3. Simplify each power of i.

a. i^{37}

b. i^{14}

c. i^{-8}

Example 3 Simplifying Powers of *i*

Simplify each power of i.

a. $i^{45} = i^{44} \cdot i = \left(i^4\right)^{11} \cdot i = 1^{11} \cdot i = i$ $i = 0 + i$ in standard form.

b. $i^{58} = i^{56} \cdot i^2 = \left(i^4\right)^{14} \cdot i^2 = 1^{14} \cdot (-1) = -1$ $1 = -1 + 0i$ in standard form.

c. $i^{-7} = \frac{1}{i^7} = \frac{1}{i^7} \cdot \frac{i}{i} = \frac{i}{i^8} = \frac{i}{1} = i$ $i = 0 + i$ in standard form.

Now work margin exercise 3.

Margin Exercise Answers

1. a. $28 + 4i$ **b.** $2 + 26i$ **c.** 3 **2. a.** $\frac{8}{5} + \frac{16}{5}i$ **b.** $\frac{1}{9} - \frac{4\sqrt{5}}{9}i$ **c.** $-\frac{1}{4} - i$ **d.** $\frac{13}{19} - \frac{8\sqrt{3}}{19}i$ **3. a.** i **b.** -1 **c.** 1

8.9 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. When two complex numbers are multiplied, the _____ method can be used.

2. When two complex conjugates are multiplied, the answer will always be a nonnegative _____ _____.

3. There are 4 possible values for any power of i: ___, ___, ___, and ___.

4. The expressions $a + bi$ and $a - bi$ are _____ of each other.

5. The product of $(a + bi)(a - bi)$ is _____.

6. The expression $a + bi$ is called the _____ form of a complex number.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Regardless of the value of the exponent, the only possible values for any power of i are i and $-i$.

8. The product $\sqrt{a} \cdot \sqrt{b}$ can be rewritten as $\sqrt{ab}$ as long as a and b are real numbers.

9. When i is squared, the product is 1.

10. The conjugate of $4 - 5i$ is $4 + 5i$.

Practice

Perform the indicated operations and write each result in standard form. See Examples 1 and 2.

1. $8(2+3i)$
2. $-3(7-4i)$
3. $-7\left(\sqrt{2}-i\right)$
4. $\sqrt{3}\left(\sqrt{3}+2i\right)$
5. $3i(4-i)$
6. $-4i(6-7i)$
7. $-i\left(\sqrt{3}+i\right)$
8. $2i\left(\sqrt{5}+2i\right)$
9. $\sqrt{3}\,i\left(2-\sqrt{3}\,i\right)$
10. $5i\left(2-\sqrt{2}\,i\right)$
11. $(5+3i)(1+i)$
12. $(2+7i)(6+i)$
13. $(-3+5i)(-1+2i)$
14. $(6+2i)(3-i)$
15. $(2-3i)(2+3i)$
16. $(4+5i)(4-5i)$
17. $(4+3i)(7-2i)$
18. $(-2+5i)(i-1)$
19. $(5+7i)^2$
20. $(3+2i)^2$
21. $\left(\sqrt{3}+i\right)\left(\sqrt{3}-2i\right)$
22. $\left(2\sqrt{5}+3i\right)\left(\sqrt{5}-i\right)$
23. $\left(5-\sqrt{2}\,i\right)\left(5-\sqrt{2}\,i\right)$
24. $\left(\sqrt{7}+3i\right)\left(\sqrt{7}+i\right)$
25. $\left(4+\sqrt{5}\,i\right)\left(4-\sqrt{5}\,i\right)$
26. $\left(7+2\sqrt{3}\,i\right)\left(7-2\sqrt{3}\,i\right)$
27. $\left(\sqrt{5}+2i\right)\left(\sqrt{2}-i\right)$
28. $\left(2\sqrt{3}+i\right)(4+3i)$
29. $\left(3+\sqrt{5}\,i\right)\left(3+\sqrt{6}\,i\right)$
30. $\left(2-\sqrt{3}\,i\right)\left(3-\sqrt{2}\,i\right)$
31. $\dfrac{-3}{i}$
32. $\dfrac{7}{i}$
33. $\dfrac{5}{4i}$
34. $\dfrac{-3}{2i}$
35. $\dfrac{2+i}{-4i}$
36. $\dfrac{3-4i}{3i}$
37. $\dfrac{-4}{1+2i}$
38. $\dfrac{7}{5-2i}$
39. $\dfrac{6}{4-3i}$
40. $\dfrac{-8}{6+i}$
41. $\dfrac{2i}{5-i}$
42. $\dfrac{-4i}{1+3i}$
43. $\dfrac{2-i}{2+5i}$
44. $\dfrac{6+i}{3-4i}$
45. $\dfrac{2-3i}{-1+5i}$
46. $\dfrac{-3+i}{7-2i}$
47. $\dfrac{1+4i}{\sqrt{3}+i}$
48. $\dfrac{9-2i}{\sqrt{5}+i}$

49. $\dfrac{\sqrt{3}+2i}{\sqrt{3}-2i}$

50. $\dfrac{\sqrt{6}-3i}{\sqrt{6}+3i}$

Simplify the following powers of i and write each result in standard form. Assume k is a positive integer. See Example 3.

51. i^{13}

52. i^{20}

53. i^{30}

54. i^{15}

55. i^{-3}

56. i^{-5}

57. i^{4k}

58. i^{4k+2}

59. i^{4k+3}

60. i^{4k+1}

Find the indicated products and simplify.

61. $(x+3i)(x-3i)$

62. $(y+5i)(y-5i)$

63. $\left(x+\sqrt{2}\,i\right)\left(x-\sqrt{2}\,i\right)$

64. $\left(2x+\sqrt{7}\,i\right)\left(2x-\sqrt{7}\,i\right)$

65. $\left(\sqrt{5}y+2i\right)\left(\sqrt{5}y-2i\right)$

66. $\left(y-\sqrt{3}\,i\right)\left(y+\sqrt{3}\,i\right)$

67. $\left[(x+2)+6i\right]\left[(x+2)-6i\right]$

68. $\left[(x+1)-\sqrt{8}\,i\right]\left[(x+1)+\sqrt{8}\,i\right]$

69. $\left[(y-3)+2i\right]\left[(y-3)-2i\right]$

70. $\left[(x-1)+5i\right]\left[(x-1)-5i\right]$

Writing & Thinking

71. Explain why the product of every complex number and its conjugate is a nonnegative real number.

72. Explain why $\sqrt{-4}\cdot\sqrt{-4}\neq 4$. What is the correct value of $\sqrt{-4}\cdot\sqrt{-4}$?

73. What condition is necessary for the conjugate of a complex number, $a + bi$, to be equal to the reciprocal of this number?