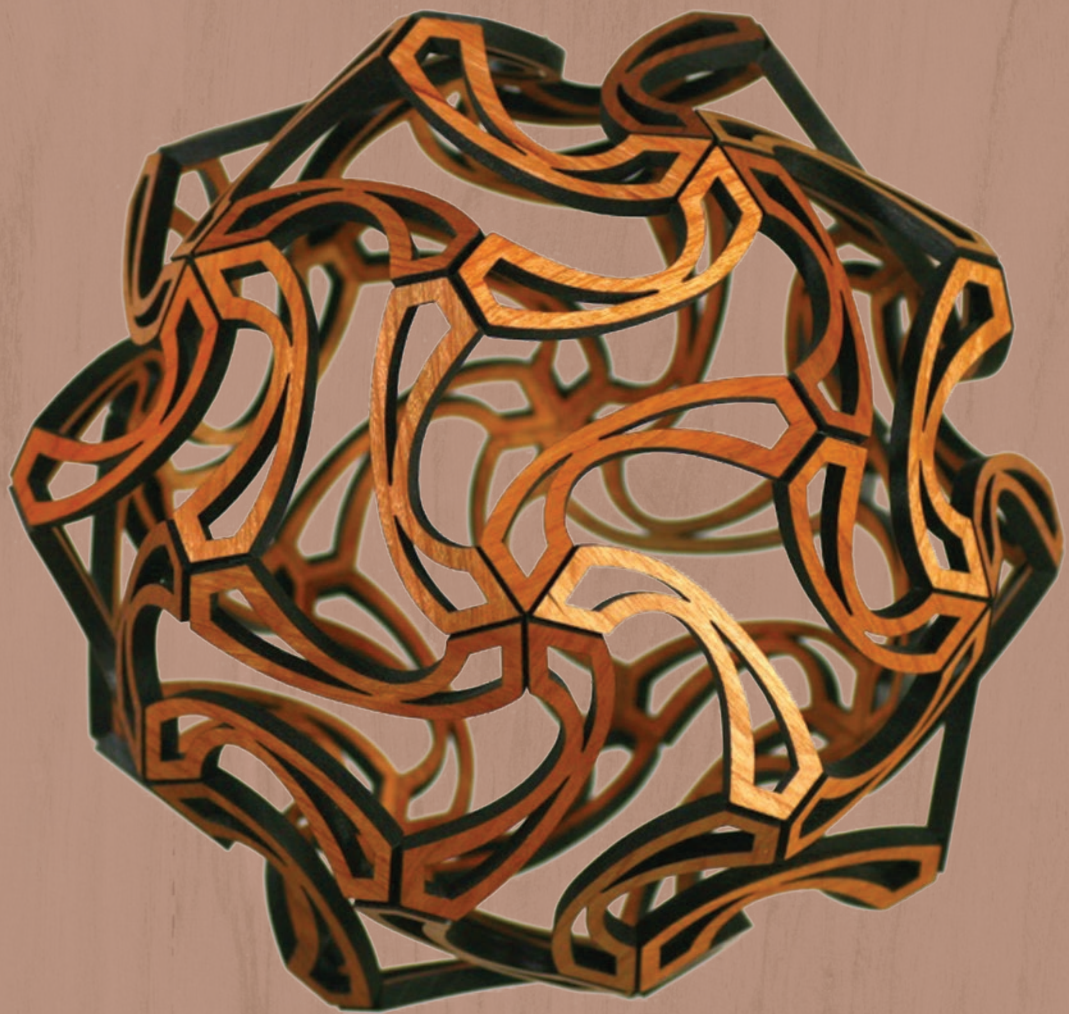


ALGEBRA *and* TRIGONOMETRY

CHAPTER PROJECTS



PAUL SISSON



CHAPTER 1 PROJECT

Polynomials



A chemistry professor calculates final grades for her class using the polynomial

$$A = 0.3f + 0.15h + 0.4t + 0.15p,$$

where A is the final grade, f is the final exam, h is the homework average, t is the chapter test average, and p is the semester project.

The following is a table containing the grades for various students in the class.

