CHAPTER R

Review of Chapters 1 – 7

Did You Know?

Throughout history, teachers of mathematics have tried to develop calculation methods that were easy to use or to memorize. One of the more interesting of these techniques is the rule of double false position. As a student of algebra, it may seem strange to you that such a complicated method would be developed to solve a simple first-degree equation of the form $ax + b = 0$. But remember that you have modern symbolism at your disposal. To use the rule, we will make two guesses as to the solution of the equation. We shall designate the guesses as g_1 and g_2. Now we will let $e_1 = ag_1 + b$ and $e_2 = ag_2 + b$, where e_1 and e_2 represent the amount of error in our guesses. Then the solution is

$$x = \frac{e_1 g_2 - e_2 g_1}{e_1 - e_2}.$$

We now illustrate the rule of double false position with an example: $3x + 6 = 0$. Suppose we guess that the solution is either 1 or 2. That is, let $g_1 = 1$ and $g_2 = 2$. Then $e_1 = 3(1) + 6 = 9$ and $e_2 = 3(2) + 6 = 12$, and the solution to the equation would be

$$\frac{9(2) - 12(1)}{9 - 12} = \frac{18 - 12}{-3} = \frac{6}{-3} = -2.$$

This unnecessarily complicated method of solving first-degree equations was taught until the nineteenth century. One of the most popular English texts of the sixteenth century, *The Grounde of Artes* by Robert Recorde (1510? – 58), gives the rule of double false position in verse:

Recorde

Gesse at this woorke as happe doth leade.
By chance to truthe you may procede.
And firste woorke by the question,
Although no truthe therein be don.
Suche falsehode is so good a grounde,
That truth by it will soone be founde.
From many bate to many mo,
From to fewe take to fewe also.
With to much ioyne to fewe againe,
To to fewe adde to manye plaine.
To crossewaies multiplye contrary kinde,
All truthe by falsehode for to fynde.

Students memorized the poem as a method of remembering the rule, which they generally did not understand. Can you figure out why making two false guesses can lead to the correct answer? After reviewing Chapters 1 – 7, you may be able to verify the rule of double false position.

"But it should always be required that a mathematical subject not be considered exhausted until it has become intuitively evident."

Felix Klein (1849 – 1925)

The Review Chapter provides a review of the topics generally covered in a one semester beginning algebra course. Each section corresponds to one of the first seven chapters in the *Introductory and Intermediate Algebra* text. For more detailed discussions of particular topics, students should refer to the corresponding section number and page number listed in the left margin.

R.1 Chapter 1 Review: Real Numbers

Section 1.1
page 2

The Real Number Line and Absolute Value

Number lines are used to graph sets of numbers. The **graph** of a number is the point that corresponds to the number and the number is called the **coordinate** of the point.

The graph of the set $A = \left\{-2.5, -1, 0, \frac{2}{3}, \sqrt{3}, \pi\right\}$ is shown in Figure 1.

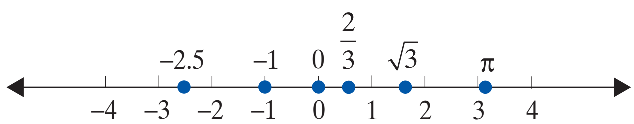


Figure 1

Various types of numbers are given names as shown below.

Types of Numbers

Natural Numbers: $\mathbb{N} = \{1, 2, 3, 4, 5, ...\}$

Whole Numbers: $\mathbb{W} = \{0, 1, 2, 3, 4, 5, ...\}$

Integers: $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

The set of integers consists of the whole numbers and their **opposites**. The opposites of positive integers are called **negative integers**. Zero is its own opposite and is neither positive nor negative.

Rational Numbers: $\mathbb{Q} = \left\{ \begin{array}{l} \text{Numbers that can be written in the form } \dfrac{a}{b} \\ \text{where } a \text{ and } b \text{ are integers and } b \neq 0 \end{array} \right\}$

In decimal form, rational numbers are either terminating decimals or infinite repeating decimals.

Irrational Numbers: $\left\{ \begin{array}{l} \text{Numbers that can be written as infinite} \\ \text{nonrepeating decimal numbers} \end{array} \right\}$

Real Numbers: $\mathbb{R} = \{\text{All rational and irrational numbers}\}$

All rational and irrational numbers are classified as **real numbers**, and number lines are called **real number lines**.

On a horizontal number line, smaller numbers are always to the left of larger numbers. The following table of symbols indicates ways in which numbers are compared.

Table of Symbols

$=$	is equal to	$\neq$	is not equal to
$<$	is less than	$>$	is greater than
$\leq$	is less than or equal to	$\geq$	is greater than or equal to

Example 1: Inequalities

a. Determine whether each of the following statements is true or false.

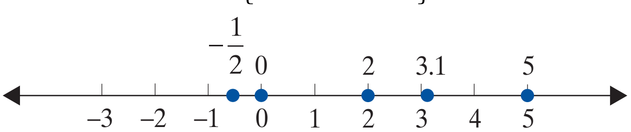
$4 < -10$ False, since 4 is greater than -10 (or -10 is less than 4).

$6 \geq 6$ True, since 6 is equal to 6.

$-3.5 > -8.1$ True, since -3.5 is greater than -8.1 (or -8.1 is less than -3.5).

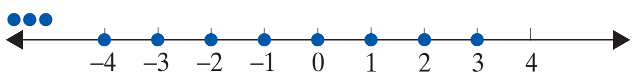
b. Graph the set of real numbers $\left\{-\frac{1}{2}, 0, 2, 3.1, 5\right\}$.

Solution:



c. Graph the set of integers x, where $x < 4$.

Solution:



The **distance a number is from 0 on a number line** is called its **absolute value** and is symbolized by two vertical bars, $|\ \ |$.

Example 2: Absolute Value

a. If $|x| = 6$, what are the possible values for x?

Solution: $x = 6$ or $x = -6$ since $|6| = 6$ and $|-6| = 6$.

Continued on the next page...

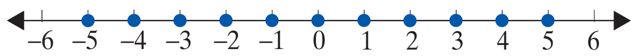
b. If $|y| = -3.2$, what are the possible values for y?

Solution: There are no values of y for which $|y| = -3.2$. The absolute value is never negative.

c. If $|x| \le 5$, what are the possible integer values for x? Graph these integers on a real number line.

Solution: The integers must be within or equal to 5 units from 0. They are

$$\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$



Sections 1.2 and 1.3
pages 16 and 21

Addition and Subtraction with Real Numbers

As stated below, the formal statements of the rules of addition with real numbers involve the use of absolute value.

Rules for Addition with Real Numbers

1. To add two real numbers with **like signs**,
   - **a.** add their absolute values, and
   - **b.** use the common sign.
2. To add two real numbers with **unlike signs**,
   - **a.** subtract their absolute values (the smaller from the larger), and
   - **b.** use the sign of the number with the larger absolute value.

Example 3: Addition

a. $(-11)+(+6) = -(|-11|-|+6|) = -(11-6) = -5$

b. $(-2.8)+(-1.5) = -(|-2.8|+|-1.5|) = -(2.8+1.5) = -4.3$

c.
$$\begin{array}{r} -3 \\ +\ 17 \\ \hline 14 \end{array}$$

d. $-20+13+(-2) = -9$

The formal definition of subtraction can be stated as follows.

Subtraction

For any real numbers a and b,

$$\boldsymbol{a - b = a + (-b)}.$$

In words, to subtract b from a, **add** the **opposite** of b to a.

Example 4: Subtraction

a. $(-7)-(+10) = -7+(-10) = -17$

b. $-1.3-(-5.7) = -1.3+(+5.7) = 4.4$

c. $-27+23+4 = -4+4 = 0$

d.

$$\begin{array}{r} 42 \\ \underline{-(-76)} \end{array} \xrightarrow[\text{change}]{\text{sign}} \begin{array}{r} 42 \\ \underline{+76} \\ 118 \end{array}$$

To find the **change in value** between two numbers, take the end value and subtract the beginning value. Symbolically,

Change in Value = (End Value) – (Beginning Value).

The **net change** in a measure is the algebraic sum of several numbers. For example, the net change in sales over a period of time might be a sum of positive and negative numbers.

Section 1.4 Multiplication and Division with Real Numbers

page 31

The rules for multiplication and division with real numbers can be summarized as follows and are illustrated in Example 5.

Rules for Multiplication and Division with Positive and Negative Real Numbers

1. The product (or quotient) of two real numbers with like signs is positive.
2. The product (or quotient) of two real numbers with unlike signs is negative.
3. The product of any number and 0 is 0.
4. The quotient $\dfrac{\mathbf{0}}{\boldsymbol{a}} = \mathbf{0}$, but the quotient $\dfrac{\boldsymbol{a}}{\mathbf{0}}$ is **undefined**.

Example 5: Multiplication and Division

a. $(-15)(-3) = 45$ **b.** $-6(3.2) = -19.2$ **c.** $-14(16)(0) = 0$

d. $\frac{36}{-12} = -3$ **e.** $\frac{17}{0} =$ undefined **f.** $\frac{-5.6}{-4} = 1.4$

The **arithmetic average** (or **mean**) of a set of data is reported frequently in newspapers and in scientific journals. Your grade in this course is probably related to your average score on exams.

