5.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The quotient rule for exponents says that when dividing two powers with the same base, keep the base and _____ the exponents.
2. The product rule for exponents says that when multiplying two powers with the same base, keep the base and _____ the exponents.
3. An expression is considered simplified if each base appears only once and each base has only _____ exponents.
4. The expression 0^0 is _____.
5. For all real values of a, a^1 = ___.
6. For all real values of a, a^0 = ___.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If a constant does not have an exponent written, it is assumed that the exponent is 0.
8. If a is a nonzero real number and n is an integer, then $a^{-n} = -a^n$.
9. Since the product rule is stated for integer exponents, the rule is also valid for 0 and negative exponents.
10. When using the quotient rule, you should subtract the smaller exponent from the larger exponent.

Practice

Simplify each expression. The final form of the expressions with variables should contain only positive exponents. Assume that all variables represent nonzero numbers. See Examples 1 though 7.

1. $3^2 \cdot 3$
2. $7^2 \cdot 7^3$
3. $8^3 \cdot 8^0$
4. $5^0 \cdot 5^2$
5. 3^{-1}
6. 4^{-2}
7. 5^{-2}
8. 6^{-3}
9. $(-2)^4(-2)^0$
10. $(-4)^3(-4)^0$
11. $3(2^3)$
12. $6(3^2)$
13. $-4(5^3)$
14. $-2(3^3)$
15. $3(2^{-3})$
16. $4(3^{-2})$
17. $-3(5^{-2})$
18. $-5(2^{-2})$
19. $x^2 \cdot x^3$
20. $x^3 \cdot x$
21. $y^2 \cdot y^0$
22. $y^3 \cdot y^8$
23. x^{-3}
24. y^{-2}

