

Example 4 Graphing Systems of Linear Inequalities

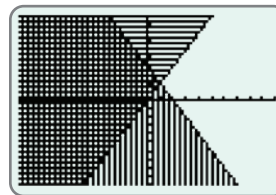
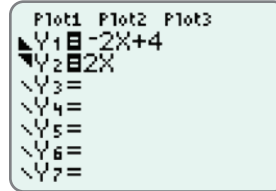
Use a graphing calculator to graph the system of linear inequalities: $\begin{cases} 2x + y < 4 \\ 2x - y \leq 0 \end{cases}$

Solution

Step 1: First, solve each inequality for y : $\begin{cases} y < -2x + 4 \\ y \geq 2x \end{cases}$

Note: Solving $2x - y \leq 0$ for y can be written as $2x \leq y$ and then as $y \geq 2x$.

Step 2: Press the $\boxed{Y=}$ key and enter both functions and the corresponding symbols as they appear here. Remember, to shade your graphs, position the cursor over the slash next to Y_1 (or Y_2) and press $\boxed{\text{ENTER}}$ repeatedly until the desired graphing symbol is displayed.



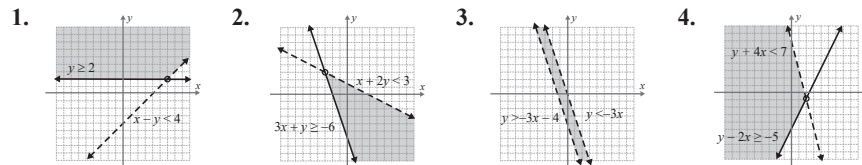
Step 3: Press $\boxed{\text{GRAPH}}$. The graph should appear as shown. The solution is the cross-hatched region and the points on the line $2x - y = 0$, where $x < 1$.

4. Use a TI-84 Plus graphing calculator to graph the following system of linear inequalities.

$$\begin{cases} y - 2x \geq -5 \\ y + 4x < 7 \end{cases}$$

Now work margin exercise 4.

Margin Exercise Answers



4.10 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- To check the graph of a system of linear inequalities, a/an _____ - _____ should be chosen to determine whether or not it satisfies both inequalities.
- When graphing an inequality such as $y > mx + b$, the line $y = mx + b$ is called the _____ line.
- If a boundary line is included in the solution, the half-plane is _____.
- When a boundary line is not included in the solution, the half-plane is _____.
- If the boundary lines are parallel, there are _____ possible types of solutions.
- The solution set of a system of two linear inequalities consists of the points in the _____ of the two half-planes and portions of the boundary lines.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. When boundary lines are parallel, the system of linear inequalities has no solution.
8. If two half-planes overlap, that region is the union of the graphs.
9. Half-planes are the graphs of linear inequalities.
10. If the graphs of two linear inequalities have no intersection, then the system has no solution.

Practice

Solve the systems of two linear inequalities graphically. See Example 1 through 3.

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|---|---|---|
| 1. $\begin{cases} y > 2 \\ x \geq -3 \end{cases}$ | 9. $\begin{cases} x - y \geq 0 \\ 3x - 2y \geq 4 \end{cases}$ | 17. $\begin{cases} x + y \geq 0 \\ x - 2y \geq 6 \end{cases}$ |
| 2. $\begin{cases} 2x + 5 < 0 \\ y \geq 2 \end{cases}$ | 10. $\begin{cases} y \geq x - 2 \\ x + y \geq -2 \end{cases}$ | 18. $\begin{cases} y \geq 2x + 3 \\ y \leq x - 2 \end{cases}$ |
| 3. $\begin{cases} x < 3 \\ y > -x + 2 \end{cases}$ | 11. $\begin{cases} 3x + y \leq 10 \\ 5x - y \geq 6 \end{cases}$ | 19. $\begin{cases} x + 3y \leq 9 \\ x - y \geq 5 \end{cases}$ |
| 4. $\begin{cases} y \leq -5 \\ y \geq x - 5 \end{cases}$ | 12. $\begin{cases} y > 3x + 1 \\ -3x + y < -1 \end{cases}$ | 20. $\begin{cases} x - y \geq -2 \\ x + 2y < -1 \end{cases}$ |
| 5. $\begin{cases} x \leq 3 \\ 2x + y > 7 \end{cases}$ | 13. $\begin{cases} 3x + 4y \geq -7 \\ y < 2x + 1 \end{cases}$ | 21. $\begin{cases} y \leq x + 3 \\ x - y \leq -5 \end{cases}$ |
| 6. $\begin{cases} 2x - y > 4 \\ y < -1 \end{cases}$ | 14. $\begin{cases} 2x - 3y \geq 0 \\ 8x - 3y < 36 \end{cases}$ | 22. $\begin{cases} y \geq 2x - 5 \\ 3x + 2y > -3 \end{cases}$ |
| 7. $\begin{cases} x - 3y \leq 3 \\ x < 5 \end{cases}$ | 15. $\begin{cases} x + y < 4 \\ 2x - 3y < 3 \end{cases}$ | 23. $\begin{cases} y \leq -2x \\ y > -2x - 6 \end{cases}$ |
| 8. $\begin{cases} 3x - 2y \geq 8 \\ y \geq 0 \end{cases}$ | 16. $\begin{cases} 2x + 3y < 12 \\ 3x + 2y > 13 \end{cases}$ | 24. $\begin{cases} y > x - 4 \\ y < x + 2 \end{cases}$ |

