

Chapter 12 Project

Constructing Fibonacci's Legacy

An activity to use sequences, series, and binomial coefficients.

In our modern era, it may be difficult to imagine anything from 10 years ago still being extremely relevant and important to us today, let alone something from over 800 years ago. However, in 1202, Leonardo Fibonacci imagined and developed a sequence of numbers based on a relatively simple concept.

1. To develop the Fibonacci sequence, start with two numbers, 0 and 1. To get the third number, add the first two numbers together: $0 + 1 = 1$. To get the fourth number, add the second and third numbers together: $1 + 1 = 2$. Continue this pattern to find the first 14 terms of the sequence starting with $F_0 = 0$ and $F_1 = 1$.

F_0	0
F_1	1
F_2	$0 + 1 = 1$
F_3	$1 + 1 = 2$
F_4	$1 + 2 =$
F_5	
F_6	
F_7	
F_8	
F_9	
F_{10}	
F_{11}	
F_{12}	
F_{13}	

2. Were the terms found in Problem 1 found explicitly or recursively? Explain your answer. (**Hint:** When terms are found recursively, one or more of the previous terms are used. Finding terms explicitly is done with a single direct formula that does not require other terms.)
3. To find an explicit rule for the terms of a sequence, it can be helpful to determine if this sequence is arithmetic or geometric. Determine the difference and ratio between each pair of consecutive terms in the table. Is there a common difference or common ratio?
4. The ratios between consecutive pairs appear to be approaching a certain decimal value. This value is called the **golden ratio**. Perform an internet search to find the golden ratio accurate to four decimal places and state two examples of where the golden ratio is observed in nature.
5. Find the partial sum of the first 14 numbers in the Fibonacci sequence.
6. The n^{th} value in the Fibonacci sequence can be written using binomial coefficients as shown in the following formula for positive integers n , h , and k , where $n - h \geq k$.

$$F_n = \binom{n-1}{0} + \binom{n-2}{1} + \binom{n-3}{2} + \cdots + \binom{n-h}{k}$$

Use the formula to determine the values of terms F_4 , F_5 , and F_6 . (**Hint:** For each sum, keep writing terms until $k > n - h$.) Do they match the values found in Problem 1?
7. Suppose you were asked to find the 15th term in Fibonacci's sequence (F_{14}) without knowing terms F_0 through F_{13} . Would it be easier to find this term using the method from Problem 1 or using the formula given in Problem 6? Explain your answer. Then, find F_{14} using your preferred method.