12.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The factorial $n!$ is defined to be the product of all positive integers from ___ through ___.
2. The notation $\binom{n}{r}$ is called a _____ coefficient.
3. _____ triangle is formed using the coefficients in the binomial expansions of $(a+b)^n$, where n is a positive integer.
4. The _____ Theorem written in sigma notation is $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$.
5. Another term for writing out the simplified product of $(a+b)^n$ is _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. The factorial $0!$ is defined to be 0.
7. The values of $\binom{6}{2}$ and $\binom{6}{4}$ are equal.
8. The sum of the exponents of a and b in each term of the expansion of $(a+b)^n$ is equal to n.

Practice

Simplify. See Examples 1 and 2.

1. $\dfrac{8!}{6!}$
2. $\dfrac{11!}{7!}$
3. $\dfrac{3!8!}{10!}$
4. $\dfrac{5!7!}{8!}$
5. $\dfrac{5!4!}{6!}$
6. $\dfrac{7!4!}{10!}$
7. $\dfrac{n!}{n}$
8. $\dfrac{n!}{(n-3)!}$
9. $\dfrac{(k+3)!}{k!}$
10. $\dfrac{n(n+1)!}{(n+2)!}$
11. $\binom{6}{3}$
12. $\binom{5}{4}$
13. $\binom{7}{3}$
14. $\binom{8}{5}$
15. $\binom{10}{0}$
16. $\binom{6}{2}$

Determine the first four terms of the expansion of each binomial expression.

17. $(x+y)^7$

18. $(x+y)^{11}$

19. $(x+1)^9$

20. $(x+1)^{12}$

21. $(x+3)^5$

22. $(x-2)^6$

23. $(x+2y)^6$

24. $(x+3y)^5$

25. $(3x-y)^7$

26. $(2x-y)^{10}$

27. $(x^2-4y)^9$

28. $(x^2-2y)^7$

Expand each expression using the Binomial Theorem. See Example 3.

29. $(x+y)^6$

30. $(x+y)^8$

31. $(x-1)^7$

32. $(x-1)^9$

33. $(3x+y)^5$

34. $(2x+y)^6$

35. $(x+2y)^4$

36. $(x+3y)^5$

37. $(3x-2y)^4$

38. $(5x+2y)^3$

39. $(x^2+2y)^4$

40. $(3x^2-y)^5$

Find the specified term of each expression. See Example 4.

41. $(x-2y)^{10}$; fifth term

42. $(x+3y)^{12}$; third term

43. $(2x+3)^{11}$; fourth term

44. $(4x-1)^9$; seventh term

45. $(5x^2-y^2)^{12}$; tenth term

46. $(2x^2+y^2)^{15}$; eleventh term

Approximate the value of each expression to the nearest thousandth. See Example 5.

47. $(1.01)^6$

48. $(0.96)^8$

49. $(0.97)^7$

50. $(1.02)^{10}$

51. $(2.3)^5$

52. $(2.8)^6$

53. $(0.98)^8$

54. $(1.03)^9$

Writing & Thinking

55. Use the Binomial Theorem to factor $x^4 + 8x^3 + 24x^2 + 32x + 16$.