Name	Final Exam	Homework Avg.	Test Avg.	Project
Alex	77	95	79	85
Ashley	91	95	88	90
Barron	82	85	81	75
Elizabeth	75	100	84	80
Gabe	94	90	90	85
Lynn	88	85	80	75

1. Find the final grade for each student, rounded to one decimal place.
2. Who has the highest total score?
3. Why is the final grade raised more with a grade of 100 on the final exam than with a grade of 100 on the semester project?
4. Assume you are a student in this class. With one week until the final exam, you have a homework average of 85, a test average of 85, and a 95 on the semester project. What score must you make on the final exam to achieve at least a 90.0 overall? Round your answer to one decimal place.

CHAPTER 2 PROJECT

Purchasing a New Car



There are many financing options for new car buyers, and sometimes comparing offers between dealerships can be confusing. Newspaper and television ads often seem much more complicated once the fine print is read. If you decide to purchase a new car, be sure to get all the details and remember that the dealerships might be negotiating on different variables. To do a thorough comparison, you must take all the variables into consideration.

Assume you have decided to purchase a new car with a manufacturer's suggested retail price (MSRP) of \$22,000, including all the options you have selected. There are two local dealerships that carry this car, and you have collected offers from both of them. You plan to use the trade-in value of your old car as a down payment. The dealership offers and the assessed values for your car are listed in the table below.

Dealership	Factory MSRP	Dealer Incentive	Trade-in Value	Financed Amount	Term of Loan	Annual Rate of Interest
City Motors	\$22,000	\$1200	\$2500	\$18,300	48 months	11%
City Motors	\$22,000	\$1000	\$2500	\$18,500	36 months	4.5%
City Motors	\$22,000	\$1000	\$2500	\$18,500	48 months	7.9%
Arrow Imports	\$22,000	\$900	\$3000	\$18,100	48 months	9.9%
Arrow Imports	\$22,000	\$500	\$3000	\$18,500	24 months	3.9%

1. Your monthly payment can be calculated using the formula

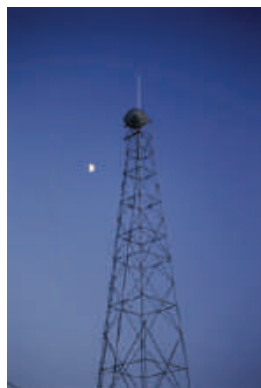
$$P = A \left[\frac{1 - (1 + r)^{-n}}{r} \right]^{-1},$$

where P represents your payment, A is the financed amount, r is the monthly interest rate in decimal form ($r = \text{annual rate} / 12$), and n is the duration of the loan in months. Compute the monthly payment for each of the scenarios above.

2. What is the total cost of the car for each of the scenarios?
3. How much interest is paid in total for each of the scenarios?
4. Which of these scenarios is the best for you? What is best for you may not be what is best for everyone, so explain the reasons for your selection.

CHAPTER 3 PROJECT

Using the Pythagorean Theorem

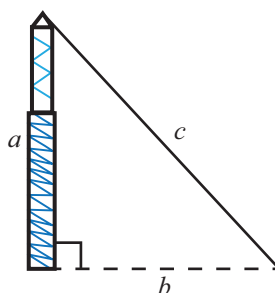


Assume that a company that builds radio towers has hired you to supervise the installation of steel support cables for several newly built structures. Your task is to find the point at which the cables should be secured to the ground. Assume that the cables reach from ground level to the top of each tower. The cables have been precut by a subcontractor and have been labeled for each tower. Finding the correct distance from the base is necessary because each cable must be grounded before being attached to a tower to avoid damaging the equipment by electric shock.

The following is your work list for this week.

Tower Name	Tower Height	Cable Length	Distance from Base
Shelbyville Tower	58 ft	75 ft	_____
Brockton Tower	100 ft	125 ft	_____
Springfield Tower	77 ft	98 ft	_____
Ogdenville Tower	130 ft	170 ft	_____

1. Use the Pythagorean formula ($a^2 + b^2 = c^2$) to determine how far from the base of the towers to attach the cables to the ground.



