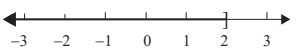
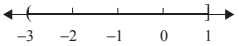
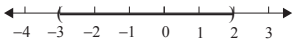
11. $(-\infty, 2]$ 

12. 

$(-3, 1]$

13. 

$(-3, 2)$ **14.** The maximum final dosage that can be administered must be less than 400 milligrams. **15.** Ashley can buy at most 8 rose centerpieces.

2.7 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. If a and b are real numbers where $a < b$, the set of all real numbers between a and b is called a/an _____ of real numbers.

2. In a/an _____ interval, neither endpoint is included.

3. In a/an _____ interval, both end points are included.

4. Linear inequalities are inequalities that relate two _____ _____.

5. If A and B are algebraic expressions and C is a real number, then the _____ principle for solving linear inequalities states that $A < B$ and $A + C < B + C$ are equivalent.

6. If A and B are algebraic expressions and C is a real number, then the _____ principle for solving linear inequalities states that $A < B$ and $AC < BC$ are equivalent.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If only one endpoint is included in an interval, it is called a half-open interval.

8. When both sides of a linear inequality are multiplied by a negative constant, the sense of the inequality should stay the same.

9. To check the solution set of a linear inequality, every solution in the solution set must be checked in the original inequality.

10. The infinity symbol ∞ does not represent a specific number.

Practice

Graph each interval on a real number line. See Example 1.

1. $(-1, \infty)$

2. $[-2, 4)$

3. $(-\infty, 5]$

4. $[0, 3]$

5. $[-5, -1]$

6. $(3, 8)$

7. $[-7, -4)$

8. $(-\infty, -6]$

Graph each interval on a real number line and tell what type of interval it is. See Examples 2 through 4.

9. $x \le -3$

10. $x \ge -0.5$

11. $x > 4$

12. $x < -\frac{1}{10}$

13. $0 < x \le 2.5$

14. $-1.5 \le x < 3.2$

15. $-2 \le x \le 0$

16. $-1 \le x \le 1$

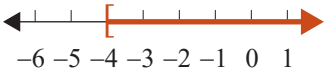
17. $4 > x \ge 2$

18. $0 > x \ge -5$

Represent each of the following graphs **a.** using algebraic notation and **b.** using interval notation and state what kind of interval it is.

19.


20.


21.


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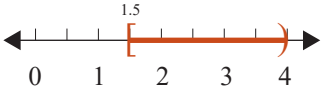

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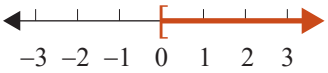

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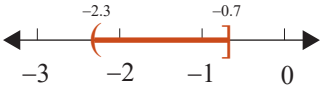

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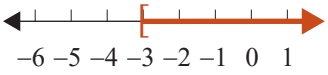

Solve each inequality and graph the solution set. Write each solution set using interval notation.