Average

The **average** (or **mean**) of a set of numbers is the value found by adding the numbers in the set and then dividing the sum by the number of numbers in the set.

Example 6: Average (or Mean)

The rainfall for three days in Costa Mesa, CA was 1.4", 2.5", and 1.5". Find the average rainfall for those three days.

Solution: First, find the sum: $1.4 + 2.5 + 1.5 = 5.4$

Now divide the sum by 3: $5.4 \div 3 = 1.8$

The average rainfall was 1.8 inches per day.

Section 1.5 Exponents, Prime Numbers, and LCM

page 42

The result of multiplication is called the **product**, and the numbers being multiplied are called **factors** of the product. **Exponents** are used to indicate repeated multiplication. For example if 3 is used as a factor 4 times, we can write:

4 is the exponent

$$3 \cdot 3 \cdot 3 \cdot 3 = 3^4 = 81$$

3 is the base 81 is the 4^{th} power of 3

The expression 3^4 is an **exponential expression** and is read “3 to the 4^{th} power.”

Exponent and Base

A whole number n is an **exponent** if it is used to tell how many times another whole number a is used as a factor. The repeated factor a is called the **base** of the exponent. Symbolically,

$$\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a \cdot a}_{n \text{ factors}} = a^n.$$

exponent

base

In expressions with exponent 2, the base is said to be **squared**. In expressions with exponent 3, the base is said to be **cubed**.

Every counting number, except 1, has **at least two** factors. Those with exactly two different factors are called **prime numbers**.

Prime and Composite Numbers

A **prime number** is a counting number greater than 1 that has exactly two different factors (or divisors), namely 1 and itself.

A **composite number** is a counting number with more than two different factors (or divisors).

Here is a list of the prime numbers less than 100. Recognizing these numbers is very helpful in working with fractions.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Finding the factors of a composite number so that all the factors are prime numbers is called finding the **prime factorization** of that number. For example,

$$300 = 3 \cdot 100 = 3 \cdot 10 \cdot 10 = 3 \cdot 2 \cdot 5 \cdot 2 \cdot 5 = 2^2 \cdot 3 \cdot 5^2$$

and $2^2 \cdot 3 \cdot 5^2$ is the prime factorization of 300.

There is only one prime factorization of any composite number. The following procedure can be used to find prime factorizations.

To Find the Prime Factorization of a Composite Number

1. Factor the composite number into any two factors.
2. Factor each factor that is not prime into two more factors.
3. Continue this process until all factors are prime.

The **prime factorization** is the product of all the prime factors.

The **least common multiple (LCM)** of a set of counting numbers is the smallest counting number that is divisible by every number in the set. The following technique involving prime numbers can be used to find the LCM of a set of counting numbers.

To Find the LCM of a Set of Counting Numbers

1. Find the prime factorization of each number.
2. List the prime factors that appear in any one of the prime factorizations.
3. Find the product of these primes using each prime the greatest number of times it appears in any one of the prime factorizations.

Example 7: Least Common Multiple (LCM)

Find the LCM of 18, 20, and 30.

Solution:

$$\left.\begin{aligned} 18 &= 2\cdot 3^2 \\ 20 &= 2^2\cdot 5 \\ 30 &= 2\cdot 3\cdot 5 \end{aligned}\right\} \text{LCM} = 2^2\cdot 3^2\cdot 5 = 180$$

Example 8: LCM with Variables

Find the LCM for $6x,\ 4x^2y,$ and $10y^3$.

Solution: Treat each variable in the same manner as a prime number.

$$\left.\begin{aligned} 6x &= 2\cdot 3\cdot x \\ 4x^2y &= 2^2\cdot x^2\cdot y \\ 10y^3 &= 2\cdot 5\cdot y^3 \end{aligned}\right\} \text{LCM} = 2^2\cdot 3\cdot 5\cdot x^2\cdot y^3 = 60x^2y^3$$

Sections 1.6 and 1.7

pages 54 and 65

Operations (Multiplication, Division, Addition, Subtraction) with Fractions

Fractions of the form

$$\frac{a}{b} \begin{array}{l} \leftarrow \text{numerator} \\ \leftarrow \text{denominator} \end{array}$$

where a and b are integers and $b \neq 0$ are called **rational numbers**. There are fractions that are not rational numbers. However, for now when we deal with fractions of the form $\frac{a}{b}$ we will assume the fraction represents a rational number and will use the terms **fraction and rational number interchangeably**. See Sections 1.6 and 1.7 for listings of the principles and definitions for operating with fractions..

Example 9: Operations with Fractions

a. $\frac{4a}{9b} \cdot \frac{3b}{8a} = \frac{\cancel{4} \cdot \cancel{a} \cdot \cancel{3} \cdot \cancel{b} \cdot 1}{\cancel{3} \cdot 3 \cdot \cancel{b} \cdot \cancel{4} \cdot 2 \cdot \cancel{a}} = \frac{1}{6}$

b. $\left(-\frac{3}{5}\right) \div \left(-\frac{3}{10}\right) = \left(-\frac{3}{5}\right) \cdot \left(-\frac{10}{3}\right) = \frac{\cancel{3} \cdot 2 \cdot \cancel{5}}{\cancel{5} \cdot \cancel{3} \cdot 1} = \frac{2}{1} = 2$

c. $\frac{1}{5} + \frac{3}{8} + \frac{3}{10} = \frac{1}{5} \cdot \frac{8}{8} + \frac{3}{8} \cdot \frac{5}{5} + \frac{3}{10} \cdot \frac{4}{4} = \frac{8}{40} + \frac{15}{40} + \frac{12}{40} = \frac{35}{40} = \frac{\cancel{5} \cdot 7}{\cancel{5} \cdot 8} = \frac{7}{8}$

d. $\frac{3}{y} - \frac{2}{7} = \frac{3}{y} \cdot \frac{7}{7} - \frac{2}{7} \cdot \frac{y}{y} = \frac{21}{7y} - \frac{2y}{7y} = \frac{21 - 2y}{7y}$

Section 1.8 Order of Operations

page 73

Many numerical expressions include exponents and parentheses (or other grouping symbols) and involve several operations. Mathematicians have agreed on the following **rules for order of operations** so that there is only one correct answer for the value of an expression.

Rules for Order of Operations

1. Simplify within grouping symbols, such as parentheses ( ), brackets [ ], and braces { }, working from the innermost grouping outward.
2. Find any powers indicated by exponents.
3. Moving from **left to right**, perform any multiplications **or** divisions **in the order they appear**.
4. Moving from **left to right**, perform any additions **or** subtractions **in the order they appear**.

Example 10: Using the Rules for Order of Operations

a. $20 \div 4 - 5 \cdot 2^3 = 20 \div 4 - 5 \cdot 8 = 5 - 40 = -35$

b. $3\left(4^2 - 1\right) + 2 \cdot 3^2 = 3(16 - 1) + 2 \cdot 9 = 3(15) + 2 \cdot 9 = 45 + 18 = 63$

c. $\left(2\frac{1}{2}\right)^2 \div \left(\frac{1}{2} + \frac{1}{3}\right) = \left(\frac{5}{2}\right)^2 \div \left(\frac{3}{6} + \frac{2}{6}\right) = \frac{25}{4} \div \frac{5}{6} = \frac{25}{4} \cdot \frac{6}{5} = \frac{\cancel{5} \cdot 5 \cdot \cancel{2} \cdot 3}{\cancel{2} \cdot 2 \cdot \cancel{5}} = \frac{15}{2}$

Section 1.9 Properties of Real Numbers

page 81

The various properties of real numbers under the operations of addition and multiplication are summarized here. These properties are used throughout algebra in developing formulas and general concepts.

Properties of Addition and Multiplication

In this table a, b, and c are real numbers.

For Addition	Property Name	For Multiplication
$a + b = b + a$	Commutative property	$ab = ba$
$(a + b) + c = a + (b + c)$	Associative property	$a(bc) = (ab)c$
$a + 0 = 0 + a = a$	Identity	$a \cdot 1 = 1 \cdot a = a$
$a + (-a) = 0$	Inverse	$a \cdot \frac{1}{a} = 1$ (for $a \neq 0$)

Zero-Factor Law: $a \cdot 0 = 0 \cdot a = 0$

Distributive Property of Multiplication over Addition: $a(b + c) = ab + ac$

Example 11: Illustrations of Properties of Real Numbers

a. $8.35 \cdot 1 = 8.35$ Multiplicative identity

b. $5(2 + x) = 10 + 5x$ Distributive property

c. $6 + (9 + 7) = (6 + 9) + 7$ Associative property of addition

d. $18 + y = y + 18$ Commutative property of addition

e. $-16 \cdot 0 = 0$ Zero-factor law

R.1 Exercises

1. Given the set of numbers $A = \left\{-2.48, -\frac{1}{3}, 0, 2.3, \pi, 4, \sqrt{7}, \frac{13}{5}\right\}$ tell which of the numbers are **a.** integers, **b.** rational numbers, **c.** irrational numbers, and **d.** real numbers.

Graph each set of real numbers on a real number line.

