

## 10.2 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

- To evaluate a function at an algebraic expression, replace the \_\_\_\_\_ with the expression everywhere the \_\_\_\_\_ appears.
- Given two functions  $f(x)$  and  $g(x)$  a new function  $f(g(x))$ , called the composition of  $f$  and  $g$ , is found by substituting the \_\_\_\_\_ for  $g(x)$  into the place of  $x$  in the \_\_\_\_\_  $f$ .
- In general,  $f(g(x))$  \_\_\_\_\_  $g(f(x))$ .
- The domain of  $f \circ g$  consists of those values of  $x$  in the \_\_\_\_\_ of  $g$  for which  $g(x)$  is in the \_\_\_\_\_ of  $f$ .
- Functions that have only one  $x$ -value for each  $y$ -value in the range are said to be \_\_\_\_\_ functions.
- In general, every \_\_\_\_\_ function has a/an \_\_\_\_\_ function.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- The vertical line test is used to determine whether a graph represents a vertical line.
- In a one-to-one function, each  $x$ -value corresponds to exactly one  $y$ -value.
- The horizontal line test is used to determine whether a graph of a function is one-to-one.
- The notation  $f^{-1}(x)$  means  $\frac{1}{f(x)}$ .

### Practice

Find the indicated function values for each function given. See Example 1.

- |                      |                           |
|----------------------|---------------------------|
| 1. $f(x) = 8x - 5$   | 4. $h(y) = y^4 + 8$       |
| a. $f(r)$            | a. $h(3p)$                |
| b. $f(3a - 1)$       | b. $h(2s^2)$              |
| 2. $r(x) = 4x - 6$   | 5. $f(c) = 3c^2 + 6c - 9$ |
| a. $r(g - 5)$        | a. $f(n - 2)$             |
| b. $r(h^2 + 8)$      | b. $f(4y^3)$              |
| 3. $g(y) = 5y^2 + 4$ | 6. $b(t) = t^2 - 2t + 7$  |
| a. $g(x - 2)$        | a. $b(5k)$                |
| b. $g(3n^2)$         | b. $b(x + 1)$             |

Find the following function compositions.


7.  $f(x) = 3x + 5$ ,  $g(x) = \frac{x+4}{2}$  Find **a.**  $f(g(2))$  and **b.**  $g(f(2))$ .
8.  $f(x) = \frac{1}{4}x + 1$ ,  $g(x) = 6x - 7$  Find **a.**  $f(g(4))$  and **b.**  $g(f(4))$ .
9.  $f(x) = x^2$ ,  $g(x) = 2x + 3$  Find **a.**  $(f \circ g)(-5)$  and **b.**  $(g \circ f)(-1)$ .
10.  $f(x) = x^2 + 1$ ,  $g(x) = x - 6$  Find **a.**  $(f \circ g)(3)$  and **b.**  $(g \circ f)(-2)$ .

Form the compositions  $f(g(x))$  and  $g(f(x))$  for each pair of functions. See Examples 2 through 4.

- |   |  |
|---|--|
| 11. $f(x) = \sqrt{x}$ , $g(x) = x^2$            | 19. $f(x) = \frac{1}{\sqrt{x}}$ , $g(x) = x^2$     |
| 12. $f(x) = \frac{1}{x}$ , $g(x) = \frac{1}{x}$ | 20. $f(x) = \frac{1}{\sqrt{x}}$ , $g(x) = x^2 - 4$ |
| 13. $f(x) = \sqrt{x}$ , $g(x) = x - 2$          | 21. $f(x) = \frac{1}{x}$ , $g(x) = x^2 + 7x - 8$   |
| 14. $f(x) = \sqrt{x}$ , $g(x) = x^2 - 9$        | 22. $f(x) = \frac{1}{x+1}$ , $g(x) = x^2 + x - 3$  |
| 15. $f(x) = x - 1$ , $g(x) = \frac{1}{x^2}$     | 23. $f(x) = x^{3n}$ , $g(x) = 2x - 6$              |
| 16. $f(x) = \frac{1}{x^2}$ , $g(x) = x^2 + 1$   | 24. $f(x) = x^{\frac{1}{3}}$ , $g(x) = 4x + 7$     |
| 17. $f(x) = x^3 + x + 1$ , $g(x) = x + 1$       | 25. $f(x) = x^3$ , $g(x) = \sqrt{x - 8}$           |
| 18. $f(x) = x^3$ , $g(x) = 2x - 1$              | 26. $f(x) = x^3 + 1$ , $g(x) = \frac{1}{x}$        |

