frac{5}{2}, \frac{5}{3}, \pm\frac{10}{9}$

45. –2

47. $\frac{381}{512}$

49. –12

51. 1

53.

Basic Step: $n = 1, (3(1)+2) = 5$ and $\frac{1(3(1)+7)}{2} = 5$;

Induction Step:

If $5+8+11+\cdots+(3k+2) = \frac{k(3k+7)}{2}$, then

$$5+8+11+\cdots+(3k+2)+(3(k+1)+2)$$
$$= \frac{k(3k+7)}{2} + (3k+5)$$
$$= \frac{3k^2+7k+6k+10}{2}$$
$$= \frac{(k+1)(3k+10)}{2}$$
$$= \frac{(k+1)(3(k+1)+7)}{2}$$

55. Basic Step: $n = 1$, $11^1 - 7^1 = 4$, which is divisible by 4;

Induction Step:

If $\frac{11^k - 7^k}{4} = p$ or $11^k - 7^k = 4p$ for some integer p, then

$$11^{k+1} - 7^{k+1} = 11 \cdot 11^k - 7 \cdot 7^k$$
$$= 4 \cdot 11^k + 7 \cdot 11^k - 7 \cdot 7^k$$
$$= 4 \cdot 11^k + 7(11^k - 7^k)$$
$$= 4 \cdot 11^k + 7(4p) = 4(11^k + 7p)$$

57. $8 \cdot 10^3 \cdot 26^3 = 140,608,000$

59. $\frac{8!}{3!2!} = 3360$

61. ${}_{21}P_5 = 2,441,880$

63. $-32y^5 + 80y^4 - 80y^3 + 40y^2 - 10y + 1$

65. $3125x^{10} - 6250x^8 y + 5000x^6 y^2 - 2000x^4 y^3 + 400x^2 y^4 - 32y^5$

67. a. $\frac{1}{9}$ **b.** $\frac{2}{3}$

69. a. $\frac{2}{7}$ **b.** $\frac{4}{7}$

71. a. $\frac{1}{10}$ **b.** $\frac{4}{5}$

73. $\frac{33}{108,290}$