2. $A = \left\{-5, -3.2, -0.3, \frac{1}{4}, 2\frac{1}{2}\right\}$

3. $B = \left\{-4.2, -3, -1.5, 0, 3\frac{1}{4}, 6\right\}$

4. Graph the sets of integers that satisfy the following conditions.

a. $|x| = 7$ **b.** $|x| < 7$ **c.** $|x| > 7$

Determine whether each statement is true or false. If a statement is false, rewrite it in a form that is a true statement. (There may be more than one way to correct a statement.)

5. $|3.5| \le 3.5$ **6.** $|6| < -6$ **7.** $\frac{3}{5} > \frac{3}{4}$

8. $|-1.8| < -2$ **9.** $-|-5| \ge -5$

Perform the indicated operations and simplify each answer.

10. $(-13)+3$ **11.** $(-9)+(-6)+5$ **12.** $-13-1-13$

13. $-15-(-4)$ **14.** $36-(-2)$ **15.** $-15.7+(-13.3)$

16. $-\frac{1}{2}+\left(-\frac{1}{2}\right)$ **17.** $\frac{3}{16}-\left(-\frac{3}{8}\right)$ **18.** $-6.3-7.1-3.9$

19. $\frac{-54}{6}$ **20.** $\frac{78}{-39}$ **21.** $43(-3)$

22. $-6.1(-2)$ **23.** $3^2 \div (-9) + (4-2^2) - 10^2$

24. $5(13-15)^3 \cdot 8 \div 2$ **25.** $8-6\left[(-22)\div 11\cdot 2-(-5)\right]$

26. $10\cdot 3 \div (2^2-5)+14-(-3)$

27. Find the sum of -15 and -12. Then subtract -3.

28. Find the change in temperature if the temperature drops from 20° C to 3° C.

29. Find the mean of the following set of integers: 32, 27, 72, 87, and 102.

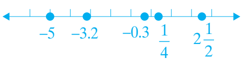
30. Explain, in your own words, why division by 0 is not possible under any circumstances.

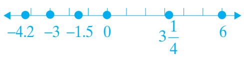
1. a. 0, 4

b. $-2.48, -\frac{1}{3}, 0, 2.3, 4, \frac{13}{5}$

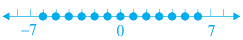
c. $\pi, \sqrt{7}$

d. $-2.48, -\frac{1}{3}, 0, 2.3, \pi, 4, \sqrt{7}, \frac{13}{5}$

2. 

3. 

4. a. 

b. 

c. 

5. True

6. False; $|6| > -6$

7. False; $\frac{3}{5} < \frac{3}{4}$

8. False; $|-1.8| > -2$

9. True **10.** -10

11. -10 **12.** -27

13. -11 **14.** 38

15. -29 **16.** -1

17. $\frac{9}{16}$ **18.** -17.3

19. -9 **20.** -2

21. -129 **22.** 12.2

23. -101 **24.** -160

25. 2 **26.** -13

27. -24

28. $-17°$ C

29. 64

30. Answers will vary.

31. a. $2^2 \cdot 3^2 \cdot 5$
b. $2 \cdot 3^3 \cdot 5$
c. $2^4 \cdot 5^4$
32. a. 720
b. 900
c. 12,600
d. $270x^3y$
33. $-2x$ 34. $\frac{11}{2}$
35. $\frac{5m}{16n}$
36. 720 women; 480 men
37. a. More than 120
b. Less than 120
c. 160 passengers
38. $\frac{36}{25}$ 39. $-\frac{1}{3}$
40. $\frac{10}{11}$ 41. $\frac{53}{60}$
42. $\frac{5}{42}$ 43. $\frac{13}{12}$
44. $-\frac{10}{9x}$
45. $-\frac{31}{8x}$ 46. 0
47. $\frac{5}{12}$ 48. $\frac{18}{5}$
49. $-\frac{b}{4a}$ 50. $\frac{63y}{160x}$
51. $\frac{7}{36}$ 52. $-\frac{49}{18}$
53. $\frac{20}{9}$
54. 24.1456
55. 0.0596

31. Find the prime factorization of each of the following numbers.
a. 180 **b.** 270 **c.** 10,000

32. Find the LCM (least common multiple) of each of the following sets of numbers/expressions.
a. $\{16, 20, 45\}$ **b.** $\{25, 30, 36\}$ **c.** $\{20, 70, 90, 200\}$ **d.** $\{10x^3, 27x, 30x^2y\}$

Multiply or divide as indicated and reduce each answer to lowest terms.

33. $\frac{8x}{-40} \cdot \frac{15x}{3y} \cdot \frac{2y}{x}$ **34.** $\frac{46}{7a} \div \frac{92}{77a}$ **35.** $\frac{25m}{21n} \div \frac{40mn}{9m} \cdot \frac{14n}{12}$

36. Gender: A study shows that $\frac{3}{5}$ of the students at a certain school are women. If the school has an enrollment of 1200 students, how many are women? How many are men?

37. Airplanes: An airplane is carrying 120 passengers. This is $\frac{3}{4}$ of the capacity of the airplane.
a. Is the capacity of the airplane more or less than 120?
b. If you were to multiply 120 by $\frac{3}{4}$, would the product be more or less than 120?
c. What is the capacity of the airplane?

38. If the product of $\frac{5}{8}$ with another number is $\frac{9}{10}$, what is the other number?

Perform the indicated operations. Reduce each answer to lowest terms.

39. $\frac{2}{15} - \frac{7}{15}$ **40.** $\frac{5}{11} + \frac{2}{11} + \frac{3}{11}$ **41.** $\frac{5}{12} + \frac{7}{15}$ **42.** $\frac{3}{14} - \frac{2}{21}$

43. $\frac{1}{3} + \frac{3}{4}$ **44.** $\frac{8}{9x} - \frac{2}{x}$ **45.** $\frac{1}{8x} - \frac{4}{x}$ **46.** $\frac{1}{2} - \frac{1}{3} - \frac{1}{6}$

47. $\frac{3}{8} \cdot \frac{2}{5} + \frac{4}{15}$ **48.** $\frac{5}{4a} \div \frac{5}{16a} - \frac{2b}{3} \cdot \frac{3}{5b}$ **49.** $\left(\frac{1}{6a} - \frac{2}{3a}\right) \div \left(\frac{5}{5b} + \frac{3}{3b}\right)$

50. $\left(\frac{1}{2x} - \frac{1}{5x}\right) \div \left(\frac{3}{7y} + \frac{1}{3y}\right)$ **51.** $\left(\frac{1}{3}\right)^2 + \left(\frac{3}{4} - \frac{2}{3}\right)$

52. $\left(-\frac{5}{8}\right) \div \frac{3}{16} \cdot \frac{2}{3} - \left(\frac{1}{5} + \frac{3}{10}\right)$ **53.** $\frac{2}{5} \cdot \frac{15}{6} \div \left(\frac{1}{2}\right)^2 - \left(\frac{4}{3}\right)^2$

Use a calculator to find the values accurate to four decimal places.

54. $78.956 \div 3.27$ **55.** $0.93 \div 15.61$

56. 15.4847
57. –0.8275
58. 2.5
59. 297.8879
60. Commutative property of multiplication
61. Associative property of addition
62. Additive identity
63. Multiplicative identity
64. Distributive property
65. Multiplicative inverse
66. Associative property of multiplication
67. Additive inverse
68. 183 lb
69. $4760
70. 161.4 calls

56. $\dfrac{93.56+7.4}{6.52}$

57. $\dfrac{7.15-14.68}{3.4+5.7}$

58. $\dfrac{-5-18}{-7.9-1.3}$

59. $(1.3)^5+17.5(16.81)$

Name the property of real numbers illustrated.

60. $5\cdot y=y\cdot 5$

61. $9+(6+x)=(9+6)+x$

62. $35+0=35$

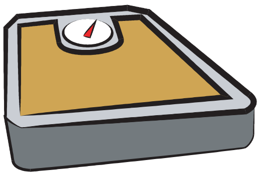
63. $35\cdot 1=35$

64. $6(x+y)=6x+6y$

65. $5\cdot\dfrac{1}{5}=1$

66. $3(xy)=3x(y)$

67. $-14+(+14)=0$

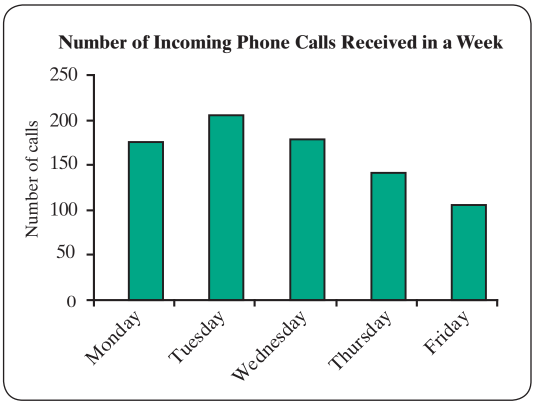


68. Dieting: Jorge wanted to lose weight and went on a 6 week diet plan. He lost 5 lb during the first week, gained 1 lb the second week, lost 4 lb the third week, lost 2 lb the fourth week, gained 3 lb the fifth week, and lost 5 lb the sixth week. If he weighed 195 lb when he started his diet, what did he weigh at the end of the six week period?