Solve.

27. For the functions  $f(x) = 6x - 3$  and  $g(x) = \frac{1}{3}x + 3$ , find:
  - a.  $f(g(3))$
  - b.  $g(f(0))$
  - c. Does it appear that  $f$  and  $g$  are inverses of each other? Explain.
28. For the functions  $h(x) = -2x + 4$  and  $g(x) = \frac{4-x}{2}$ , find:
  - a.  $h(g(6))$
  - b.  $g(h(-4))$
  - c. Does it appear that  $h$  and  $g$  are inverses of each other? Explain.
29. Given  $f(x) = \frac{1}{2x+1}$  and  $g(x) = -\frac{1}{x}$ , find:
  - a.  $g(f(4))$
  - b.  $f(g(2))$
  - c. Explain the different results from parts a. and b.
30. Given  $f(x) = \sqrt{x-9}$  and  $g(x) = x - 9$ , find:
  - a.  $g(f(109))$
  - b.  $f(g(9))$
  - c. Explain the different results from parts a. and b.

 Show that the given one-to-one functions are inverses of each other. Then graph both functions on the same set of axes and show the line  $y = x$  as a dotted line on each graph. (You may use a calculator as an aid in finding the graphs.) See Examples 6 and 7.

31.  $f(x) = 3x + 1$  and  $g(x) = \frac{x-1}{3}$

37.  $f(x) = x^3 + 2$  and  $g(x) = \sqrt[3]{x-2}$

32.  $f(x) = -2x + 3$  and  $g(x) = \frac{3-x}{2}$

38.  $f(x) = \sqrt[5]{x+6}$  and  $g(x) = x^5 - 6$

33.  $f(x) = \sqrt[3]{x-1}$  and  $g(x) = x^3 + 1$

39.  $f(x) = \frac{2}{x}$  and  $g(x) = \frac{2}{x}$

34.  $f(x) = x^3 - 4$  and  $g(x) = \sqrt[3]{x+4}$

40.  $f(x) = \frac{3}{x}$  and  $g(x) = \frac{3}{x}$

35.  $f(x) = x^2$  for  $x \geq 0$  and  
 $g(x) = \sqrt{x}$

36.  $f(x) = \sqrt{x+3}$  and  
 $g(x) = x^2 - 3$  for  $x \geq 0$

Find the inverse of the given function. Then graph both functions on the same set of axes and show the line  $y = x$  as a dotted line on the graph. See Examples 9 and 10.