2. What length of cable would you need for a 150-foot tower if the grounding point has to be 100 feet from the base?
3. You have 400 feet of cable and wish to attach two lengths opposite one another to the top of a tower 200 feet tall, securing both lengths to the ground at a required distance of 75 feet from the tower's base. Do you have enough cable to do so? If so, how much is left after attaching the two cables? If not, you decide in advance to run one length to the top of the tower and use the remaining length of cable on the other side, attaching it to the highest point possible on the tower. How far down from the top would that attachment point be?
4. The tallest radio tower in the United States is in the Oro Valley near Tucson, Arizona. A cable from its top attached to the ground 260 feet from its base is 700 feet long. How tall is the radio tower?

CHAPTER 4 PROJECT

Demand Curves and Revenue

The graph of the relationship between the price p of a given product and the quantity x of that product sold is called the **demand curve** for the product. It typically reflects the fact that an increase in price results in lower sales and a decrease in price results in higher sales. The revenue realized by selling quantity x of the product at price p is then given by the function $R(x) = xp$.

1. One common relationship between p and x is an equation of the form

$$p + bx = a,$$

where a and b are positive constants.

- With the horizontal axis representing x and the vertical axis representing p , sketch the general shape of the demand curve associated with this relationship. (**Hint:** It may help to initially pick specific values for a and b in order to identify the behavior.)
- In words, how would you describe the dependence of p on x ?
- What value of x corresponds to the largest feasible value for p ?
- What value of p corresponds to the largest feasible value for x ?
- Find a formula for the revenue function $R(x)$ as a function of x alone.
- What class of function is $R(x)$?
- How would you describe the graph of $R(x)$?

2. Another common relationship between p and x is an equation of the form

$$px = a,$$

where again a is a positive constant.

- With the horizontal axis representing x and the vertical axis representing p , sketch the general shape of the demand curve associated with this relationship. (It may again help to initially pick a specific value for a in order to identify the behavior.)
- What is the smallest feasible value for x in this relationship?
- What value of p corresponds to the smallest feasible value for x ?
- Is there a smallest feasible value for p in this relationship?
- Find a formula for the revenue function $R(x)$.
- What class of function is $R(x)$?
- How would you describe the graph of $R(x)$?

CHAPTER 5 PROJECT

The Ozone Layer

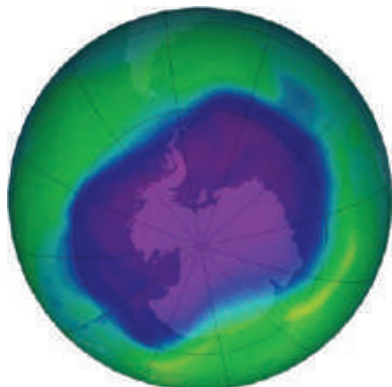


Photo Courtesy of NASA

The level of ozone in Earth's atmosphere has been closely watched since the 1970s, with the level of ozone depletion at the south pole a good indicator of global trends. One model of the region of polar depletion assumes the region is circular and that its radius, over a certain period of time, grows at a constant rate of 2.6 kilometers per hour.

1. Write the area of the circle as a function of the radius r .
2. Assuming that t is measured in hours, that $t = 0$ corresponds to the start of the annual growth of the hole, and that the radius of the hole is initially 0 kilometers, write the radius as a function of time t .
3. Write the area of the circle as a function of time t .
4. What is the radius after 3 hours?
5. What is the radius after 5.5 hours?
6. What is the area of the circle after 3 hours?
7. What is the area of the circle after 5.5 hours?
8. What is the average rate of change of the area from 3 hours to 5.5 hours?
9. What is the average rate of change of the area from 5.5 hours to 8 hours?
10. Is the average rate of change of the area increasing or decreasing as time passes?

CHAPTER 6 PROJECT

Polynomial Functions

Ace Electric Motorcycles is a new business that has just finished the design phase for its first product, and they're ready to begin production. The monthly costs for their staff, utilities, and plant lease is projected to be \$729,000, and the cost to produce each motorcycle will be \$4500. Their business plan projects that the revenue from selling x motorcycles will be $-3x^2 + 10,000x$.

1. Given that profit is equal to revenue minus cost, what is their profit for producing and selling x motorcycles per month?
2. What monthly interval of production will be profitable for the company?
3. Given that revenue is equal to the number of items sold times the price per item, what price range for the motorcycle corresponds to the profitable production interval?
4. What number of motorcycles produced and sold each month will maximize their profit?
5. Their executive team is prepared for monthly fixed costs to be as much as \$300,000 higher. In that worst case scenario, what does their monthly profitable production interval become?

