6. Rewrite the polynomial expression $f(x) = 8x - 27$ by substituting for x as indicated by the function notation $f(3a-1)$.

Example 6 Evaluating Polynomials

Rewrite the polynomial expression $f(x) = 6x + 13$ by substituting for x as indicated by the function notation $f(2a+1)$.

Solution

Substitute $2a + 1$ for x throughout the polynomial.

$$\begin{aligned} f(2a+1) &= 6(2a+1)+13 \\ &= 12a+6+13 \\ &= 12a+19 \end{aligned}$$

Now work margin exercise 6.

Completion Example Answers

2. $-2x^2 + 4x + 6$; second-degree polynomial **5.** $5(3)^2 + 6(3) - 10 = 45 + 18 - 10 = 53$

Margin Exercise Answers

1. a. $9x^2$; second-degree monomial **b.** $9x^2 - 5x$; second-degree binomial **c.** $4y^3 - \frac{8}{3}y^2 + 8$; third-degree trinomial **d.** $4x^2 - 8$; second-degree binomial **e.** not a polynomial
2. $13x^3 - 2x^2 + 4$; third-degree trinomial **3.** 23 **4.** -6 **5.** 68 **6.** $24a - 35$

5.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. If the terms in a polynomial are written so that the exponents on the variable decrease in order from left to right, it is said that the expression is in _____ order.
2. A monomial or an indicated sum or difference of monomials is known as a/an _____.
3. In a polynomial, the coefficient of the term of the largest degree is called the _____ coefficient.
4. Monomials may not have fractional or _____ exponents.
5. A polynomial with two terms is called a/an _____.
6. A trinomial is a polynomial with ___ terms.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A nonzero constant is a monomial with no degree.
8. A polynomial with four terms is a quadrinomial.
9. A monomial is a polynomial with one term.
10. A polynomial must have at least two terms.

Practice

Identify each expression as a monomial, binomial, trinomial, or not a polynomial.

1. $3x^4$

2. $5y^2 - 2y + 1$

3. $8x^3 - 7$

4. $-2x^{-2}$

5. $14a^7 - 2a - 6$

6. $17x^{\frac{2}{3}} + 5x^2$

7. $6a^3 + 5a^2 - a^{-3}$

8. $-3y^4 + 2y^2 - 9$

9. $\frac{1}{2}x^3 - \frac{2}{5}x$

10. $\frac{5}{8}x^5 + \frac{2}{3}x^4$

Simplify the polynomials. Write the polynomials in descending order and state the degree and type of each polynomial. Then, state the leading coefficient. See Examples 1 and 2.

11. $y + 3y$

12. $4x^2 - x + x^2$

13. $x^3 + 3x^2 - 2x$

14. $3x^2 - 8x + 8x$

15. $x^4 - 4x^2 + 2x^2 - x^4$

16. $2 - 6y + 5y - 2$

17. $-x^3 + 6x + x^3 - 6x$

18. $11x^2 - 3x + 2 - 7x^2$

19. $6a^5 + 2a^2 - 7a^3 - 3a^2$

20. $2x^2 - 3x^2 + 2 - 4x^2 - 2 + 5x^2$

21. $4y - 8y^2 + 2y^3 + 8y^2$

22. $2x + 9 - x + 1 - 2x$

23. $5y^2 + 3 - 2y^2 + 1 - 3y^2$

24. $13x^2 - 6x - 9x^2 - 4x$

25. $7x^3 + 3x^2 - 2x + x - 5x^3 + 1$

26. $-3y^5 + 7y - 2y^3 - 5 + 4y^2 + y^2$

27. $x^4 + 3x^4 - 2x + 5x - 10 - x^2 + x$

28. $a^3 + 2a^2 - 6a + 3a^3 + 2a^2 + 7a + 3$

29. $2x + 4x^2 + 6x + 9x^3$

30. $15y - y^3 + 2y^2 - 10y^2 + 2y - 16$

Find the values of the functions as indicated. See Examples 3 through 5.