41.  $f(x) = 2x - 3$

48.  $f(x) = -\frac{1}{2}x - 3$

42.  $f(x) = 2x - 5$

49.  $f(x) = -x - 2$

43.  $g(x) = x$

50.  $f(x) = -2x + 4$

44.  $g(x) = 1 - 4x$

51.  $f(x) = x^2 + 1, x \geq 0$

45.  $f(x) = 5x + 1$

52.  $f(x) = x^2 - 1, x \geq 0$

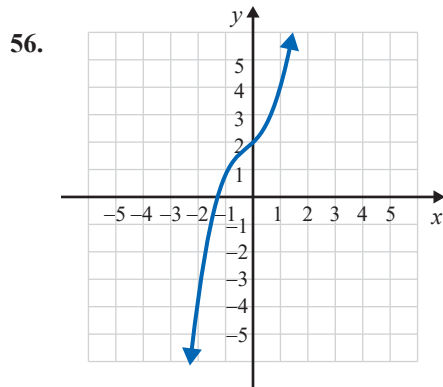
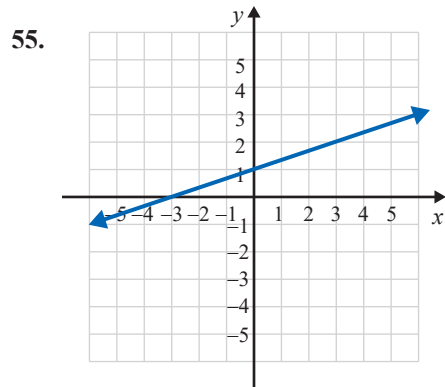
46.  $g(x) = -3x + 1$

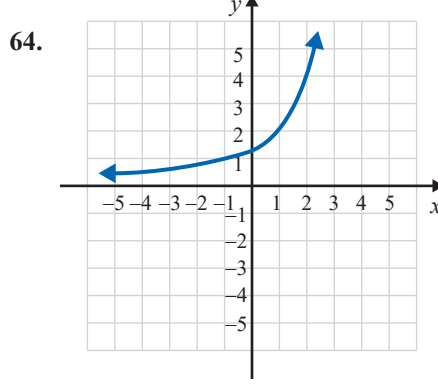
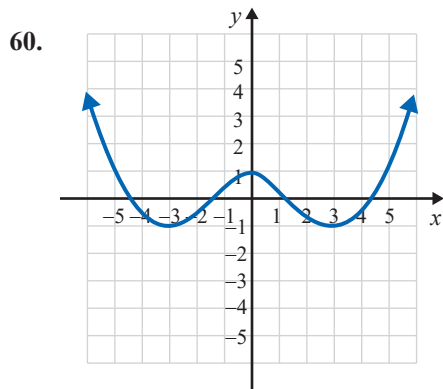
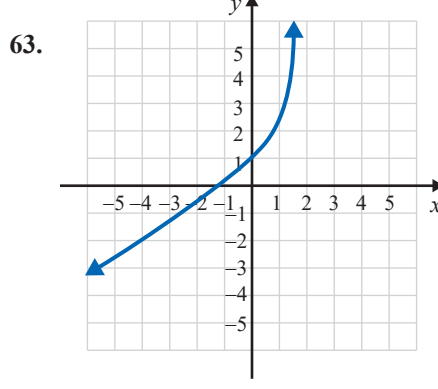
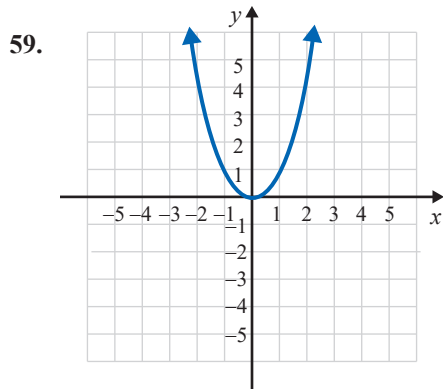
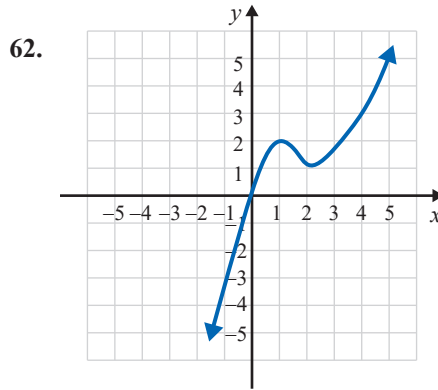
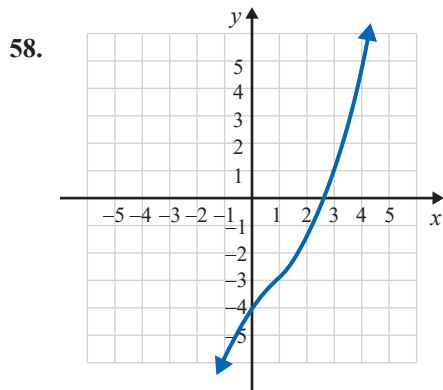
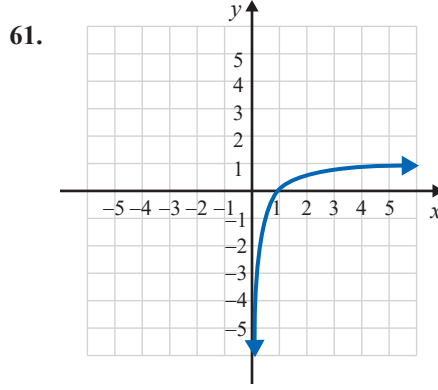
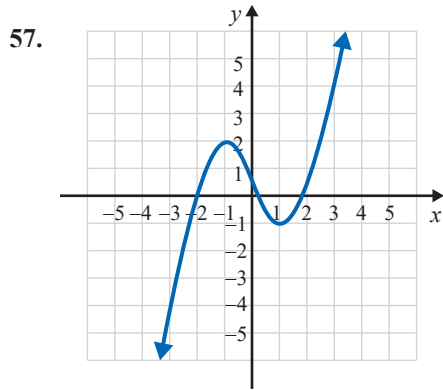
53.  $f(x) = -\sqrt{x}, x \geq 0$