CHAPTER 7 PROJECT

Exponential Functions



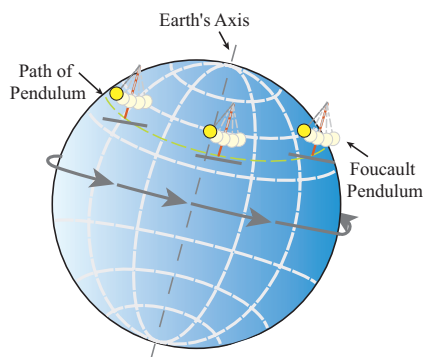
Computer viruses cause enormous economic harm to businesses through costs associated with preventive measures, data recovery, and damaged equipment and reputation. The speed at which viruses can spread makes it difficult to manage an attack.

Suppose a new virus has been created and initially infects 100 computers in a large corporation through a single email. Let $t = 0$ be the time of the initial infection, and suppose the number of computers infected in the corporation doubles every 47 minutes. The infection is finally brought under control 6 hours later.

1. How many computers are infected at the 1 hour mark?
2. How many computers are infected after another 30 minutes?
3. How many computers are infected when the virus is finally brought under control?
4. A second virus is detected a week later, and the number of computers it infects t minutes after initial infection is estimated to be $50e^{\frac{t}{110}}$. To the nearest minute, what is the doubling time for this virus?

CHAPTER 8 PROJECT

Trigonometric Applications



At the Paris Observatory in 1851, Jean Foucault used a long pendulum to prove that Earth is rotating. As it swings, the pendulum appears to change its path. However, it is not the pendulum that changes path, but the room rotating underneath it. At the North Pole, Earth revolves 360° underneath the pendulum over 24 hours. The path of a pendulum at the equator does not revolve at all; instead, the pendulum travels in a huge circle while Earth spins. At points between the two, the pendulum cannot show how far it travels, but it can show how much planet Earth is revolving underneath it. To calculate how much our planet revolves in a particular location, use the following equation.

$$\text{degrees of revolution} = 360^\circ \sin(\text{latitude of location})$$

Location	Latitude
United Nations, NY	$40^\circ 44' 58''$
California Academy of Sciences	$37^\circ 46' 12''$
Smithsonian, Washington, DC	$38^\circ 53' 19''$
St. Isaac's Cathedral, Russia	$59^\circ 53' 02''$
Paris Observatory, France	$48^\circ 48' 58''$

1. In a 24-hour period, how many degrees does Earth revolve at the locations specified in the table?

Your university has decided to install a 50-foot Foucault Pendulum in your science building and asked you to make sure that there is enough room.

2. If the pendulum swings a total of 16° , how long is the arc traced in the air by the tip of the pendulum during one swing?
3. The school plans to build a small wall encircling the swinging pendulum. What should the diameter of the circle be if they want the tip of the pendulum to come within 6 inches of the wall?
4. When the pendulum reaches the farthest point from the center, how much higher will the tip be compared to when it is at the center?
5. If the science center only has room for a circular wall of diameter 12 feet, how many degrees can the pendulum swing and still stay 6 inches from the wall?
6. The Foucault Pendulum in the United Nations building has a length of 75 feet and a period of 10 seconds. Assuming simple harmonic motion and that at $t = 0$ the pendulum is at its farthest distance away (6 feet from the center of the circle), what function models the motion of the pendulum? Graph this function.

CHAPTER 9 PROJECT

Trigonometric Identities



Lasers are used in such varied applications as video players, checkout counter scanners, surgical operations, and weaponry. A beam of light from a flashlight shines for a couple hundred yards, but a laser's narrow band of light can be reflected off the moon and detected on Earth. A laser has this ability because its light is *coherent*. Coherent light means that each light wave has exactly the same amplitude, direction, and phase. Coherence reflects the superposition principle which states that when combining two waves, the resulting wave is the sum of the two individual waves.

Let's examine how the superposition principle works.

1. Consider two waves with a difference in displacement of $\frac{\pi}{2}$.

$$y_1 = 2 \sin(kx - \omega t)$$
$$y_2 = 2 \sin\left(kx - \omega t + \frac{\pi}{2}\right)$$

Using a trigonometric identity, add these two waves to find the equation of their superposition. What is the amplitude of the resulting wave?

