$$\frac{15}{x^2-16}+\frac{x}{x+4}-\frac{x+3}{x-4}=\frac{15}{(x+4)(x-4)}+\frac{x}{x+4}\cdot\frac{(x-4)}{(x-4)}-\frac{x+3}{x-4}\cdot\frac{(x+4)}{(x+4)}$$
$$=\frac{15+x(x-4)-(x+3)(x+4)}{(x+4)(x-4)}$$
$$=\frac{15+x^2-4x-(x^2+7x+12)}{(x+4)(x-4)}$$
$$=\frac{15+x^2-4x-x^2-7x-12}{(x+4)(x-4)}$$
$$=\frac{3-11x}{(x+4)(x-4)}$$

Now work margin exercise 7.

Completion Example Answers

3. LCD $=(y-5)(y+6)$;

$$\frac{y(y+6)}{(y-5)(y+6)}+\frac{3(y-5)}{(y+6)(y-5)}=\frac{(y^2+6y)+(3y-15)}{(y+6)(y-5)}=\frac{y^2+9y-15}{(y+6)(y-5)}$$

6. $$\frac{x}{(x-2)(x+1)}-\frac{1(x+1)}{(x-2)(x+1)}=\frac{x-(x+1)}{(x-2)(x+1)}=\frac{-1}{(x-2)(x+1)}$$

Margin Exercise Answers

1. a. $\frac{1}{x-5}$; $x\neq-5,5$ **b.** $\frac{3}{x+5}$; $x\neq-5,-3$ **2. a.** $\frac{x^2+6x+6}{(x+3)(x+2)}$ **b.** $\frac{x^2+2x-25}{(x+5)^2(x-5)}$

3. $\frac{s^2+5s+12}{(s+3)(s+1)}$ **4. a.** $\frac{x+2y}{3x-y}$ **b.** $\frac{x-5}{x+3}$ **c.** $\frac{x+5}{x-4}$ **5. a.** $\frac{x+18}{x+6}$ **b.** $\frac{15x^2-13xy-4y^2}{3(x+y)^2(x-y)}$

c. $\frac{-x^2-2x+12}{(x+6)(x+3)}$ **d.** $\frac{11y+17}{(x-2)(y+1)(y+3)}$ **6.** $\frac{-4x+1}{(x-1)^2}$ **7.** $\frac{8y-1}{(5-y)(5+y)}$

7.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. To add rational expressions with a common denominator, proceed just as with fractions: add the __________ and keep the common ___________.

2. When finding the LCM for a set of polynomials, the first step is to _______ each polynomial.

3. Next, form the product of all factors that appear, using each factor the ______ number of times it appears in any one polynomial.

4. To add rational expressions with different denominators, first find the _______.

5. Then, rewrite each fraction in a/an ____________ form with the LCD as the denominator.

6. The final step when adding or subtracting rational expressions is to ______, if possible.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The LCM of a set of denominators is called the least common denominator.

8. With polynomials, it is most common to place negative signs in the denominator.

9. As with addition, when subtracting rational expressions with different denominators, the first step is to find the LCM of the denominators.

10. You should not use parentheses when subtracting rational expressions.

Practice

Perform the indicated operations and reduce, if possible. Assume that no denominator has a value of 0.