69. Checking accounts: Ms. Mariani knew that the balance in her checking account was $2300. She made deposits of $560, $3000, and $575. She wrote checks for $650, $800, and $225. What was her new balance?

70. Phone calls: The following bar graph shows the number of phone calls received by a software company during one week. What was the mean number of phone calls per day during that week?

Day of Week	Number of Calls
Monday	175
Tuesday	206
Wednesday	179
Thursday	142
Friday	105



R.2 Chapter 2 Review: Algebraic Expressions, Linear Equations, and Applications

Section 2.1 Simplifying and Evaluating Algebraic Expressions

page 102

A single number is called a **constant**. Any constant or variable or the indicated product and/or quotient of constants and variables is called a **term**. A number written next to a variable is called the **coefficient** (or the **numerical coefficient**) of the variable.

Like Terms

Like terms (or **similar terms**) are terms that are constants or terms that contain the same variables raised to the same powers.

Note: The sum of the exponents on the variables is the **degree** of the term.

An **algebraic expression** is a combination of variables and numbers using any of the operations of addition, subtraction, multiplication, or division, as well as exponents.

To simplify expressions that contain like terms we want to **combine like terms**.

Combining Like Terms

To **combine like terms**, add (or subtract) the coefficients and keep the common variable expression.

Example 1: Combining Like Terms

Combine like terms whenever possible.

a. $7x+11x=(7+11)x=18x$

b. $4(n+6)-5(n-3)=4n+24-5n+15=4n-5n+24+15=-n+39$

c. $5x^2-4a+x^2-a=5x^2+x^2-4a-a=6x^2-5a$

In most cases, if an expression is to be evaluated, like terms should be combined first, and then given values substituted for the variables. The resulting expression is evaluated by following the rules for order of operations.

In general, except for $x = 0$,

1. $-x^2$ is **negative** **2.** $(-x)^2$ is **positive** **3.** $-x^2 \neq (-x)^2$

To Evaluate an Algebraic Expression

1. Combine like terms, if possible.
2. Substitute the values given for any variables.
3. Follow the rules for order of operations. (See Section 1.8)

Example 2: Evaluating Algebraic Expressions

The following expressions are simplified and then evaluated for $x = 3$, $y = -2$, $a = -1$, and $b = 5$.

a. $3x + 5y + 7x = 10x + 5y = 10(3) + 5(-2) = 30 - 10 = 20$

b. $4ab - ab + 8a - 2a = 3ab + 6a = 3(-1)(5) + 6(-1) = -15 - 6 = -21$

c. $4(x+1) - 2(y-5) = 4x + 4 - 2y + 10 = 4x - 2y + 14$
$$= 4(3) - 2(-2) + 14 = 12 + 4 + 14 = 30$$

d. $8b - \dfrac{4a+5a}{9} = 8b - \dfrac{9a}{9} = 8b - a = 8(5) - (-1) = 40 + 1 = 41$

Section 2.2 page 110

Translating English Phrases and Algebraic Expressions

We want to be able to change English phrases into their "algebraic" equivalents and vice versa. Certain key words are used to indicate basic operations. Some of these words are listed in Section 2.2 on page 110. Example 3 shows English phrases translated into algebraic expressions.

Example 3: Translating English Phrases to Algebraic Expressions

English Phrase	Algebraic Expression (Any variable may be used.)
a. The **product** of five and a number	$5n$
b. **Twice** a number **decreased by** 17	$2x - 17$
c. **Twice** the **sum of** a number and 17	$2(x + 17)$

Example 4 shows algebraic expressions translated into English phrases. In many cases there is more than one correct translation. For example, the + sign can be interpreted as "plus", "add", "sum of", or "increased by".

Example 4: Translating Algebraic Expressions to English Phrases

	Algebraic Expression	Possible English Phrase
a.	$x+25$	25 **more than** a number
b.	$\frac{n}{-3}$	The **quotient** of a number and –3
c.	$4y-8$	8 **less than** 4 **times** a number

Section 2.3 Solving Linear Equations: $x + b = c$ and $ax = c$

page 117

An **equation** is a statement that two algebraic expressions are equal. If an equation contains a variable, any number that gives a true statement when substituted for the variable is called a **solution** to the equation. The solutions to an equation form a **solution set**. The process of finding the solution set is called **solving the equation**. A **linear equation in *x*** is an equation of the form

$$ax + b = c \qquad \text{where } a, b, \text{ and } c \text{ are constants and } a \neq 0.$$

To solve linear equations, we apply the addition and multiplication principles of equality.

The Addition and Multiplication Principles of Equality

1. **The Addition Principle of Equality**
   If A, B, and C are algebraic expressions,
   then the equations $\boldsymbol{A} = \boldsymbol{B}$ and $\boldsymbol{A} + \boldsymbol{C} = \boldsymbol{B} + \boldsymbol{C}$ are equivalent.
2. **The Multiplication Principle of Equality**
   If A and B are algebraic expressions and C is a nonzero **constant**,
   then the equations $\boldsymbol{A} = \boldsymbol{B}$, $\boldsymbol{AC} = \boldsymbol{BC}$, and $\frac{\boldsymbol{A}}{\boldsymbol{C}} = \frac{\boldsymbol{B}}{\boldsymbol{C}}$ are equivalent.

Example 5: Solving Equations of the Forms $x + b = c$ and $ax = c$

Each of the equations is solved for the unknown variable.

a.
$$\begin{aligned} x - 5 &= 8 \\ x - 5 + 5 &= 8 + 5 \\ x &= 13 \end{aligned}$$

b.
$$\begin{aligned} 3.7 &= 4y - 3y + 0.9 \\ 3.7 &= y + 0.9 \\ 3.7 - 0.9 &= y + 0.9 - 0.9 \\ 2.8 &= y \end{aligned}$$

c.

$$-3z = -21$$
$$\frac{-3z}{-3} = \frac{-21}{-3}$$
$$z = 7$$

d.

$$\frac{5x}{7} = \frac{5}{8}$$
$$\frac{7}{5} \cdot \frac{5}{7}x = \frac{7}{5} \cdot \frac{5}{8}$$
$$x = \frac{7}{8}$$

Section 2.4 Solving Linear Equations: $ax + b = c$

page 127

The **general procedure for solving linear equations** is a combination of the use of the addition and multiplication principles of equality.

Solving Linear Equations that Simplify to the Form $ax + b = c$

1. Combine like terms on both sides of the equation.
2. Use the **addition principle of equality** and add the opposite of the constant b to both sides.
3. Use the **multiplication** (or **division**) **principle of equality** to multiply both sides by the reciprocal of the coefficient of the variable (or divide both sides by the coefficient itself). The coefficient of the variable will become +1.
4. Check your answer by substituting it for the variable in the original equation.

Example 6: Solving Equations of the Form $ax + b = c$

The following equations are solved for the unknown variables.

a.

$$3y + 12 - 5y = -20$$
$$-2y + 12 = -20$$
$$-2y + 12 - 12 = -20 - 12$$
$$-2y = -32$$
$$\frac{-2y}{-2} = \frac{-32}{-2}$$
$$y = 16$$

b.

$$\frac{2}{3}x + \frac{1}{4}x + \frac{5}{8} = 0$$
$$24\left(\frac{2}{3}x + \frac{1}{4}x + \frac{5}{8}\right) = 24 \cdot 0$$
$$24 \cdot \frac{2}{3}x + 24 \cdot \frac{1}{4}x + 24 \cdot \frac{5}{8} = 24 \cdot 0$$
$$16x + 6x + 15 = 0$$
$$22x + 15 = 0$$
$$22x + 15 - 15 = 0 - 15$$
$$22x = -15$$
$$\frac{22x}{22} = \frac{-15}{22}$$
$$x = -\frac{15}{22}$$

Example 7: An Application

The perimeter of a rectangle is the sum of twice the length and twice the width and can be represented with the formula $P = 2l + 2w$. If the perimeter of a rectangular home lot is 430 feet and the length of the lot is 125 feet, solve the equation $430 = 2(125) + 2w$ to find the width.

Solution:

$$430 = 2(125) + 2w$$
$$430 = 250 + 2w$$
$$180 = 2w$$
$$90 = w$$

The lot is 90 feet wide.

Section 2.5 Solving Linear Equations: $ax + b = cx + d$

page 134

If a linear equation has variables on both sides of the equation, then the addition principle is applied so that the variable terms are on one side and constant terms are on the other side.

Example 8: Solving an Equation of the Form $ax + b = cx + d$

Each of the following equations is solved for the unknown variable. Follow the steps carefully. Explain to yourself what happens at each step.

a.

$$8y + 7.5 = 6y - 4.6$$
$$8y + 7.5 - 6y = 6y - 4.6 - 6y$$
$$2y + 7.5 = -4.6$$
$$2y + 7.5 - 7.5 = -4.6 - 7.5$$
$$2y = -12.1$$
$$y = -6.05$$

b.

$$4(x + 3) + x = 2(x - 7) + 32$$
$$4x + 12 + x = 2x - 14 + 32$$
$$5x + 12 = 2x + 18$$
$$5x + 12 - 2x = 2x + 18 - 2x$$
$$3x + 12 = 18$$
$$3x + 12 - 12 = 18 - 12$$
$$3x = 6$$
$$\frac{3x}{3} = \frac{6}{3}$$
$$x = 2$$

Every linear equation has one solution and is a **conditional equation**. However, in some cases simplifying an equation will result in a statement that is always true. In such a case, the original equation is called an **identity** and has an infinite number of solutions which can be written as all real numbers or $\mathbb{R}$. If the simplification results in a statement that is never true, then the original equation is called a **contradiction** and there is **no solution**.