29. $x + 1 > 5$

30. $x - 3 < 2$

31. $3 + x \le 7$

32. $5 + x \ge 11$

33. $3 < 4 + x$

34. $9 > 6 + x$

35. $4 \ge x - 3$

36. $12 \le x + 8$

37. $4x > 16$

38. $3x < 27$

39. $5x \le 15$

40. $-2x \ge 6$

41. $10 > -5x$

42. $12 < 8x$

43. $14 \ge 2x$

44. $9 \le -3x$

45. $2x + 3 < 5$

46. $4x - 7 \ge 9$

47. $14 - 5x < 4$

48. $23 < 7x - 5$

49. $6x - 15 > 1$

50. $9 - 2x < 8$

51. $5.6 + 3x \ge 4.4$

52. $12x - 8.3 < 6.1$

53. $1.5x + 9.6 < 12.6$

54. $0.8x - 2.1 \geq 1.1$

55. $2 + 3x \geq x + 8$

56. $x - 6 \leq 4 - x$

57. $3x - 1 \leq 11 - 3x$

58. $5x + 6 \geq 2x - 2$

59. $4 - 2x < 5 + x$

60. $4 + x > 1 - x$

61. $x - 6 > 3x + 5$

62. $4 + 7x \leq 4x - 8$

63. $\frac{x}{2} - 1 \leq \frac{5x}{2} - 3$

64. $\frac{x}{4} + 1 \leq 5 - \frac{x}{4}$

65. $\frac{x}{3} - 2 > 1 - \frac{x}{3}$

66. $\frac{5x}{3} + 2 > \frac{x}{3} - 1$

67. $6x + 5.91 < 1.11 - 2x$

68. $4.3x + 21.5 \geq 1.7x + 0.7$

69. $6.2x - 5.9 > 4.8x + 3.2$

70. $0.9x - 11.3 < 3.1 - 0.7x$

71. $4(6 - x) < -2(3x + 1)$

72. $-3(2x - 5) \leq 3(x - 1)$

73. $-(3x + 8) \geq 2(3x + 1)$

74. $6(3x + 1) < 5(1 - 2x)$

75. $11x + 8 - 5x \geq 2x - (4 - x)$

76. $1 - (2x + 8) < (9 + x) - 4x$

77. $5 - 3(4 - x) + x \leq -2(3 - 2x) - x$

78. $x - 2(x + 3) \geq 7 - (4 - x) + 11$

79. $\frac{2(x-1)}{3} < \frac{3(x+1)}{4}$

80. $\frac{3(x-2)}{2} \geq \frac{4(x-1)}{3}$

81. $\frac{x-2}{4} > \frac{x+2}{2} + 6$

82. $\frac{x+4}{9} \leq \frac{x}{3} - 2$

83. $\frac{2x+7}{4} \leq \frac{x+1}{3} - 1$

84. $\frac{4x}{7} - 3 > \frac{x-6}{2} - 4$

85. $-4 < x + 5 < 6$

86. $2 \leq -x + 2 \leq 6$

87. $3 \geq 4x - 3 \geq -1$

88. $13 > 3x + 4 > -2$

89. $1 \leq \frac{2}{3}x - 1 \leq 9$

90. $-2 \leq \frac{1}{2}x - 5 \leq -1$

91. $14 > -2x - 6 > 4$

92. $-11 \geq -3x + 2 > -20$

93. $-1.5 < 2x + 4.1 < 3.5$

94. $0.9 < 3x + 2.4 < 6.9$

Represent each of the following statements as an inequality involving a variable x, and graph its solution set on a number line.

95. You must be at least 58 inches in height to ride this roller coaster.

96. There are fewer than 12 days left before final exams.

97. Gifts worth \$5 or less do not need to be declared.

98. Arsenic levels over 10 parts per billion may be dangerous.

Applications

Solve.

99. A statistics student has grades of 82, 95, 93, and 78 on four hour-long exams. He must average 90 or higher to receive an A for the course. What scores can he receive on the final exam and earn an A if:

a. The final is equivalent to a single hour-long exam (100 points maximum)?

b. The final is equivalent to two hourly exams (200 points maximum)?

100. To receive a grade of B in a chemistry class, Melissa must average 80 or more but less than 90. If her five hour-long exam scores were 75, 82, 90, 85, and 77, what score does she need on the final exam (100 points maximum) to earn a grade of B?

101. A car salesman makes $1000 each week that he works and makes approximately $250 commission for each car he sells. If a car salesman wants to make at least $3500 in one week, how many cars does he need to sell?

102. Allison is ordering boxes of 24 tea bags from a website. The website is having a promotion where each box of tea comes with 2 free sample packs, and each sample pack contains 3 tea bags. If Allison has an empty container that holds 150 tea bags, what is the largest number of boxes of tea Allison can order and not overfill the container?

103. WildLily Florist is creating arrangements for a wedding this weekend. The large arrangements use 8 flowers and the small arrangements use 5 flowers.

a. Let x represent the number of large arrangements. Write an algebraic expression for the number of small arrangements, if there are 15 tables that need an arrangement.

b. Write an algebraic expression representing the total number of flowers used in the 15 arrangements.

c. If the bride has paid for 100 flowers, use an inequality to determine the maximum number of the 15 arrangements that can be large.