1. $\frac{3x}{x+4}+\frac{12}{x+4}$

2. $\frac{7x}{x+5}+\frac{35}{x+5}$

3. $\frac{x-1}{x+6}+\frac{x+13}{x+6}$

4. $\frac{3x-1}{2x-6}+\frac{x-11}{2x-6}$

5. $\frac{3x+1}{5x+2}+\frac{2x+1}{5x+2}$

6. $\frac{x^2+3}{x+1}+\frac{4x}{x+1}$

7. $\frac{x-5}{x^2-2x+1}+\frac{x+3}{x^2-2x+1}$

8. $\frac{2x^2+5}{x^2-4}+\frac{3x-1}{x^2-4}$

9. $\frac{13}{7-x}-\frac{1}{x-7}$

10. $\frac{6x}{x-6}+\frac{36}{6-x}$

11. $\frac{3x}{x-4}+\frac{16-x}{4-x}$

12. $\frac{20}{x-10}-\frac{3}{10-x}$

13. $\frac{x^2+2}{x^2+x-12}+\frac{x+1}{12-x-x^2}$

14. $\frac{10}{x^2-x-6}-\frac{5x}{6+x-x^2}$

15. $\frac{x^2+2}{x^2-4}-\frac{4x-2}{x^2-4}$

16. $\frac{2x+5}{2x^2-x-1}-\frac{4x+2}{2x^2-x-1}$

17. $\frac{x+3}{7x-2}+\frac{2x-1}{14x-4}$

18. $\frac{3x+1}{4x+10}+\frac{4-x}{2x+5}$

19. $\frac{5}{x-3}+\frac{x}{x^2-9}$

20. $\frac{x+1}{x^2-3x-10}+\frac{x}{x-5}$

21. $\frac{x}{x-1}-\frac{4}{x+2}$

22. $\frac{x-1}{3x-1}-\frac{8+4x}{x+2}$

23. $\frac{x+2}{x+3}-\frac{4}{3-x}$

24. $\frac{x-1}{4-x}+\frac{3x}{x+5}$

25. $\frac{x+2}{3x+9}+\frac{2x-1}{2x-6}$

26. $\frac{x}{4x-8}-\frac{3x+2}{3x+6}$

27. $\frac{3x}{6+x}-\frac{2x}{x^2-36}$

28. $\frac{4}{x+5}-\frac{2x+3}{x^2+4x-5}$

29. $\frac{4x+1}{7-x}+\frac{x-1}{x^2-8x+7}$

30. $\frac{3x-4}{x^2-x-20}-\frac{2}{5-x}$

31. $\frac{4x}{x^2+3x-28}+\frac{3}{x^2+6x-7}$

32. $\frac{3x}{x^2+2x+1}-\frac{x}{x^2+9x+8}$

33. $\frac{3x+4}{2x^2-23x+30}-\frac{x+5}{2x^2-19x+24}$

34. $\frac{x+1}{x^2-3x+2}+\frac{6}{x^2-6x+8}$

35. $\frac{4x-1}{x^2-5x+4}+\frac{2x+7}{x^2-11x+28}$

36. $\frac{7x+3}{5x^2+27x+36}+\frac{3x-2}{5x^2+22x+24}$

37. $\frac{x-6}{7x^2-3x-4}+\frac{7-x}{7x^2+18x+8}$

38. $\frac{x+10}{x^2+5x+4}-\frac{4}{x^2+6x+8}$

39. $\frac{x-3}{4x^2-5x-6}-\frac{4x+10}{2x^2+x-10}$

40. $\frac{2x+1}{8x^2-37x-15}+\frac{2-x}{8x^2+11x+3}$

41. $\frac{3x}{4-x}+\frac{7x}{x+4}-\frac{x-3}{x^2-16}$

42. $\frac{x}{x+3}+\frac{x+1}{3-x}+\frac{x^2+4}{x^2-9}$

43. $-\frac{1}{2}+\frac{x-5}{x-3}+\frac{x-1}{x^2-5x+6}$

44. $-4+\frac{1-2x}{x+6}+\frac{x^2+1}{x^2+4x-12}$

45. $\frac{2}{x^2-4}-\frac{3}{x^2-3x+2}+\frac{x-1}{x^2+x-2}$

46. $\frac{5}{x^2+3x+2}+\frac{4}{x^2+6x+8}-\frac{6}{x^2+5x+4}$

47. $\frac{x}{x^2+4x-21}+\frac{1-x}{x^2+8x+7}+\frac{3x}{x^2-2x-3}$

48. $\frac{3x}{x^2+4x-5}-\frac{2}{3x^2+17x+10}-\frac{3}{3x^2-x-2}$

49. $\frac{3x+9}{x^2-5x+4}+\frac{49}{12+x-x^2}+\frac{3x+21}{x^2+2x-3}$

50. $\frac{5x+22}{x^2+8x+15}+\frac{4}{x^2+4x+3}+\frac{6}{x^2+6x+5}$

51. $\frac{x}{xy+x-2y-2}+\frac{x+2}{xy+x+y+1}$

52. $\frac{4x}{xy-3x+y-3}+\frac{x+2}{xy+2y-3x-6}$

53. $\frac{3y}{xy+2x+3y+6}+\frac{x}{x^2-2x-15}$

54. $\frac{2}{xy-4x-2y+8}+\frac{5y}{y^2-3y-4}$

55. $\frac{x+6}{2x-1}-\frac{3x^2+x-4}{2x^2-3x+1}$

56. $\frac{2x-5}{2x^2+2}+\frac{x^2-2x+5}{x^3+x^2+x+1}$

57. $\frac{x+1}{x^3-3x^2+x-3}+\frac{x^2-5x-8}{x^4-8x^2-9}$

58. $\frac{x+4}{x^3-5x^2+6x-30}-\frac{x-7}{x^3-2x^2+6x-12}$

59. $\frac{x-6}{3x^2+10x+3}-\frac{2x}{x^2+2x-3}+\frac{6x}{3x^2-2x-1}$

60. $\frac{x+1}{2x^2-x-1}+\frac{2x}{2x^2+5x+2}-\frac{2x}{x^2+x-2}$

Applications

Solve.