47.  $g(x) = \frac{2}{3}x + 2$

54.  $f(x) = -\sqrt{x-2}, x \geq 2$

Using the horizontal line test, determine which of the graphs are graphs of one-to-one functions. If the graph represents a one-to-one function, graph its inverse by reflecting the graph of the function across the line  $y = x$ . (**Hint:** If a function is one-to-one, label a few points on the graph and use the fact that the  $x$ - and  $y$ -coordinates are interchanged on the graph of the inverse.) See Example 5.





 Use a graphing calculator to graph each of the functions and determine which of the functions are one-to-one by inspecting the graph and using the horizontal line test.

65.  $f(x) = 2x + 3$

66.  $f(x) = 7 - 4x$

67.  $g(x) = x^2 - 2$

68.  $g(x) = 9 - x^2$

69.  $f(x) = 4 - x^3$

70.  $f(x) = x^3 + 2$

71.  $f(x) = \frac{4}{x}$


72.  $g(x) = \frac{1}{x}$

73.  $g(x) = \sqrt{x-3}$

74.  $f(x) = \sqrt{x+5}$

75.  $f(x) = |x+1|$

76.  $f(x) = |x-5|$

 Find the inverse of the given function. Then use a graphing calculator to graph both the function and its inverse. Set the WINDOW so that it is "square."

77.  $f(x) = x^3$

78.  $f(x) = (x+1)^3$

79.  $f(x) = \frac{1}{x-3}$

80.  $f(x) = \frac{1}{x}$

81.  $f(x) = x^2, x \geq 0$

82.  $f(x) = x^2 + 2, x \geq 0$

83.  $g(x) = x^3 + 2$

84.  $g(x) = 6 - x^3$

85.  $f(x) = \sqrt{x+5}, x \geq -5$

86.  $g(x) = \sqrt{x-3}, x \geq 3$

87.  $f(x) = -x^2 + 1, x \geq 0$

88.  $g(x) = -x^2 - 2, x \geq 0$

## Writing & Thinking

89. Explain in your own words why the domains of the two composite functions  $f(g(x))$  and  $g(f(x))$  might not be the same. Give an example of two functions that illustrate this possibility.
90. Explain briefly why a function must be one-to-one to have an inverse.