

Step 3: Press $\boxed{2\text{nd}}$ MATRIX again and on the NAMES menu, choose [A] 3×3 .
Press $\boxed{\text{ENTER}}$ and press $\boxed{\square}$.

A calculator display showing the text "det([A])" in a monospaced font. The display is light green with a thin black border.

Step 4: Press $\boxed{\text{ENTER}}$ and the display will appear with the solution, $\det(A) = -67$.

A calculator display showing the text "det([A])" followed by "-67" on the right side. The display is light green with a thin black border.

Now work margin exercise 5.

Margin Exercise Answers

1. a. -18 ; b. -8 ; c. 0 2. a. $\begin{vmatrix} 4 & 8 \\ 7 & -6 \end{vmatrix}$; b. $\begin{vmatrix} 4 & 0 \\ -1 & 3 \end{vmatrix}$; c. $\begin{vmatrix} 4 & 8 \\ -1 & 2 \end{vmatrix}$ 3. a. 26 b. 175 4. -7 ; 5. 229

4.8 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- In a/an _____ matrix, the number of rows is equal to the number of columns.
- A determinant is indicated by closing the array of real numbers between two _____.
- Every determinant with real-number entries simplifies to a/an _____ value.
- One method of evaluating a 3×3 determinant is called _____ by minors.
- When evaluating a 3×3 determinant, each minor is multiplied by its corresponding entry and the value ___ or ___.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- A 2×2 determinant has three rows and three columns.
- To find the value of a 3×3 determinant, you need to find the value of several 2×2 determinants.
- A minor of a_{13} of a determinant is found by deleting the third row and the first column.

Practice

Find the determinant of each 2×2 matrix. See Example 1.

$$1. A = \begin{bmatrix} 2 & 7 \\ 4 & 3 \end{bmatrix}$$

$$2. A = \begin{bmatrix} 7 & 3 \\ 8 & 5 \end{bmatrix}$$

$$3. A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$4. A = \begin{bmatrix} 7 & 2 \\ 3 & -6 \end{bmatrix}$$

$$5. A = \begin{bmatrix} 6 & 3 \\ -11 & -5 \end{bmatrix}$$

$$6. A = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$$

$$7. A = \begin{bmatrix} 9 & 4 \\ 4 & 7 \end{bmatrix}$$

$$8. A = \begin{bmatrix} 3 & -4 \\ 8 & -6 \end{bmatrix}$$

Given the following determinant, find the minor of each chosen entry. See Example 2.

$$\det(A) = \begin{vmatrix} 3 & -5 & 9 \\ 2 & 0 & -7 \\ 4 & 1 & 1 \end{vmatrix}$$

$$9. a_{31}$$

$$10. a_{12}$$

$$11. a_{32}$$

$$12. a_{23}$$

Evaluate the value of each 3×3 determinant. See Example 3.

$$13. \begin{vmatrix} 0 & -1 & 2 \\ 3 & 5 & -7 \\ -3 & 4 & 1 \end{vmatrix}$$

$$14. \begin{vmatrix} 1 & 0 & -1 \\ -2 & 3 & 5 \\ 6 & -3 & 4 \end{vmatrix}$$

$$15. \begin{vmatrix} 1 & -1 & 2 \\ -2 & 5 & -7 \\ 6 & 4 & 1 \end{vmatrix}$$

$$16. \begin{vmatrix} 2 & -1 & -3 \\ 5 & 9 & 4 \\ 7 & 6 & -2 \end{vmatrix}$$

$$17. \begin{vmatrix} 2 & 1 & 3 \\ 3 & 4 & 5 \\ 1 & 7 & 2 \end{vmatrix}$$

$$18. \begin{vmatrix} -3 & 2 & 1 \\ 1 & -4 & -1 \\ 2 & 5 & 3 \end{vmatrix}$$

$$19. \begin{vmatrix} 2 & 1 & -1 \\ 4 & 3 & 2 \\ 1 & 5 & 5 \end{vmatrix}$$

$$20. \begin{vmatrix} 6 & 7 & 1 \\ 0 & 3 & 3 \\ 4 & 1 & -5 \end{vmatrix}$$

$$21. \begin{vmatrix} 3 & -1 & -1 \\ 2 & 4 & 1 \\ -1 & 1 & 2 \end{vmatrix}$$

$$22. \begin{vmatrix} 2 & 3 & 2 \\ 1 & -1 & 5 \\ 0 & 5 & 1 \end{vmatrix}$$

Solve for the variable. See Example 4.