31. Given $f(x) = 3x - 10$, find

- **a.** $f(2)$
- **b.** $f(-2)$
- **c.** $f(0)$

32. Given $g(x) = -4x + 7$, find

- **a.** $g(-3)$
- **b.** $g(6)$
- **c.** $g(0)$

33. Given $p(x) = x^2 + 14x - 3$, find $p(-1)$.

34. Given $h(x) = -5x^2 - 8x + 7$, find $h(-3)$.

35. Given $f(x) = 3x^3 - 9x^2 - 10x - 11$, find $f(3)$.

36. Given $f(y) = y^3 - 5y^2 + 6y + 2$, find $f(2)$.

37. Given $p(y) = -4y^3 + 5y^2 + 12y - 1$, find $p(-10)$.

38. Given $h(a) = a^3 + 4a^2 + a + 2$, find $h(-5)$.

39. Given $p(a) = 2a^4 + 3a^2 - 8a$, find $p(-1)$.

40. Given $g(x) = 8x^4 + 2x^3 - 6x^2 - 7$, find $g(-2)$.

41. Given $g(x) = x^5 - x^3 + x - 2$, find $g(-2)$.

42. Given $p(x) = 3x^6 - 2x^5 + x^4 - x^3 - 3x^2 + 2x - 1$, find $p(1)$.

A polynomial function is given. Rewrite the polynomial function by substituting for the variable as indicated in the given function notation.

43. Given $p(x) = 3x^4 + 5x^3 - 8x^2 - 9x$, find $p(a)$.

44. Given $p(x) = 6x^5 + 5x^2 - 10x + 3$, find $p(c)$.

45. Given $f(x) = 3x + 5$, find $f(a + 2)$.

46. Given $f(x) = -4x + 6$, find $f(a - 2)$.

47. Given $g(x) = 5x - 10$, find $g(2a + 7)$.

48. Given $g(x) = -4x - 8$, find $g(3a + 1)$.

Applications

Solve.

49. A car on an amusement park ride starts with an initial velocity (or initial speed) of 5 feet per second and accelerates at a rate of 15 feet per second squared. The speed of the car during the ride can be modeled by the polynomial $p(x) = 15x + 5$, where x is the time, in seconds, since the car started moving.

- **a.** Identify the degree of the polynomial.
- **b.** Determine how fast the car is moving after 0 seconds.
- **c.** Determine how fast the car is moving after 3 seconds.
- **d.** Determine how fast the car is moving after 6 seconds.

50. A ball is thrown from the top of a building towards the ground with an initial velocity (or initial speed) of 12 feet per second. The force of gravity causes the ball to accelerate at a rate of 32 feet per second squared. The distance of the ball from the top of the building can be modeled by the polynomial $p(x) = 12x + 32x^2$ where x is the time, in seconds, since the ball was thrown.

- **a.** Identify the degree of the polynomial.
- **b.** Determine how far the ball is from the top of the building at 1 second.
- **c.** Determine how far the ball is from the top of the building at 2 seconds.
- **d.** Determine how far the ball is from the top of the building at 3 seconds.

51. Chelsea is competing in a double–elimination softball tournament, which means that a team is eliminated once they lose two games. If the winning team goes undefeated, the total number of games played will be $G(t) = 2t - 1$, where t is the number of teams participating in the tournament.

a. Identify the degree of the polynomial.

b. Determine how many games will be played if 5 teams are participating.

c. Determine how many games will be played if 6 teams are participating.

d. Determine how many games will be played if 15 teams are participating.

52. Max runs a no-kill dog shelter. To help manage finances for the upcoming year, Max uses data from the previous years to construct a mathematical model that can predict the number of dogs the shelter will have during each month of the upcoming year. The model he constructs is $D(x) = x^2 - 12x + 80$, where x is the number of the month of the year (this means that January = 1, February = 2, etc.).

a. Identify the degree of the polynomial.

b. How many animals are predicted to be in the shelter in January?

c. How many animals are predicted to be in the shelter in July?

d. How many animals are predicted to be in the shelter in December?

53. The value of a car starts to depreciate the moment you drive it off of the car dealership's lot. After two years, the approximate value of a car that originally costs \$25,000 is calculated by $V = \$25{,}000x^2$, where x is the average percent that the car retains its value per year. Cars that are well taken care of and driven sparingly will retain more value than cars that are poorly maintained and driven frequently.

a. Suppose that a car that is well maintained and rarely driven will retain an average of 90% of its value each year. What will be the approximate value of the car after 2 years?

b. Suppose that a car that is not well maintained will retain an average of 80% of its value each year. What will be the approximate value of the car after 2 years?