104. John's algebra test consists of 19 questions, 13 equations and 6 word problems. Each equation is worth 4 points, and each word problem is worth 8 points. Assume there is no partial credit on this test.

a. Let w be the number of word problems John gets correct. Write an expression for the number of points John will get from the word problem part of his test.

b. Assuming John gets every equation correct, write an inequality that will help determine the fewest number of word problems he can get correct and still make an 80 on the test. What is the fewest number he can get correct?

c. Let x be the number of equations John gets correct. Write an expression for the number of points John will get from the equation part of his test.

d. Assuming John gets every word problem correct, write an inequality that will help determine the fewest number of equations he can get correct and still get an 80 on the test. What is the fewest number of equations he can get correct?

105. Dr. Smyth has an attendance clause in his course syllabus that a student loses 5 points on his or her final grade average for every unexcused absence the student has after his or her first three unexcused absences. If Kara must have a 70 to pass the course, determine the largest number of unexcused absences Kara can have and still have any chance to pass the course.

106. Tracy needs to purchase 25 pastries for the PTA Teachers' Breakfast. Bear Claws cost $1.75 each and Apple Turnovers cost $2.15 each. If Tracy's budget is $50, find the maximum number of Apple Turnovers that Tracy can purchase.

107. The maximum occupancy for a concert in Thompson-Boling Arena is 24,000 people. However, for every 15 tickets sold, there must be one worker present (security, food service, admissions, etc.). Determine the maximum number of tickets than can be sold.

108. Phineas wants to build a nuclear-powered submarine to take his friends on a tour of the Arctic Circle. At least twice as much titanium must be used in the construction of the shell of the sub as the amount of stainless steel used in its construction. The cost of titanium is $500 per pound and the cost of steel is $300 per pound. If Phineas has only $1,000,000 to spend on metal for the sub, determine the greatest number of pounds of metal (both together) that can be used to construct his submarine. (Round your answer to the nearest pound.)

109. Nicole has just moved to Orlando and discovered that Florida residents can purchase 4-day tickets to Disney World for $55 per day. Annual passes (with certain restrictions) for Florida residents are $390. Nicole is trying to decide if she thinks she will go to the park enough times to make it worth buying an annual pass. Use the formula $55x \leq 390$, where x is the number of days spent visiting at Disney World, to determine how many times she would have to go in order for the annual pass to be the better deal.

110. Fernando has already consumed 270 grams of carbohydrates and is on a diet that restricts his carbohydrate consumption to no more than 300 grams of carbohydrates per day. A serving of 6 crackers has 21 grams of carbohydrates. Solve the inequality $\frac{21}{6}c + 270 \leq 300$ to determine how many crackers (c) Fernando can eat without going over his goal. Write your answer as a whole number.

111. Jeph is in charge of buying office supplies for the nonprofit organization he works for. He has $400 to spend. He needs to buy a printer that costs $150, a box of printer paper for $60, and some ink cartridges for $12.50 each. What is the maximum number of ink cartridges that Jeph can buy? (**Note:** Tax is not included in the sales price.)

a. Set up the linear inequality. Use the variable c to represent the number of ink cartridges.

b. Solve the equation from part a. for the variable.

c. What does the answer from part b. mean? Write a complete sentence.

112. Sarah is participating in National Novel Writers Month where she has to write a rough draft of a novel with at least 50,000 words during the month of November. At the end of the day on November 20th, she has a total of 32,500 words. What is the minimum number of words that Sarah needs to write each day for the rest of the month to make the goal of 50,000 words?

a. Set up the linear inequality. Use the variable w to represent the number of words per day.

b. Solve the equation from part a. for the variable.

c. What does the answer from part b. mean? Write a complete sentence.

113. Andrew needs to earn at least a B in each class to keep his scholarship. The grade in his economics class is based on five exams that are equally weighted. On the first four exams, Andrew received the following scores: 92, 74, 80, 72. Andrew needs an average of at least 80 to earn a B for the class. What range of scores does he need on the fifth exam to keep his scholarship?

a. Set up the linear inequality. Use the variable E to represent the fifth exam score.

b. Solve the equation from part a. for the variable.

c. What does the answer from part b. mean? Write a complete sentence.

Writing & Thinking

114. **a.** Write a list of three situations where inequalities might be used in daily life.

b. Illustrate theses situations with algebraic inequalities and appropriate numbers.