 Use a graphing calculator to solve the systems of linear inequalities. See Example 4.

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| 25. $\begin{cases} y \geq 0 \\ 3x - 5y \leq 10 \end{cases}$ | 29. $\begin{cases} 3x - 4y \geq -6 \\ 3x + 2y \leq 12 \end{cases}$ | 33. $\begin{cases} y \leq x \\ y < 2x + 1 \end{cases}$ |
| 26. $\begin{cases} y \leq 0 \\ 3x + y \leq 11 \end{cases}$ | 30. $\begin{cases} 3y \leq 2x + 2 \\ x + 2y \leq 11 \end{cases}$ | 34. $\begin{cases} x - y \geq -2 \\ 4x - y < 16 \end{cases}$ |
| 27. $\begin{cases} 4x - 3y \geq 6 \\ 3x - y \leq 3 \end{cases}$ | 31. $\begin{cases} x + y \leq 8 \\ 3x - 2y \geq -6 \end{cases}$ | |
| 28. $\begin{cases} 3x + 2y \leq 15 \\ 2x + 5y \geq 10 \end{cases}$ | 32. $\begin{cases} x + y \leq 7 \\ 2x - y \leq 8 \end{cases}$ | |

Applications

Solve.

35. Barbara's Bombtastic Bakery sells cookie bouquets where the price depends on the arrangement. Each completed bouquet arrangement needs to weigh less than 5 pounds for shipping purposes. The small cookies weigh 0.1 pounds and the large cookies weigh 0.3 pounds. The flower pot and Styrofoam weigh 1.2 pounds. The cost of each arrangement needs to be less than \$30. The small cookies cost \$1 each and the large cookies cost \$2 each. (The cost of the flower pot and foam are included in the cookie prices.)
- Write two linear inequalities to describe the situation. Use the variable x to represent the number of small cookies and the variable y to represent the number of large cookies in a bouquet.
 - Graph the two linear inequalities on the same coordinate plane.
 - Describe the solution set for the situation.
 - Do any of the values in the solution set not make sense in the context of the problem? Explain why or why not.
36. Robin is planning a charity ball to raise money for her favorite charity. There are two different ticket options. The VIP option includes dinner, dancing, and cocktails for \$150 per ticket. The regular option includes dancing and cocktails for \$75 per ticket. Robin wants to make at least \$14,000 in ticket sales. The ballroom that is being used for the charity event has a maximum capacity of 150 people.
- Write two linear inequalities to describe the situation. Let the variable x represent the number of VIP tickets sold and let the variable y represent the number of regular tickets sold.
 - Graph the two linear inequalities on the same coordinate plane.
 - Describe the solution set for the situation.
 - Can Robin reach her sales goal if she only sells tickets for the regular option? Explain why or why not.

Writing & Thinking

37. Graph the inequalities and explain how you can tell that there is no solution.

$$\begin{cases} y \leq 2x - 5 \\ y \geq 2x + 3 \end{cases}$$