Example 9: Solutions of Equations

Determine whether each of the following equations is a conditional equation, an identity, or a contradiction.

a. $-2(x-3)+x=6-x$

Solution:
$$\begin{aligned} -2(x-3)+x &= 6-x \\ -2x+6+x &= 6-x \\ -x+6 &= 6-x \\ 6 &= 6 \end{aligned}$$

The last equation is always true. Therefore, the original equation is an **identity** and has an infinite number of solutions. Every real number is a solution.

b. $3x+1=3(x+1)+2$

Solution:
$$\begin{aligned} 3x+1 &= 3(x+1)+2 \\ 3x+1 &= 3x+3+2 \\ 3x+1 &= 3x+5 \\ 1 &= 5 \end{aligned}$$

The last equation is never true. Therefore the original equation is a **contradiction** and has no solution.

Section 2.6 Introduction to Problem Solving

page 142

We can use Pólya's four-step process to analyze word problems.

Basic Steps for Solving Applications

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Look back over the results.

Example 10: Applications (Numbers, Consecutive Integers)

a. Four times the difference between a number and 6 is equal to twice the number plus 16. What is the number?

Solution: Let $x =$ the unknown number.

$$\underbrace{4(x-6)}_{\text{4 times the difference between a number and 6}} \quad \underbrace{=}_{\text{is equal to}} \quad \underbrace{2x+16}_{\text{twice the number plus 16}}$$

$$4x - 24 = 2x + 16$$
$$2x - 24 = 16$$
$$2x = 40$$
$$x = 20$$

The number is 20.

b. Find three consecutive integers such that twice the first plus three times the second is thirty-three more than the third.

Solution: Represent the three consecutive integers as n, $n+1$, and $n+2$.

$$2n + 3(n+1) = n + 2 + 33$$
$$2n + 3n + 3 = n + 35$$
$$5n + 3 = n + 35$$
$$4n = 32$$
$$n = 8$$
$$n + 1 = 9$$
$$n + 2 = 10$$

The three consecutive integers are 8, 9, and 10.

Section 2.7 Applications with Percent

page 153

There are three basic types of percent problems. The formula

$$R \cdot B = A$$

where $R =$ rate or percent, $B =$ base, and $A =$ amount, represents these problems and can help you decide whether to multiply or divide in a particular problem. (For a general review of percents see Appendix A.1.)

Example 11: Percent of a Number

63% of what number is 52.723? Round you answer to the nearest tenth.

Solution: $R = 0.63$, B is unknown, and A is 52.723.

$$R \cdot B = A$$
$$\downarrow \ \downarrow \quad \downarrow$$
$$0.63 \cdot B = 52.723$$
$$\frac{0.63 \cdot B}{0.63} = \frac{52.723}{0.63}$$
$$B = 83.7 \qquad \text{To the nearest tenth}$$

So 63% of **83.7** is 52.723.

Example 12: Application (Commission)

An electronics salesman earns a salary of \$800 per month plus a commission of 6% of his sales over \$6000. What did he earn the month he sold \$75,000 in electronics equipment?

Solution: First subtract \$6000 from \$75,000 to determine the amount on which the commission is based: $75{,}000 - 6000 = 69{,}000$

Now find the amount of the commission:

$$0.06 \times 69{,}000 = 4140 \qquad \text{Commission}$$

Add the commission to the salary to find how much he earned that month:

$$4140 + 800 = 4940$$

His income was \$4940 that month.

R.2 Exercises

Simplify each expression by combining like terms.

1. $7x + 5x - 14y - y$ **2.** $-3a + 14b + a - 2b$ **3.** $-10 + 5x - x + 6$

4. $15 + 3y + y + 4 - 4y$ **5.** $-4(y+5)+6y$ **6.** $\frac{4(5x-x)}{8} - 2x + 7$

7. $6ab + 7a^3 - a^3 + 4ab - 9a + a$ **8.** $3a^2b - 5ab + 9a^2b + a^2b + 3ab - 4a$

1. $12x - 15y$
2. $-2a + 12b$
3. $4x - 4$ 4. 19
5. $2y - 20$ 6. 7
7. $6a^3 + 10ab - 8a$
8. $13a^2b - 2ab - 4a$

9. a. $-3x^2-2x+18$
b. 10
10. a. $-y^3+5y^2+10y+42$
b. 92
11. a. $3.9y^2+2.4$ **b.** 99.9
12. a. $3.4x^2+2.3x-8.9$
b. 0.1
13. a. $\frac{3}{4}x^2-\frac{3}{2}x+\frac{5}{8}$
b. $\frac{53}{8}$
14. a. $\frac{7}{8}x^3-\frac{1}{3}x+\frac{3}{5}$
b. $-\frac{86}{15}$
15. a. $x+8$ **b.** 6
16. a. $7x+y$ **b.** -9
17. $2x+5$ **18.** $7x-3$
19. $x-6$ **20.** $3x-4$
21. $2(x+5)$
22. $8x+10$
23. $3x-20$
24. $20-3x$
25. $\frac{x}{5}+x$ **26.** $\frac{x}{7}+15$
27. 7 more than 8 times a number
28. 1.5 times a number decreased by 8
29. 5 times the sum of a number and 10
30. 4 times the difference of a number and 13
31. Sum of 6 times a number and 7 times the same number
32. Product of $\frac{1}{2}$ and a number, increased by $\frac{3}{4}$
33. 5 less than the quotient of a number and 14

For the following expressions, ***a.*** *simplify, and* ***b.*** *evaluate the simplified expression for $x=-2$ and $y=5$.*

9. $5x^2-8x^2+15-2x+3$

10. $-y^3+5y^2+13y-3y+42$

11. $1.4y^2+2.5y^2-3.2+5.6$

12. $1.6x^2+2.3x-7.5+1.8x^2-1.4$

13. $\frac{3}{4}x^2-\frac{1}{2}x-x+\frac{5}{8}$

14. $\frac{7}{8}x^3+\frac{1}{6}x-\frac{1}{2}x+\frac{3}{5}$

15. $3(x+2)-2(x-1)$

16. $4(x+y)+3(x-y)$

Translate each English phrase into an equivalent algebraic expression.

17. 5 more than twice a number
18. 3 less than 7 times a number
19. 6 less than a number
20. 3 times a number decreased by 4
21. Twice the sum of a number and 5
22. The product of a number and 8 increased by 10
23. 20 less than three times a number
24. 20 less three times a number
25. The quotient of a number and 5, plus the number
26. 15 more than the quotient of a number and 7

Translate each algebraic expression into an equivalent English phrase. (There may be more than one correct translation.)

27. $7+8x$ **28.** $1.5x-8$ **29.** $5(n+10)$ **30.** $4(n-13)$

31. $6a+7a$ **32.** $\frac{1}{2}x+\frac{3}{4}$ **33.** $\frac{y}{14}-5$ **34.** $3-\frac{16}{n}$

35. $-9x+11$ **36.** $-2(x+20)$

Solve each of the following linear equations.

37. $x+6=-9$ **38.** $x-10=-4$ **39.** $2x=15$

40. $-3x=-18$ **41.** $4x+3=-5$ **42.** $17=5x-18$

43. $-3=3y-20$ **44.** $14=-2y+11$ **45.** $-16=4-(6-x)$

46. $5-(x+3)=21$ **47.** $2x=-5x+7$ **48.** $-x+4+2x=7+2$

34. 3 decreased by the quotient of 16 and a number
35. 11 more than the product of –9 and a number
36. –2 times the sum of a number and 20
37. $x = -15$
38. $x = 6$ **39.** $x = \frac{15}{2}$
40. $x = 6$ **41.** $x = -2$
42. $x = 7$ **43.** $y = \frac{17}{3}$
44. $y = -\frac{3}{2}$
45. $x = -14$
46. $x = -19$
47. $x = 1$ **48.** $x = 5$
49. $y = -4$
50. $n = \frac{1}{2}$
51. $y = 0$ **52.** $y = \frac{15}{2}$
53. $x = -4$
54. $x = -3.725$
55. $a = 18.5$
56. $a = 1.5$
57. 12 **58.** 112.5
59. 72 **60.** 12.5%
61. 150 **62.** 76
63. 62.5%
64. 82.56
65. \$527.72
66. **b.** is better. 8% is better than 7%, so a profit of \$1200 on an investment of \$15,000 is better
67. 19, 21, 23
68. 31, 32, 33, 34
69. –7
70. 8, 11, 15
71. Width = 30 meters; Length = 75 meters

49. $2y - 6 = 5y + 2 - y$

50. $n + 8 - 2n + 10 = 7n + 14$

51. $\frac{2}{3}y + \frac{1}{2}y = \frac{3}{4}y$

52. $1 + \frac{1}{5}y = \frac{2}{15}y + \frac{3}{2}$

53. $4.6 + 0.6x = 1.4 - 0.2x$

54. $-2(5x + 1.2) - 0.5 = -6(x - 2)$

55. $2 + 3(4 - a) = 1.5 - 2(a + 3)$

56. $3(a + 0.4) - 0.2a = 1.8a + 2.7$

Find the missing rate, base, or amount.