54. Forensic scientists can use a simple formula to approximate the time of death. This formula is based on the average body temperature of humans being 37 °C and the fact that a deceased body will lose an average of 1.5 °C per hour until the body temperature matches the temperature of the surrounding environment. The formula is $f(t) = 37 - 1.5t$, where t is the time in hours since death.

a. What is the approximate body temperature of a person that died 4 hours ago?

b. What is the approximate body temperature of a person that died 14 hours ago?

c. If the temperature of the environment in which the body was found was 20 °C, would it be reasonable for the body temperature to be the temperature from part b.? Explain why or why not.

55. A sled going down a hill has an initial speed of 5 feet per second and a constant acceleration of 1 foot per second squared. The distance of the sled in feet from the top of the hill can be modeled by the polynomial $d(t) = 5t + \frac{1}{2}t^2$, where t is the time in seconds after the sled leaves the top of the hill.

a. Determine the distance the sled is from the top of the hill after 2 seconds.

b. Determine the distance the sled is from the top of the hill after 4 seconds.

c. Determine the distance the sled is from the top of the hill after 8 seconds.

d. Does the distance that the sled travels double when the time doubles? Explain why or why not.

56. Camilla is creating square baby quilts. She determines that the sale price of each quilt should be $p(x) = \$1.80x^2 + 4(\$0.50)x + \$15$, where x is the side length of each square blanket in feet, \$1.80 is the cost per square foot of material, \$0.50 is the cost per foot of border material, and \$15 is the amount of profit Camilla wants to make on each blanket.

a. How much will a blanket that has a side length of 3 feet cost?

b. How much will a blanket that has a side length of 4 feet cost?

57. The value of an automobile depreciates linearly over time. A new 2018 Mercedes E300 sells for \$52,950 and its value, V, after t years is given by the equation $V = 52{,}950 - 3500t$. Find the value of the 2018 Mercedes in 2020 (e.g. after 2 years), and then in 2026.

58. PDQ Tennis Shoe Co. follows a profit model of $P = 3x^2 - 15x + 2$, where P is the profit in hundreds of dollars after selling x hundred pairs of tennis shoes. Find each of the profits for PDQ after selling 100 pairs, 500 pairs, and 1000 pairs of tennis shoes. What does a negative value for profit represent?

59. The number of protozoa in a biology laboratory experiment is given by the polynomial function $p(t) = 0.04t^4 + 0.3t^3 + 2t^2$, where p is the number of protozoa after t hours. Determine the number of protozoa after 4 hours. What is the number of protozoa after 2 days? (Round both values to the nearest whole number.)

60. The percent p of material retained by a student x days after hearing a lecture is given by the polynomial function $p(x) = 100 - 5x^2$. What percent of the material is still remembered by a student 4 days after hearing the lecture?

Writing & Thinking

61. Tony was classifying expressions for a homework assignment. He said that $7y^2 + 12y - 3$ was a polynomial. Was he correct or not? Justify your answer.

62. Jeanne thought that $10a - 9 + 6a^2$ was in descending order. Explain Jeanne's error and what the correct descending order should be.

First-degree polynomials are also called linear polynomials, second-degree polynomials are called quadratic polynomials, and third-degree polynomials are called cubic polynomials. The related functions are called linear functions, quadratic functions, and cubic functions, respectively.

63. Use a graphing calculator to graph the following linear functions. (See Section 3.5 to review graphing functions on a graphing calculator.)

a. $p(x) = 2x + 3$

b. $p(x) = -3x + 1$

c. $p(x) = \frac{1}{2}x$

64. Use a graphing calculator to graph the following quadratic functions.

a. $p(x) = x^2$

b. $p(x) = x^2 + 6x + 9$

c. $p(x) = -x^2 + 2$

65. Use a graphing calculator to graph the following cubic functions.

a. $p(x) = x^3$

b. $p(x) = x^3 - 4x$

c. $p(x) = x^3 + 2x^2 - 5$

66. Make up a few of your own linear, quadratic, and cubic functions and graph these functions with your calculator. Using the results from Exercises 63, 64, and 65, and your own functions, describe in your own words:

a. the general shape of the graphs of linear functions.

b. the general shape of the graphs of quadratic functions.

c. the general shape of the graphs of cubic functions.