$$23. \begin{vmatrix} 1 & 3 & 4 \\ 2 & x & 3 \\ 1 & 3 & 5 \end{vmatrix} = 1$$

$$24. \begin{vmatrix} -2 & -1 & 1 \\ x & 1 & -1 \\ 4 & 3 & -2 \end{vmatrix} = 7$$

$$25. \begin{vmatrix} -2 & 0 & x \\ 0 & 4 & -8 \\ 6 & 1 & 3 \end{vmatrix} = -16$$

$$26. \begin{vmatrix} 5 & 3 & -2 \\ -1 & 0 & x \\ 2 & 1 & -1 \end{vmatrix} = 3$$

$$27. \begin{vmatrix} -4 & 1 & 3 \\ 2 & x & x \\ 0 & 5 & -3 \end{vmatrix} = -28$$

$$29. \begin{vmatrix} x & x & 1 \\ 1 & 5 & 0 \\ 0 & 1 & -2 \end{vmatrix} = -15$$

$$28. \begin{vmatrix} 1 & x & x \\ 2 & -2 & 1 \\ -1 & 3 & 2 \end{vmatrix} = 0$$

$$30. \begin{vmatrix} 3 & 1 & -2 \\ 1 & x & 4 \\ 2 & x & 0 \end{vmatrix} = 38$$

The equation $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$ is an equation of a line passing through two points

$P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Find an equation in standard form for the line determined by the following pairs of points.

$$31. (3, 2), (-1, 4)$$

$$33. (4, -4), (0, 6)$$

$$32. (-2, 1), (5, 3)$$

$$34. (1, 3), (-2, -1)$$

The area of a triangle having the vertices $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, and $P_3(x_3, y_3)$ is given by

the absolute value of the expression $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$. Draw the triangle with the given

points as vertices and then find the area of the triangle.

$$35. (3, 1), (1, -1), (5, 2)$$

$$37. (-1, 3), (-4, -1), (3, -2)$$

$$36. (4, 0), (5, -2), (7, 1)$$

$$38. (1, 5), (-1, -2), (3, 0)$$

Use a graphing calculator to find the value of each determinant.

$$39. \begin{vmatrix} 3 & -4 & 6 \\ 2 & 4 & -1 \\ 7 & 9 & -1 \end{vmatrix}$$

$$41. \begin{vmatrix} 1.6 & \frac{1}{2} & -5.9 \\ 0.7 & \frac{3}{4} & 1.7 \\ 5.0 & 8.2 & -4.1 \end{vmatrix}$$

$$40. \begin{vmatrix} 2.1 & 3.5 & -3.4 \\ 2.6 & 5.0 & 1.2 \\ -1.0 & 3.4 & 6.3 \end{vmatrix}$$

$$42. \begin{vmatrix} -10 & 15 & 25 \\ 0 & -7 & 5 \\ 16 & -12 & 8 \end{vmatrix}$$

Writing & Thinking

43. Explain in your own words the position the three points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$,

and $P_3(x_3, y_3)$ if the expression $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ has a value of 0. (**Hint:** Refer to the discussion before Exercise 35.)

44. Suppose that in a 2×2 determinant two rows are identical. What will be the value of this determinant? Give two specific examples and a general example to back up your conclusion.

45. Suppose that in a 3×3 determinant one row is all 0s. What will be the value of this determinant? Give two specific examples and a general example to back up your conclusion.
46. In each part, give two specific examples and a general example to back up your conclusion.
- Suppose that in a 2×2 determinant two rows (or columns) are switched. How will the value of this new determinant relate to the value of the original determinant?
 - Suppose that in a 3×3 determinant two rows (or columns) are switched. How will the value of this new determinant relate to the value of the original determinant?