61. A landscaper is hired to place large flowering bushes along the borders of a botanical garden. The property is in the shape of a rectangle that measures $7x^2 + 3$ feet long by $4x^2 + 5$ feet wide. The bushes are to be placed every $x + 2$ feet across the width of the property and every $x - 2$ feet along the length of the property.

a. Write a rational expression to determine how many bushes will go along one length of the property.

b. Write a rational expression to determine how many bushes will go along one width of the property.

c. Use the rational expressions from parts a. and b. to create a rational expression to determine how many bushes will be needed to line the entire property.

62. Two teams of set designers are jointly creating a set for a scene in a movie. In one hour, the first team can create $\frac{1}{x}$ of the set and the second team can create $\frac{1}{2x-3}$ of the set. If the two teams work together, how much of the set will be completed in one hour?

63. Three janitors work the night shift at the local hospital. Working alone, it takes Marla three more hours than it takes Tom, and it takes Bob twice as long as it takes Marla. So in one hour, Tom can clean $\frac{1}{x}$ of the building, Marla can clean $\frac{1}{x+3}$ of the building, and Bob can clean $\frac{1}{2x+6}$ of the building. If all three janitors work together, how much of the building can they clean in one hour?

64. Anna's average running speed is three times faster than her walking speed. Since $\text{time} = \frac{\text{distance}}{\text{rate}}$, the time it takes Anna to run 30 km is $\frac{30}{3x}$ and the time it takes Anna to walk 30 km is $\frac{30}{x}$. Find the difference between Anna's walking time and running time for the 30 km.

65. A car and a truck are both traveling to the same destination. The car is traveling 15 mph more than twice the speed of the truck. (Use the formula $\text{time} = \frac{\text{distance}}{\text{rate}}$.)

a. Write a rational expression to describe the time it will take the truck to travel 100 miles.

b. Write a rational expression to describe the time it will take the car to travel 100 miles.

c. Find a rational expression to describe the difference in travel time between the truck and the car for the 100 miles.

66. During Expedition 34 to the International Space Station, three crew members are tasked with unloading supplies from the SpaceX Dragon spacecraft. In one hour, Chris Hadfield can unload $\frac{1}{x}$ of the supplies, Thomas Marshburn can unload $\frac{1}{x+4}$ of the supplies, and Oleg Novitskiy can unload $\frac{1}{x-2}$ of the supplies. If they work together, what portion of the supplies will they unload in one hour?

a. Find the sum of the fractions of the supplies each crew member can unload in one hour.

b. If $x = 10$, what fraction of the supplies will be unloaded after one hour?

c. When $x = 10$, will more than half of the supplies be unloaded in one hour? Explain your answer.

67. Barbara's Bombtastic Bakery was a cupcake shop when it first opened up. The bakery space that Barbara rented came with most of the equipment that was needed, such as a commercial oven and a display case. This meant that she only had to buy a mixer for $5000, cupcake pans for $250, and various utensils for $500. Barbara estimated that the ingredients for each cupcake would cost $0.35.

a. What was the total amount that Barbara spent on equipment to bake the cupcakes?

b. The total cost to bake the cupcakes is equal to the sum of the total amount spent on equipment plus the total amount spent on ingredients. Create a function $C(x)$ to describe the total cost to bake the cupcakes and use the variable x to represent the number of cupcakes baked.

c. The average cost to bake each cupcake is calculated by dividing the total cost by the number of cupcakes baked. Create a formula $A(x)$ to describe the average cost to create each cupcake.

d. After Barbara's Bombtastic Bakery's first week of business, Barbara baked a total of 950 cupcakes. What was the average cost to bake each cupcake after the first week? Round your answer to the nearest cent.

e. After one month of business, Barbara baked a total of 3500 cupcakes. What was the average cost to bake each cupcake after the first month? Round your answer to the nearest cent.

Writing & Thinking

68. Discuss the steps in the process you go through when adding two rational expressions with different denominators. That is, discuss how you find the least common denominator when adding rational expressions and how you use this LCD to find equivalent rational expressions that you can add.