

Comparing the denominators, we see that  $2^n < 2^{n+1}$ . This means that

$$\frac{1}{2^n} > \frac{1}{2^{n+1}} \text{ and } a_n > a_{n+1}.$$

Therefore, the sequence  $\{a_n\} = \left\{\frac{1}{2^n}\right\}$  is decreasing.

- b. The sequence  $\{b_n\} = \{2 + (-1)^n\}$  has a term of  $(-1)^n$ , which means it is likely an alternating sequence. Consider the first four terms of the sequence  $\{b_n\}$ .

$$b_1 = 2 + (-1)^1 = 2 - 1 = 1$$

$$b_2 = 2 + (-1)^2 = 2 + 1 = 3$$

$$b_3 = 2 + (-1)^3 = 2 - 1 = 1$$

$$b_4 = 2 + (-1)^4 = 2 + 1 = 3$$

From this pattern, we see that the sequence is 1, 3, 1, 3, ..., which is neither increasing nor decreasing.

- c. In formula form, we have  $c_n = n + 3$  and  $c_{n+1} = (n+1) + 3 = n + 4$ . Because  $n + 3 < n + 4$ , we have  $c_n < c_{n+1}$ , which means the sequence is increasing.

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**Now work margin exercise 5.**

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**Margin Exercise Answers**

1. a.  $c_1 = 0, c_2 = \frac{1}{2}, c_3 = \frac{2}{3}, c_{50} = \frac{49}{50}$  b.  $d_1 = 3, d_2 = 8, d_3 = 13, d_{50} = 248$

2. a.  $a_n = \frac{1}{n}$  b.  $a_n = 2^{n-1}$  3. \$960, \$768, \$614.40 4. -3, 5, -7, 9, -11

5. a. neither b. decreasing c. increasing

## 12.1 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

1. A sequence is a list of \_\_\_\_\_ that occur in a certain order.
2. The general term  $a_n$  is called the \_\_\_\_\_ term of the sequence.
3. A/An \_\_\_\_\_ sequence is formed by each successive term in a sequence being found by referring to a previous term.
4. An alternating sequence has terms that alternate in \_\_\_\_\_.
5. If a sequence is \_\_\_\_\_, each successive term becomes smaller.
6. If a sequence is \_\_\_\_\_, each successive term becomes larger.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Alternating sequences generally involve expressions with a factor of  $(-1)^n$  or  $(-1)^{n+1}$ .
8. In a sequence,  $a_2$  is the second term of the sequence.
9. The terms in an alternating sequence are always negative.

## Practice

Write the first five terms of each sequence. See Example 1.

- |  |  |
|--|--|
| 1. $\{3n-1\}$  | 11. $\{(-1)^n(n^2+1)\}$                                |
| 2. $\{4n+1\}$  | 12. $\{(-1)^{n-1}(3^n)\}$                              |
| 3. $\left\{1+\frac{1}{n}\right\}$                    | 13. $\left\{(-1)^n\left(\frac{n}{n+1}\right)\right\}$  |
| 4. $\left\{\frac{n+3}{n+1}\right\}$                  | 14. $\left\{(-1)^n\left(\frac{1}{2n+3}\right)\right\}$ |
| 5. $\{n^2+n\}$                                       | 15. $\left\{\frac{2}{n(n+1)}\right\}$                  |
| 6. $\{n-n^2\}$                                       | 16. $\left\{\frac{2n}{n+1}\right\}$                    |
| 7. $\{2^n\}$   | 17. $\left\{\frac{n(n-1)}{2}\right\}$                  |
| 8. $\{2^n-n^2\}$                                     | 18. $\left\{\frac{1+(-1)^n}{2}\right\}$                |
| 9. $\left\{\left(\frac{1}{2}\right)^n\right\}$       |  |
| 10. $\left\{\left(-\frac{1}{2}\right)^{n+1}\right\}$ |  |

Find the formula for the general term of each sequence. See Example 2.

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|--|--|
| 19. 2, 5, 8, 11, 14, ...   | 24. $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \dots$ |
| 20. 5, 9, 13, 17, 21, ...  | 25. 6, 12, 18, 24, 30, ...   |
| 21. 1, 4, 9, 16, 25, ...   | 26. 5, 10, 20, 40, 80, ...   |
| 22. 2, 5, 10, 17, 26, ...  | 27. 1, -3, 5, -7, 9, ...   |
| 23. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots$ | 28. -3, 7, -11, 15, -19, ...   |

Determine whether each sequence is decreasing, increasing, or neither. See Example 5.

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|----------------|------------------------------------|
| 29. $\{n+4\}$  | 31. $\{n^0\}$                      |
| 30. $\{1-2n\}$ | 32. $\left\{\frac{1}{n+3}\right\}$ |

33.  $\left\{ \frac{1}{3^n} \right\}$

34.  $\left\{ \frac{2n+1}{n} \right\}$

35.  $\left\{ (-1)^{n+1} \left( \frac{2}{n+1} \right) \right\}$

36.  $\left\{ \frac{n}{n+1} \right\}$

## Applications

Write the terms of the finite sequence described, then answer the stated question.



37. A certain automobile costs \$40,000 new and depreciates at a rate of  $\frac{3}{10}$  of its current value each year. What will be its value after 3 years?
38. A culture of bacteria triples every day. If there were 100 bacteria in the original culture, how many would be present after 4 days?
39. A ball is dropped from a height of 10 meters. Each time it bounces, it rises to  $\frac{2}{5}$  of its previous height. How high will it bounce on its fourth bounce?
40. A local university is experiencing a declining enrollment of 6% per year. If the present enrollment is 20,000, what is the projected enrollment after 5 years?
41. A certain medication has a half-life of 1 day, which means that after 1 day, half of the dosage remains in a person's system. The general formula for the remaining amount of medication in a person's system in this situation is  $h_n = A \left( \frac{1}{2} \right)^n$ , where  $A$  is the original amount of medication administered and  $n$  is the number of days that have passed. How much medication remains in a person's system after 3 days if the original dosage was 100 mg?
42. The population of a certain city is expected to grow by 3.5% of its current value each year. If the current population is 15,500 people, how many people are expected to reside in the city after 4 years? (Round each of your answers up to the nearest person.)

## Writing & Thinking

43. Show that the sequence of digits from the irrational number  $\pi = 3.1415926535\dots$  is neither increasing nor decreasing. (**Note:** There is no formula for  $a_n$ .)
44. Show that the sequence  $\left\{ \frac{(-1)^{n+1}}{3n} \right\}$  is neither increasing nor decreasing. Is this an alternating sequence?
45. The famous Fibonacci sequence is an example of a recurrence sequence. That is, each term depends on previous terms.
- Write the first eight terms of the Fibonacci sequence defined as  $F_{n+2} = F_{n+1} + F_n$ , where  $F_1 = 1$  and  $F_2 = 1$ .
  - Form the sequence of the differences of successive terms. What do you notice?