Example 9 Solving Absolute Value Inequalities

Solve the absolute value inequality and graph the solution set: $|2x-5|-5\geq 4$

Solution

$$|2x-5|-5\geq 4$$
$$|2x-5|\geq 9$$
$$2x-5\leq -9 \quad \text{or} \quad 2x-5\geq 9$$
$$2x\leq -4 \qquad\qquad 2x\geq 14$$
$$x\leq -2 \qquad\qquad x\geq 7$$

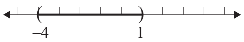
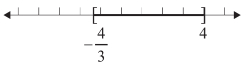
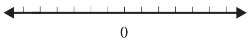
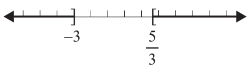
Add 5 to both sides in order to get the inequality in standard form.

So, x is in $(-\infty,-2]\cup[7,\infty)$.

Now work margin exercise 9.

9. Solve the absolute value inequality $|3x+2|+1\geq 8$ and graph the solution set. Write the solution set using interval notation.

Margin Exercise Answers

1.  $(-3,3)$
2.  $[-5,3]$
3.  $(-4,1)$
4.  no solution
5.  $\left[-\frac{4}{3},4\right]$
6.  $(-\infty,-2)\cup(2,\infty)$
7.  $(-\infty,-5]\cup[-2,\infty)$
8.  $(-\infty,\infty)$
9.  $(-\infty,-3]\cup\left[\frac{5}{3},\infty\right)$

2.8 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. If an absolute value expression is isolated on one side of an inequality, the inequality is in __________ form.

2. The inequality $|x-6|>5$ means that the _______ between x and 6 is ____ than 5.

3. If $|x|>c$ then $x<-c$ _______ $x>c$.

4. If an inequality is always true, such as $|3x-8|>-6$, then the solution is all _______ numbers.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

5. If the solution is a union, there are two statements or inequalities, both of which must be true.

6. If the solution to a compound inequality is $-4<x<6$, then the solution is a union.

7. For a number to have absolute value greater than 2, its distance from 0 must be less than 2.

8. The inequality $|2x+9|<-2$ has no solution.

Practice

Solve each of the absolute value inequalities and graph the solution sets. Write each solution using interval notation. See Examples 1 through 9.

1. $|x| \geq -2$
2. $|x| \geq 3$
3. $|x| \leq \frac{4}{5}$
4. $|x| \geq \frac{7}{2}$
5. $|x-3| > 2$
6. $|y-4| \leq 5$
7. $|x+6| \leq 4$
8. $|x+2| \leq -4$
9. $|x+5| \geq 3$
10. $|x-1| < 6$
11. $|2x-1| \geq 2$
12. $|3x+4| > -8$
13. $|3-2x| < -2$
14. $|4+3x| > 5$
15. $|5+4x| \leq 3$
16. $|5x-2| < 8$
17. $|3x+4|-1 < 0$
18. $|2x-3|-3 \leq 0$
19. $\left|\frac{3x}{2}-4\right| \geq 5$
20. $\left|\frac{3}{7}y+\frac{1}{2}\right| > 2$
21. $|2x-9|-7 \leq 4$
22. $|3x-7|+4 \leq 4$
23. $-4 < |6x-1|+4$
24. $4 \leq |3x+1|-6$
25. $5 > |4-2x|+2$
26. $7 > |8-5x|+3$
27. $3|4x+5|-5 > 10$
28. $6|4x-7|+7 > 19$
29. $4|7x+9|-3 < 17$
30. $2|7x-3|+4 \geq 12$

Writing & Thinking

A set of real numbers is described. **a**. Sketch a graph of the set on a real number line. **b**. Represent each set using absolute value notation. **c**. Represent each set using interval notation. If the set is one interval, state what type of interval it is.

31. The set of real numbers between –10 and 10, inclusive
32. The set of real numbers within 7 units of 4
33. The set of real numbers more than 6 units from 8
34. The set of real numbers greater than or equal to 3 units from –1
35. The set of real numbers within 2 units of –5