57. 15% of 80 is _____.

58. 125% of 90 is _____.

59. What is 72% of 100?

60. What percent of 180 is 22.5?

61. 86% of what number is 129?

62. 92% of what number is 69.92?

63. What percent of 64 is 40?

64. Find 24% of 344.

65. Computers: Computers are on sale at a 35% discount. Sales tax is at 8.25%. What total amount will you pay for a computer that is originally priced at \$750?

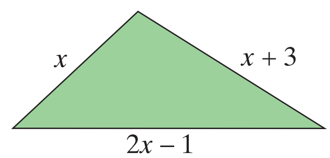
66. Investing: Which is the better investment: **a.** a profit of \$700 on an investment of \$10,000 or **b.** a profit of \$1200 on an investment of \$15,000? Explain in terms of percent.

67. Find three consecutive odd integers such that the sum of the first and twice the second is equal to 15 more than twice the third.

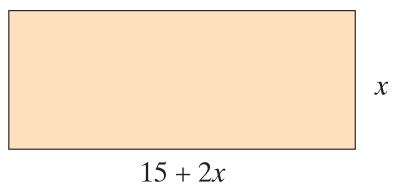
68. Find four consecutive integers such that the sum of the first, second, and fourth is 64 more than the third.

69. If twice a certain number is decreased by 25, the result is 18 less than three times the number. What is the number?

70. Triangles: The triangle shown here indicates that the sides can be represented by x, $x + 3$, and $2x - 1$. What is the length of each side if the perimeter is 34 feet?



71. Rectangles: The perimeter of a rectangle is 210 meters. If the length is 15 more than twice the width, what are the dimensions of the rectangle?



R.3 Chapter 3 Review: Formulas and Linear Inequalities

Section 3.1 page 178 Working with Formulas

Formulas are general rules or principles stated mathematically, generally as equations with two or more variables. A list of eight formulas and their meanings is given on pages 178-179 in Section 3.1.

If values are known for all but one of the variables in a formula, you can substitute those values in the formula and solve the resulting equation for the unknown variable. Example 1 illustrates how this is done.

Example 1: Evaluating a Formula

Renee borrowed \$4000 from her sister for 6 months and agreed to pay her back at a 5% rate of interest at the end of the 6 months. How much will she pay her sister?

Solution: Here, $P = \$4000$,

$r = 5\% = 0.05$, and

$t = 6 \text{ months} = \frac{1}{2} \text{ year}$

Substitute these values in the formula $I = Prt$ and evaluate.

$$I = 4000 \cdot 0.05 \cdot \frac{1}{2} = \$100$$

The interest is \$100 and the amount to be paid at the end of 6 months is

$$\text{principal} + \text{interest} = \$4000 + \$100 = \$4100.$$

Many times a formula is easier to use if the form is changed. To "solve" a formula for a variable, other than the one given, we treat the remaining variables as if they were constants and solve the equation accordingly.

Example 2: Solving for Different Variables

a. Given $y = mx + b$, solve for m.

Solution:

$$y = mx + b$$
$$y - b = mx + b - b$$
$$\frac{y-b}{x} = \frac{mx}{x}$$
$$\frac{y-b}{x} = m \quad \text{or} \quad m = \frac{y-b}{x}$$

b. Given $v = v_0 - 32t$, solve for t.

Solution:

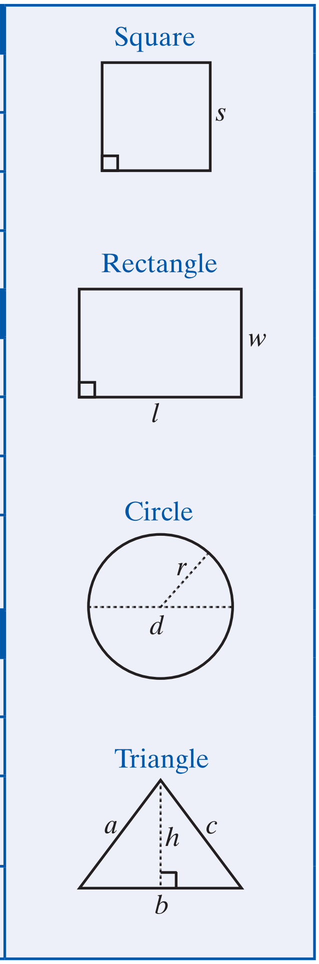
$$v = v_0 - 32t$$
$$v - v_0 = v_0 - 32t - v_0$$
$$\frac{v - v_0}{-32} = \frac{-32t}{-32}$$
$$\frac{v_0 - v}{32} = t \quad \text{or} \quad t = \frac{v_0 - v}{32}$$

Section 3.2
page 189

Formulas in Geometry

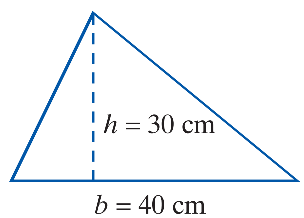
The formulas used in geometry discussed in this text deal with perimeter, area, and volume. For formulas involving π we use the rounded value $\pi = 3.14$. Some of the common geometric formulas are listed here for easy reference. Other formulas may be found in Section 3.2.

Perimeter or Circumference	
$P = 4s$	Perimeter of a square
$P = 2l + 2w$	Perimeter of a rectangle
$C = 2\pi r$ or $C = \pi d$	Circumference of a circle
$P = a + b + c$	Perimeter of a triangle
Area	
$A = s^2$	Area of a square
$A = lw$	Area of a rectangle
$A = \pi r^2$	Area of a circle
$A = \dfrac{1}{2}bh$	Area of a triangle
Volume	
$V = lwh$	Volume of a rectangular solid
$V = \pi r^2 h$	Volume of a right circular cylinder
$V = \dfrac{4}{3}\pi r^3$	Volume of a sphere
$V = \dfrac{1}{3}\pi r^2 h$	Volume of a right circular cone



Example 3: Using Geometric Formulas

a. Find the area of a triangle with a height of 30 centimeters and a base of 40 centimeters.



Solution: Sketch the figure as an aid.

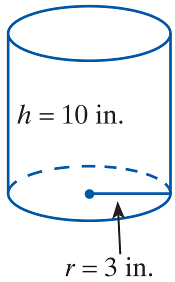
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 30 \cdot 40 = 600$$

The area is 600 cm^2.

Continued on the next page...

b. Find the volume of a right circular cylinder with a radius of 3 inches and a height of 10 inches.



Solution: Sketch the figure as an aid.

$$V = \pi r^2 h$$

$$V = 3.14 \cdot 3^2 \cdot 10 = 282.6$$

The volume is approximately 282.6 in.^3

Section 3.3 Applications: Distance-Rate-Time, Interest, Average

page 203

Applications in this section relate to numbers, distance-rate-time ($d = rt$), selling price and cost ($S - C = P$), interest ($I = Prt$), and average (or mean).

Example 4: Number Application

The difference between two numbers is 12. If twice the smaller number is equal to $\frac{1}{2}$ of the larger number, what are the two numbers?

Solution: **Analyze the problem and assign variables to the unknown quantities.**

Let $x =$ larger number

$x - 12 =$ smaller number

Set up and solve the related equation.

$$2(x-12) = \frac{1}{2}x$$

$$2 \cdot 2(x-12) = 2 \cdot \frac{1}{2}x \qquad \text{Multiply both sides by 2.}$$

$$4(x-12) = x \qquad \text{Simplify.}$$

$$4x - 48 = x \qquad \text{Use the distributive property.}$$

$$3x = 48 \qquad \text{Add } -x \text{ and } +48 \text{ to both sides.}$$

$$x = 16 \qquad \text{Divide both sides by 3.}$$

$$x - 12 = 4$$

The larger number is 16 and the smaller number is 4.

Example 5: Cost

A farmer grows raspberries. They cost him \$1.60 a basket to produce. He is able to sell only 86% of those he produces. If he sells his raspberries at \$3.50 a basket, how many did he produce if he made a profit of \$9165? How many did he actually sell?

Solution: Let $x =$ amount produced

$0.86x =$ amount actually sold

Using the formula $S - C = P$, we have

$$\begin{aligned} 3.50(0.86x) - 1.60x &= 9165 \\ 3.01x - 1.60x &= 9165 \\ 1.41x &= 9165 \\ x &= 6500 \\ 0.86x &= 5590 \end{aligned}$$

He produced 6500 baskets of raspberries and sold 5590 baskets.

Example 6: Interest

Jennifer invested \$20,000, part at 6.5% and part at 5%. In one year, the interest from the 6.5% investment was \$380 more than the interest from the 5% investment. How much did she invest at each rate?

Solution: Let x = amount invested at 6.5% = 0.065
$20{,}000 - x$ = amount invested at 5% = 0.05
The information leads to the following equation.

$$\begin{aligned} 0.065x - 0.05(20{,}000 - x) &= 380 \\ 0.065x - 1000 + 0.05x &= 380 \\ 0.115x &= 1380 \\ x &= 12{,}000 \\ 20{,}000 - x &= 8000 \end{aligned}$$

She invested \$12,000 at 6.5% and \$8000 at 5%.