25. $2x^{-1}$

26. $5y^{-4}$

27. $-8y^{-2}$

28. $-10x^{-3}$

29. $5x^{6}y^{-4}$

30. $x^{0}y^{-2}$

31. $3x^{0}+y^{0}$

32. $5y^{0}-3x^{0}$

33. $\dfrac{7^{3}}{7}$

34. $\dfrac{9^{5}}{9^{2}}$

35. $\dfrac{10^{3}}{10^{4}}$

36. $\dfrac{10}{10^{5}}$

37. $\dfrac{2^{3}}{2^{6}}$

38. $\dfrac{5^{7}}{5^{4}}$

39. $\dfrac{x^{4}}{x^{2}}$

40. $\dfrac{x^{6}}{x^{3}}$

41. $\dfrac{x^{3}}{x}$

42. $\dfrac{y^{7}}{y^{2}}$

43. $\dfrac{x^{7}}{x^{3}}$

44. $\dfrac{x^{8}}{x^{3}}$

45. $\dfrac{x^{-2}}{x^{2}}$

46. $\dfrac{x^{-3}}{x}$

47. $\dfrac{x^{4}}{x^{-2}}$

48. $\dfrac{x^{5}}{x^{-1}}$

49. $\dfrac{x^{-3}}{x^{-5}}$

50. $\dfrac{x^{-4}}{x^{-1}}$

51. $\dfrac{y^{-2}}{y^{-4}}$

52. $\dfrac{y^{3}}{y^{-3}}$

53. $3x^{3}\cdot x^{0}$

54. $3y\cdot y^{4}$

55. $x^{3}\cdot x^{2}\cdot x^{-1}$

56. $x^{-3}\cdot x^{0}\cdot x^{2}$

57. $\left(4x^{3}\right)\left(9x^{0}\right)$

58. $\left(5x^{2}\right)\left(3x^{4}\right)$

59. $\left(-2x^{2}\right)\left(7x^{3}\right)$

60. $\left(3y^{3}\right)\left(-6y^{2}\right)$

61. $\left(-4x^{5}\right)\left(3x\right)$

62. $\left(6y^{4}\right)\left(5y^{5}\right)$

63. $\dfrac{8y^{3}}{2y^{2}}$

64. $\dfrac{12x^{4}}{3x}$

65. $\dfrac{9y^{5}}{3y^{3}}$

66. $\dfrac{-10x^{5}}{2x}$

67. $\dfrac{-8y^{4}}{4y^{2}}$

68. $\dfrac{12x^{6}}{-3x^{3}}$

69. $\dfrac{x^{-1}\cdot x^{2}}{x^{3}}$

70. $\dfrac{x\cdot x^{3}}{x^{-3}}$

71. $\dfrac{10^{4}\cdot 10^{-3}}{10^{-2}}$

72. $\dfrac{10\cdot 10^{-1}}{10^{2}}$

73. $\left(9x^{2}\right)^{0}$

74. $\left(-2x^{-3}y^{5}\right)^{0}$

75. $\left(9x^{2}y^{3}\right)\left(-2x^{3}y^{4}\right)$

76. $\left(-3xy\right)\left(-5x^{2}y^{-3}\right)$

77. $\dfrac{-8x^{2}y^{4}}{4x^{3}y^{2}}$

78. $\dfrac{-8x^{-2}y^{4}}{4x^{2}y^{-2}}$

79. $\left(3a^{2}b^{4}\right)\left(4ab^{5}c\right)$

80. $\left(-6a^{3}b^{4}\right)\left(4a^{-2}b^{8}\right)$

81. $\dfrac{36a^{5}b^{0}c}{-9a^{-5}b^{-3}}$

82. $\dfrac{7x^{2}y^{-2}}{28x^{0}yz^{-2}}$

83. $\dfrac{25y^{6}\cdot 3y^{-2}}{15xy^{4}}$

84. $\dfrac{12a^{-2}\cdot 18a^{4}}{36a^{2}b^{-5}}$

Use a graphing calculator to evaluate each expression. Round quotients to the nearest ten-thousandth, if necessary. See Example 8.

85. $\left(2.16\right)^{0}$

86. $\left(-5.06\right)^{2}$

87. $\left(1.6\right)^{-2}$

88. $\left(2.5\right)^{-4}$

89. $\left(6.4\right)^{4}\left(2.3\right)^{2}$

90. $\left(-14.8\right)^{2}\left(21.3\right)^{2}$

Applications

Solve.

91. Rylee wants to move all her files to a new hard drive that has 2^{12} GB of storage on it. She wants to designate the same amount of storage for each of 2^4 projects. How much storage should be assigned to each project? Write your answer as a power of two.

92. Trey is studying patterns in bacteria. For a positive test result in his experiment, bacteria must grow in population at a minimum rate of 3^2 in 24 hours. If the initial population of the bacteria is 3^5 and his final measurement after 24 hours is 3^8, should he mark the test as positive or negative?

93. A molecule being studied under a powerful microscope is cubic in shape. What is the volume of the molecule if the length of one side is 10^{-8} cm?

94. A hurricane caused flooding in a home at the rate of 2^3 ft^3 per hour. If that home has a storage closet that is 2^1 feet wide, 2^3 feet long, and 2^4 feet high, how long will it take the storage closet to fill with water? Write your answer as a power of 2.

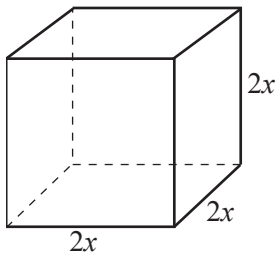
95. A conference center needs an array of gift bags set up for a meeting. There will be 2^5 gift bags per row and 2^4 rows of gift bags. The delivery truck can hold 2^9 gift bags per load. How many deliveries will the truck need to make in order to supply the gift bags needed?

96. A local children's convention receives donations of 2^8 bags of candy for use as gifts for attendees. The convention has 2^7 children attending. How many bags of candy will each child receive?

97. Molly buys land that is 3^4 yards wide and 3^5 yards long. What is the area of the land? Write your answer as a power of 3.

98. Samuel wants to buy grass seed to plant in his yard. His lawn is 2^6 feet wide and 27 feet long. Each bag of grass seed will cover 2^{10} square feet. How many bags of seed should he purchase? Write your answer as a power of 2.

99. Barbara's Bombtastic Bakery makes *petit four glaces,* which are small bite-sized cakes. Each cake is in the shape of a cube that has a side length of $2x$, where x is a positive length which varies depending on the cake flavor.



a. Write an expression using exponents to find the volume of the *petit four glaces*. Do not simplify.

b. Which exponential rule will you need to use to simplify the expression from part a.?

c. Simplify the expression from part a.

d. If $x = 2$ cm, determine the volume of the *petit four glaces* using the expression from part c.

100. A strain of the influenza virus is spreading throughout a community and the number of confirmed cases of the flu doubles every day. On day 0 (the initial day) of the outbreak, 1 person has the virus. On day 1 of the outbreak, $1 \cdot 2 = 2$ people will have the virus. On day 2 of the outbreak, $1 \cdot 2 \cdot 2 = 1 \cdot 2^2 = 4$ people will have the virus.

a. Write an exponential expression to describe how many people will have influenza virus on day 5. Write as a power of 2 and simplify.

b. Write an exponential expression to describe how many people would have the virus on day n if 3 people had the virus on day 0 of the outbreak. Write the expression in exponential form and simplify.

c. Use the expression from part b. to determine the number of people that will have the virus on day 5 of the outbreak if 3 people had the virus on day 0?

101. A standard hard drive has 2^{38} bytes of data. 1 gigabyte is equivalent to 230 bytes.

a. Write an exponential expression to determine how many gigabytes are equivalent to 2^{38} bytes?

b. Simplify the expression from part a. to determine how many gigabytes are in 2^{38} bytes.

c. What rule of exponents did you use to simplify part b.?