2. The following equations are given.

$$y_1 = A \sin(kx - \omega t)$$
$$y_2 = A \sin(kx - \omega t + \delta)$$

- a. For what values of δ would the amplitude be the largest? (This happens when two waves are coherent and it is called constructive interference.)
 - b. What is the smallest amplitude possible for $y_1 + y_2$?
 - c. For what values of δ would the smallest amplitude occur? (This is called destructive interference.)
3. Graph the following equations.

$$y_1 = 3 \sin t$$
$$y_2 = 3 \sin\left(t + \frac{\pi}{3}\right)$$

Now graph $y_1 + y_2$. Discuss the relationship between the three graphs.

CHAPTER 10 PROJECT

Trigonometric Applications



Built by King Khufu from 2589–2566 BC to serve as his tomb, the Great Pyramid of Giza covers 13 acres and weighs more than 6.5 million tons. The Great Pyramid is the oldest and the only remaining wonder of the 7 Wonders of the Ancient World. Even using modern technology, engineers in the 21st century would have difficulties recreating this impressive structure. Today, we can only put forth theories as to how this amazing structure was created.

When first built, the Egyptians called the Great Pyramid Ikhet (which means Glorious Light) because the sides of the pyramid were covered in highly polished limestone that would have shone brightly under the hot Egyptian Sun.

1. When built, the length of each side of the base was 754 ft and the distance from each corner of the base to the peak was 718 ft. What was the surface area of the four sides?

The Egyptians quarried most of the stone locally, but they also floated huge granite blocks down the Nile River from Aswan.

2. Your barge with the latest shipment of granite for King Khufu is quickly approaching Giza. The river is flowing at 195 yards per minute. You command your oarsmen to start rowing towards shore at a 65° angle from the direction of the current. They can row 260 yards per minute.
 - a. What is the resultant true velocity of the barge?
 - b. The Nile River is 840 yards wide near Giza. If the boat is in the center of the river and the dock is 750 yards ahead, will they hit the bank before or after it? By how much will they miss it? (**Hint:** (velocity)(time) = distance.)
3. Once the boat reaches the shore, the granite stones need to be moved into place.
 - a. If a granite block weighs 8300 pounds and 12 of your men are each pulling on it (in the same direction) with a force of 115 pounds, what is the total force being applied?
 - b. In order to get the stone to the necessary spot, the stone must be pulled up a ramp that has been built around the pyramid. If the ramp has a 9° grade, what force is necessary to keep the block from sliding?
 - c. If the top of the pyramid is currently 320 feet above the desert, how long does the ramp have to be?
 - d. How much work is it for the 12 men to drag the stone to the top of the pyramid? (Remember, in this case work is done in both the horizontal and vertical directions.)

CHAPTER 11 PROJECT

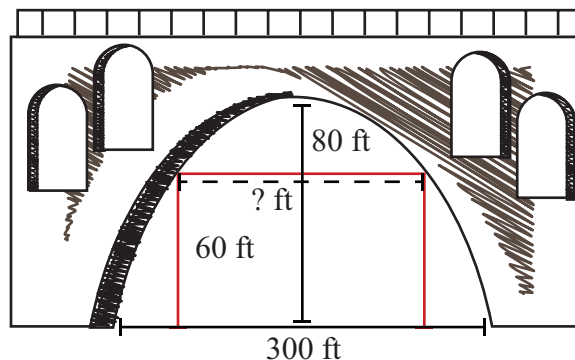


Constructing a Bridge

Plans are in process to develop an uninhabited coastal island into a new resort. Before development can begin, a bridge must be constructed joining the island to the mainland.

Two possibilities are being considered for the support structure of the bridge. The archway could be built as a parabola, or in the shape of a semiellipse.

Assume all measurements that follow refer to dimensions at high tide. The county building inspector has deemed that in order to establish a solid foundation, the space between supports must be at most 300 feet and the height at the center of the arch should be 80 feet. There is a commercial fishing dock located on the mainland whose fishing vessels travel constantly along this intracoastal waterway. The tallest of these ships requires 60 feet of clearance to pass comfortably beneath the bridge. With these restrictions, the width of a channel with a minimum height of 60 feet has to be determined for both possible shapes of the bridge to confirm that it will be suitable for the water traffic beneath it.



