Completion Example Answers

6. $$\begin{aligned} 2x^2 - 12x &= \underline{-2} \\ x^2 - 6 &= \underline{-1} \\ x^2 - 6x + \underline{9} &= \underline{-1} + \underline{9} \\ (x - \underline{3})^2 &= \underline{8} \\ x - \underline{3} &= \pm\underline{\sqrt{8}} \\ x &= \underline{3} \pm \underline{2\sqrt{2}} \end{aligned}$$

Margin Exercise Answers

1. a. $y^2 - 14y + \underline{49} = (\underline{y - 7})^2$ **b.** $x^2 + 9x + \underline{\frac{81}{4}} = \left(\underline{x + \frac{9}{2}}\right)^2$ **2.** $x = 5 \pm 2\sqrt{14}$ **3.** $x = -1 \pm \sqrt{7}$ **4.** $x = \frac{-1 \pm \sqrt{19}}{2}$ **5.** $x = -2 \pm i\sqrt{7}$ **6.** $x = 1 \pm \sqrt{6}$ **7.** $y^2 - 8y + 25 = 0$ **8.** $x^2 - 4x - 14 = 0$ **9.** $x^2 - 2x + 6 = 0$

9.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Completing the square is the process of adding terms to binomials so that the result will be a perfect square ______.

2. To solve a quadratic equation by completing the square, arrange terms with _________ on one side of the equation and _________ on the other.

3. When solving by completing the square, the quadratic equation should have a leading coefficient of ______.

4. After finding the coefficient that completes the square of the polynomial, ______ this constant to both sides of the equation.

5. When completing the square, the constant term is the ______ of $\frac{1}{2}$ of the coefficient of x.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. To get a leading coefficient of 1, multiply both sides of the equation by the reciprocal of the leading coefficient.

7. When solving a quadratic equation, there is either no solution or two solutions.

8. It's possible for the roots of a quadratic equation to be nonreal numbers.

9. The last step of solving a quadratic equation by completing the square is to use the square root property.

Practice

Add the correct constant to complete the square; then factor the trinomial as indicated. See Example 1.

1. $x^2 - 12x + \_\_\_ = (\_\_\_\_\_)^2$
2. $y^2 + 14y + \_\_\_ = (\_\_\_\_\_)^2$
3. $x^2 + 6x + \_\_\_ = (\_\_\_\_\_)^2$
4. $x^2 + 8x + \_\_\_ = (\_\_\_\_\_)^2$
5. $x^2 - 5x + \_\_\_ = (\_\_\_\_\_)^2$
6. $x^2 + 7x + \_\_\_ = (\_\_\_\_\_)^2$
7. $y^2 + y + \_\_\_ = (\_\_\_\_\_)^2$
8. $x^2 + \frac{1}{2}x + \_\_\_\_ = (\_\_\_\_\_\_\_)^2$
9. $x^2 + \frac{1}{3}x + \_\_\_\_ = (\_\_\_\_\_\_\_)^2$
10. $y^2 + \frac{3}{4}y + \_\_\_\_ = (\_\_\_\_\_\_\_)^2$
11. $2x^2 + 4x + \_\_\_\_\_ = 2(\_\_\_\_\_)^2$
12. $3x^2 + 18x + \_\_\_\_\_ = 3(\_\_\_\_\_)^2$

Solve the quadratic equations by completing the square. See Examples 2 through 6.

13. $x^2 + 4x - 5 = 0$
14. $x^2 + 6x - 7 = 0$
15. $y^2 + 2y = 5$
16. $x^2 + 3 = 8x$
17. $x^2 + 3 = 10x$
18. $z^2 + 4z = 2$
19. $x^2 - 4x - 45 = 0$
20. $x^2 - 10x + 21 = 0$
21. $x^2 - 3x - 40 = 0$
22. $x^2 + x - 42 = 0$
23. $3x^2 + x - 4 = 0$
24. $2x^2 + x - 6 = 0$
25. $x^2 - 6x + 10 = 0$
26. $x^2 - 2x + 5 = 0$
27. $x^2 + 11 = 12x$
28. $x^2 = 6 - x$
29. $y^2 = 10y - 4$
30. $x^2 = 3 - 4x$
31. $z^2 + 3z - 5 = 0$
32. $x^2 - 5x + 5 = 0$
33. $x^2 + x + 2 = 0$
34. $y^2 + 3y + 3 = 0$
35. $x^2 + 5x + 2 = 0$
36. $4x^2 + 7x + 2 = 0$
37. $3x^2 - 10x + 5 = 0$
38. $3y^2 + 5y - 3 = 0$
39. $3x^2 + 6x + 18 = 0$
40. $4x^2 + 8x + 16 = 0$
41. $2x - 3 = 4x^2$
42. $2x + 2 = -6x^2$
43. $5y^2 + 15y + 25 = 0$
44. $4x^2 + 20x + 32 = 0$
45. $3y^2 = 4 - y$
46. $2x^2 + 4 = -9x$
47. $2x^2 - 8x + 4 = 0$
48. $3x^2 - 18x + 12 = 0$

Write a quadratic equation with integer coefficients that has the given roots.
See Examples 7 through 9.

49. $x = \sqrt{7}, x = -\sqrt{7}$

50. $x = \sqrt{6}, x = -\sqrt{6}$

51. $x = 1 + \sqrt{3}, x = 1 - \sqrt{3}$

52. $z = 3 + \sqrt{2}, z = 3 - \sqrt{2}$

53. $y = -2 + 2\sqrt{5}, y = -2 - 2\sqrt{5}$

54. $x = 1 + 2\sqrt{3}, x = 1 - 2\sqrt{3}$

55. $x = 4i, x = -4i$

56. $x = 7i, x = -7i$

57. $y = i\sqrt{6}, y = -i\sqrt{6}$

58. $y = i\sqrt{5}, y = -i\sqrt{5}$

59. $x = 2 + i, x = 2 - i$

60. $x = -3 + 2i, x = -3 - 2i$

61. $x = 1 + i\sqrt{2}, x = 1 - i\sqrt{2}$

62. $x = 2 + i\sqrt{3}, x = 2 - i\sqrt{3}$

63. $x = -5 + 2i\sqrt{6}, x = -5 - 2i\sqrt{6}$

64. $y = 4 + 3i\sqrt{2}, y = 4 - 3i\sqrt{2}$

Applications

Solve.

65. A local frame shop determines that the revenue function for their custom framing service is $R(p) = 360p - 4p^2$, where p is the base price in dollars for each custom framing job.

- **a.** Set the function equal to 0 and solve for p using the method of completing the square.
- **b.** What do the solutions from part a. mean?

66. The height of a golf ball that is hit from the ground at a speed of 128 feet per second can be modeled with the expression $h(t) = -16t^2 + 128t$, where t is the time in seconds after the ball is hit.

- **a.** Set the function equal to 0 and solve for t using the method of completing the square.
- **b.** What do the solutions from part a. mean?

Writing & Thinking

67. Explain, in your own words, the steps involved in the process of solving a quadratic equation by completing the square.