Section 3.4 Linear Inequalities

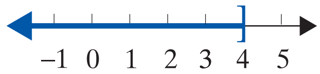
page 214

A **set** is a collection of objects or numbers. The items in the set are called **elements** of the set. The **union** of two (or more) sets consists of those elements belonging to either set or to both sets. The **intersection** of two (or more) sets consists of those elements belonging to both sets. A union is indicated with the symbol $\cup$ and the word **or**. An intersection is indicated with the symbol $\cap$ and the word **and**. The notation $\{x \mid \ ... \ \}$ is read "the set of all x such that … " and is called **set-builder notation**. A statement following the vertical bar $(\,|\,)$ gives a condition (or restriction) on the variable x.

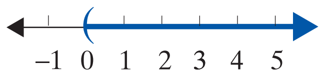
Example 7: Sets of Real Numbers Illustrating Intersection

Graph the set $\{x \mid x \leq 4 \text{ and } x > 0\}$. The word **and** implies those values that satisfy **both** inequalities. The solution graph shows the intersection $\cap$ of the first two graphs.

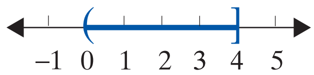
Solution: $x \leq 4$



$x > 0$



$x \leq 4$ **and** $x > 0$



Note that the third graph shows the points in common between the first two graphs in this example.

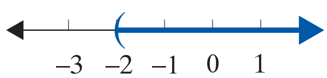
This set can also be indicated as $\{x \mid 0 < x \leq 4\}$.

If a and b are two real numbers and $a < b$, then the set of all real numbers between a and b is called an **interval** of real numbers. Table 1 on page 216 of Section 3.4 contains a list of various **types of intervals** and the corresponding **interval notation**.

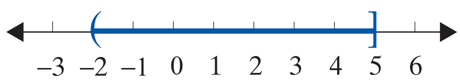
Example 8: Intervals

a. Graph the open interval $(-2, \infty)$.

Solution:



b. Represent the following graph using interval notation and state what type of interval it is.



Solution: $(-2, 5]$ is a half open interval.

Just as an equation of the form $ax + b = c$ is called a **linear equation**, an inequality of the form $ax + b < c$ is called a **linear inequality**. (**Note:** A linear inequality may have the symbol $\leq$, $>$, or $\geq$ in place of $<$.) Also, a **compound inequality** has three parts and is used to indicate a variable or a variable expression is between two numbers. The following rules are used to solve linear inequalities.

Rules for Solving Linear Inequalities

1. Simplify each side of the inequality by removing any grouping symbols and combining like terms.
2. Use the addition property of equality to add the opposites of constants or variable expressions so that variable expressions are on one side of the inequality and constants are on the other.
3. Use the multiplication property of equality to multiply both sides by the reciprocal of the coefficient of the variable (that is, divide both sides by the coefficient) so that the new coefficient is 1. **If this coefficient is negative, reverse the sense of the inequality.**
4. A quick (and generally satisfactory) check is to select any one number in your solution and substitute it into the original inequality.

Example 9: Solving Linear Inequalities

Solve the following inequalities and graph the solution sets. Write each solution set using interval notation. Assume that x is a real number.

a. $4x+5 \le 5x-1$

Solution:

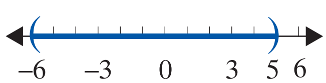
$$4x+5 \le 5x-1$$
$$4x \le 5x-6$$
$$-x \le -6$$
$$x \ge 6$$

The solution set is the half-open interval $[6, \infty)$.

b. $-9 < 2x+3 < 13$

Solution:

$$-9 < 2x+3 < 13$$
$$-9-3 < 2x+3-3 < 13-3$$
$$-12 < 2x < 10$$
$$\frac{-12}{2} < \frac{2x}{2} < \frac{10}{2}$$
$$-6 < x < 5$$



The solution set is the open interval $(-6, 5)$.

Section 3.5 Absolute Value Equations and Inequalities

page 229

Solving Absolute Value Equations

For $c > 0$:

a. If $|x| = c$, then $x = c$ or $x = -c$.

b. If $|ax + b| = c$, then $ax + b = c$ or $ax + b = -c$.

Note: If the absolute value expression is isolated on one side of the equation, we say that the **equation** is in **standard form**. You may need to manipulate the absolute value equation to get it into standard form before you can solve it.

Example 10: Solving Absolute Value Equations

a. Solve $|3x - 7| = 8$

Solution: $|3x - 7| = 8$

$$3x - 7 = 8 \quad \text{or} \quad 3x - 7 = -8$$
$$3x = 15 \qquad\qquad 3x = -1$$
$$x = 5 \qquad\qquad x = -\frac{1}{3}$$

b. Solve $4|2x + 1| - 3 = 25$

Solution: $4|2x + 1| - 3 = 25$

$$4|2x + 1| = 28$$
$$|2x + 1| = 7$$
$$2x + 1 = 7 \quad \text{or} \quad 2x + 1 = -7$$
$$2x = 6 \qquad\qquad 2x = -8$$
$$x = 3 \qquad\qquad x = -4$$

If two numbers have the same absolute value, then either they are equal or they are opposites of each other.

Solving Equations with Two Absolute Value Expressions

If $|a| = |b|$, then either $a = b$ or $a = -b$.

More generally,

if $|ax + b| = |cx + d|$, then either $ax + b = cx + d$ or $ax + b = -(cx + d)$.

Example 11: Solving Equations with Two Absolute Value Expressions

Solve $|3x-1| = |x+9|$

Solution: $|3x-1| = |x+9|$

$$\begin{aligned} 3x-1 &= x+9 \quad \text{or} \quad & 3x-1 &= -(x+9) \\ 2x &= 10 & 3x-1 &= -x-9 \\ x &= 5 & 4x &= -8 \\ & & x &= -2 \end{aligned}$$

We can use the following techniques for solving **absolute value inequalities**.

Solving Absolute Value Inequalities with < (or ≤)

For $c > 0$:

a. If $|x| < c$, then $-c < x < c$.

b. If $|ax+b| < c$, then $-c < ax+b < c$.

The inequalities in **a.** and **b.** are also true if $<$ is replaced by $\leq$.

Note: If the absolute value expression is isolated on one side of the inequality, we say that the inequality is in **standard form**. You may need to manipulate the absolute value inequality to get it into standard form before you can solve it.

Example 12: Solving Absolute Value Inequalities

Solve the following absolute value inequality and graph the solution set.

$$|3x-4| \leq 8$$

Solution: $|3x-4| \leq 8$

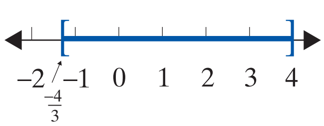
$$-8 \leq 3x-4 \leq 8$$

$$-8+4 \leq 3x-4+4 \leq 8+4$$

$$-4 \leq 3x \leq 12$$

$$\frac{-4}{3} \leq \frac{3x}{3} \leq \frac{12}{3}$$

$$-\frac{4}{3} \leq x \leq 4$$

So x is in $\left[-\frac{4}{3}, 4\right]$. 

Solving Absolute Value Inequalities with $>$ (or $\geq$)

For $c > 0$:

a. If $|x| > c$, then $x < -c$ **or** $x > c$.

b. If $|ax + b| > c$, then $ax + b < -c$ **or** $ax + b > c$.

Note: The inequalities in **a.** and **b.** are true if $>$ is replaced by $\geq$.

Example 13: Solving Absolute Value Inequalities

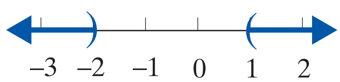
Solve the following absolute value inequality and graph the solution set.

$$|2x+1| > 3$$

Solution: $|2x+1| > 3$

$$\begin{aligned} 2x+1 < -3 \quad &\text{or} \quad 2x+1 > 3 \\ 2x < -4 \quad &\text{or} \quad 2x > 2 \\ x < -2 \quad &\text{or} \quad x > 1 \end{aligned}$$

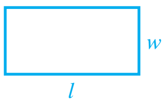
So x is in $(-\infty, -2) \cup (1, \infty)$.



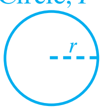
Answers will vary for Exercises 1-3.

1. Square, $P = 4a$

Rectangle, $P = 2l + 2w$

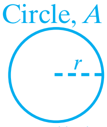


Circle, $P = 2\pi r$

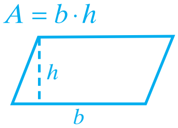


2. Rectangle, $A = l \cdot w$

Circle, $A = \pi r^2$



Parallelogram, $A = b \cdot h$



Exercises R.3

1. Write the formulas used to find the perimeter of three different geometric figures and sketch a diagram of each figure.

2. Write the formulas used to find the area of three different geometric figures and sketch a diagram of each figure.

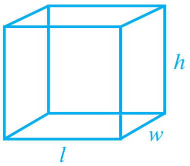
3. Write the formulas used to find the volume of three different geometric figures and sketch a diagram of each figure.

4. Write the formula for finding simple interest and explain the meaning of each variable in the formula.

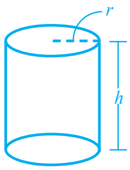
Evaluate the formula using the given information. (Use $\pi = 3.14$.)