1. Find the equation of a parabola that will fit these constraints.
2. How wide is the channel with a minimum 60-foot vertical clearance for the parabola in question 1?
3. Find the equation of a semiellipse that will fit these constraints.
4. How wide is the channel with a minimum 60-foot vertical clearance for the semiellipse in question 3?
5. Which of these bridge designs would you choose, and why?
6. Suppose the tallest fishing ship installs a new antenna which raises the center height by 12 feet. How far off of center (to the left or right) can the ship now travel and still pass under the bridge without damage to the antenna
 - a. for the parabola?
 - b. for the semiellipse?

CHAPTER 12 PROJECT

Market Share Matrix



Assume you are the sales and marketing director for Joe's Java, a coffee shop located on a crowded city street corner. There are two competing coffee shops on this block—Buck's Café and Tweak's Coffee. The management has asked you to develop a marketing campaign to increase your market share from 25% to at least 35% within 6 months. With the resulting plan to meet this goal, you predict that each month

- you will retain 93% of your customers, 4% will go to Buck's Café, and 3% will go to Tweak's Coffee;
- Buck's Café will retain 91% of their customers, 6% will come to Joe's Java, and 3% will go to Tweak's Coffee; and
- Tweak's Coffee will retain 92% of their customers, 3% will come to Joe's Java and 5% will go to Buck's Café.

The current percentage of the market is shown in this matrix:

$$x_0 = \begin{bmatrix} 0.25 \\ 0.45 \\ 0.30 \end{bmatrix} \begin{matrix} \text{Joe's} \\ \text{Buck's} \\ \text{Tweak's} \end{matrix}$$

After one month the shares of the coffee shops are

$$x_1 = Px_0 = \begin{bmatrix} 0.93 & 0.06 & 0.03 \\ 0.04 & 0.91 & 0.05 \\ 0.03 & 0.03 & 0.92 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.45 \\ 0.30 \end{bmatrix} = \begin{bmatrix} 0.2685 \\ 0.4345 \\ 0.2970 \end{bmatrix}$$

- Construct a table that lists the market share for all of the coffee shops at the end of each of the first 6 months.
- Will your campaign be successful based on this model? (Will you reach 35% market share in 6 months?)
- What actions do you think Buck's Café and Tweak's Coffee will take as your market share changes?
- What effect could their actions have on the market?

CHAPTER 13 PROJECT

Probability



You may be familiar with the casino game of roulette. But have you ever tried to compute the probability of winning on a given bet?

The roulette wheel has 38 total slots. The wheel turns in one direction and a ball is rolled in the opposite direction around the wheel until it comes to rest in one of the 38 slots. The slots are numbered 00, and 0–36. Eighteen of the slots between 1 and 36 are colored black and eighteen are colored red. The 0 and 00 slots are colored green and are considered neither even nor odd, and neither red nor black. These slots are the key to the house's advantage.

The following are some common bets in roulette:

A gambler may bet that the ball will land on a particular number, or a red slot, or a black slot, or an odd number, or an even number (not including 0 or 00). He or she could wager instead that the ball will land on a column (one of 12 specific numbers between 1 and 36), or on a street (one of 3 specific numbers between 1 and 36).

The payoffs for winning bets are

- 1 to 1 on odd, even, red, and black
- 2 to 1 on a column
- 11 to 1 on a street
- 35 to 1 any one number

1. Compute the probability of the ball landing on
 - a. a red slot.
 - b. an odd number.
 - c. the number 0.
 - d. a street (any of 3 specific numbers).
 - e. the number 2.
2. Based on playing each of the scenarios above (a.–e.) compute the winnings for each bet individually, if \$5 is bet each time and all 5 scenarios lead to winnings.
3. If \$1 is bet on hitting just one number, what would be the expected payoff? (**Hint:** Expected payoff is $[(\text{probability of winning}) \cdot (\text{payment for a win})] - [(\text{probability of losing}) \cdot (\text{payout for a loss})]$.)
4. Given the information in question 3, would you like to play roulette on a regular basis? Why or why not? Why will the casino acquire more money in the long run?