5. $C = \frac{5}{9}(F - 32)$: Find C for $F = 158$.

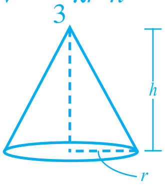
3. Rectangular solid, $V = lwh$



Right circular cylinder, $V = \pi r^2 h$



Right circular cone, $V = \frac{1}{3}\pi r^2 h$



4. $I = Prt$, where I = total simple interest, P = principal, r = rate of interest, and t = time (one year or part of a year)

5. 70 **6.** 2,592,000
7. 38.465 **8.** 2971.696
9. 5200 **10.** 18

11. $r = \frac{d}{t}$

12. $c = P - a - b$

13. $\beta = 180 - \alpha - \gamma$

14. $w = \frac{V}{lh}$

15. $m = Mn$

16. $h = \frac{2A}{b+c}$

17. $y = 3x - 14$

18. $y = \frac{20 - x}{3}$

19. $n = \frac{L-a}{d} + 1$ or $n = \frac{L-a+d}{d}$

20. $K - 273 = C$

6. $F = \frac{4}{3}Av^2$: Find F for $A = 240$ and $v = 90$.

7. $A = \pi r^2$: Find A for $r = 3.5$.

8. $V = \pi r^2 h$: Find V for $r = 13$ and $h = 5.6$.

9. $A = P + Prt$: Find A for $P = 5000$, $r = 0.08$, and $t = 0.5$.

10. $S = \frac{a}{1-r}$: Find S for $a = 12$ and $r = \frac{1}{3}$.

Solve each formula for the indicated variable.

11. $d = rt$; solve for r.

12. $P = a + b + c$; solve for c.

13. $\alpha + \beta + \gamma = 180$; solve for β.

14. $V = lwh$; solve for w.

15. $n = \frac{m}{M}$; solve for m.

16. $A = \frac{1}{2}h(b+c)$; solve for h.

17. $3x - y = 14$; solve for y.

18. $x + 3y = 20$; solve for y.

19. $L = a + (n-1)d$; solve for n.

20. $K = C + 273$; solve for C.

Solve the following problems. (Use $\pi = 3.14$.)

21. Circles: The radius of a circle is 25 cm. Find the area.

22. Circular cylinders: Find the volume of a circular cylinder with radius 10 in. and height 15 in.

23. Cones: Find the volume of a cone that has radius 10 in. and height 15 in.

24. Rectangular solids: If the volume of a rectangular solid is 175 cm^3 and its base is a square 5 cm on each side, what is the height of the solid?

25. Spheres: Find the volume of a sphere that has a radius of 3 feet.

26. Circles: What is the area between two circles with the same center if the radius of one circle is 5 m and the radius of the other is 3 m?

27. Circles: The circumference of a circle is 94.2 in. Find the radius and the area of the circle.

28. Rectangles: The perimeter of a rectangle is 60 cm. If the length of the rectangle is 3 times the width, what are the length and width of the rectangle? Find the area.

29. Trapezoids: The area of a trapezoid is 63 square inches. One base is 8 in. long and the other is 10 in. long. Find the height of the trapezoid.

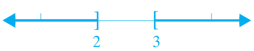
30. Rectangles: The perimeter of a rectangle is 188 feet. If the length is 50 feet, find the area of the rectangle.

31. Triangles: An isosceles triangle has a base of 15 m. If the perimeter is 125 m, what is the length of each of the two equal sides?

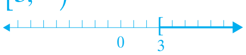
21. 1962.5 cm^2
22. 4710 in.^3
23. 1570 in.^3
24. 7 cm
25. 113.04 ft^3
26. 50.24 m^2
27. Radius = 15 in.;
Area = 706.5 in.^2
28. Width = 7.5 cm;
Length = 22.5 cm;
Area = 168.75 cm^2
29. 7 in.
30. 2200 ft^2
31. 55 m
32. $P = 56$ cm;
$A = 192 \text{ cm}^2$
33. $P = 73.12$ cm;
$A = 356.48 \text{ cm}^2$
34. 24, 3 **35.** 18, 6
36. $30,000 at 3.5% and $20,000 at 10%
37. $2522
38. 48.93 miles per hour
39. 99
40. a. 64 feet per second, the object is going up since the velocity is positive.
b. –160 feet per second, the object is falling down since the velocity is negative.
41. $15,200
42. 

43. 

44. 

45. 

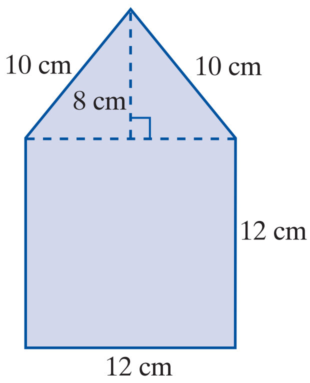
46. $(-\infty, 2]$


Half-open interval
47. $[3, \infty)$


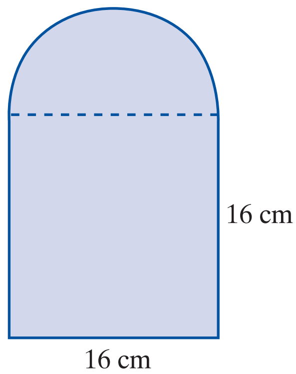
Half-open interval

Find the perimeter and area for each figure. (Use $\pi = 3.14$.)

32. $P =$ ________ $A =$ ________



33. $P =$ ________ $A =$ ________



34. The difference between two numbers is 21. Their sum is 27. What are the two numbers?

35. The larger of two numbers is three times the smaller. If 12 is subtracted from twice their sum, the result is 36. Find the two numbers.

36. Investing: Maria has had $50,000 invested for one year, some with a CD at 3.5% and the rest in the stock market which yielded 10% for the year. If her interest income last year was $3050, how much did she have in the CD and how much in the stock?

37. Jewelry: A jeweler paid $1455 for a watch. He wants to price the watch for sale so that he can give the customer a 25% discount on the selling price and still make a profit of 30% on his cost. What selling price should he mark for the watch?

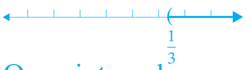
38. Traffic: The following speeds (in miles per hour) were clocked for 15 cars at a particularly dangerous intersection. Find the average speed of these cars (to the nearest hundredth).

45	55	42	50	62
50	56	48	60	36
58	40	35	52	45

39. Exam scores: Suppose that you have scores of 86, 93, 84, and 88 on four exams in your algebra class. What score will you need on the fifth exam to have an average of 90?

40. Velocity: If an object is shot upward with an initial velocity v_0 feet per second, the velocity v in feet per second is given by the formula $v = v_0 - 32t$. If the initial velocity is 160 feet per second, find **a.** the velocity at the end of 3 seconds and **b.** the velocity at the end of 10 seconds. Explain the meanings of your answers.

48. $\left(\frac{1}{3}, \infty\right)$



Open interval

49. $(-1, \infty)$

Open interval

50. $(-\infty, 0]$

Half-open interval

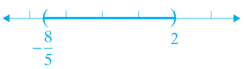
51. $(0, \infty)$

Open interval

52. $\left[-\frac{1}{3}, 3\right]$

Closed interval

53. $\left(-\frac{8}{5}, 2\right)$

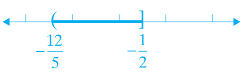


Open interval

54. $[-12, -7]$

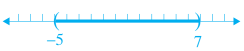
Closed interval

55. $\left(-\frac{12}{5}, -\frac{1}{2}\right]$



Half-open interval

56. $(-5, 7)$



Open interval

41. **Car depreciation:** The value V of an item after t years of "linear" depreciation is given by the formula $V = C - Crt$ where C is the original cost and r is the rate of depreciation expressed as a decimal. If you buy a car for \$38,000 and depreciate it linearly at a rate of 12%, what will be its value after 5 years?

Graph each set of indicated numbers on a real number line.

42. $\{x \mid x > 1 \text{ or } x < -1\}$

43. $\{x \mid x < 3 \text{ and } x \geq 0\}$

44. $\{x \mid x \leq -4 \text{ and } x \geq -6\}$

45. $\{x \mid x \geq 3 \text{ or } x \leq 2\}$

Solve the inequalities. Write each solution using interval notation and graph the solution sets on a number line. Tell what type of interval it is. Assume that all variables represent real numbers.

46. $3x + 5 - 3 \leq 8$

47. $5x - 11 \geq 4$

48. $-2(x+3) < x - 7$

49. $-5(x-5) < 30$

50. $14y + 3.5 \leq 2y + 3.5$

51. $6y + \frac{1}{2} > -2y + \frac{1}{2}$

52. $0 \leq 3x + 1 \leq 10$

53. $-5 < 5 - 5y < 13$

54. $0 \leq \frac{1}{2}x + 6 \leq 2\frac{1}{2}$

55. $-4 < 10x + 20 \leq 15$

56. $-8 < 2(3 - x) < 16$

57. $-2.3 < a + 0.5 \leq 2.4$

Solve each of the absolute value equations.

58. $|x + 3| = 7$

59. $|2x - 1| + 2 = 8$

60. $|3x| = |2x - 1|$

61. $3|5 - x| - 4 = 17$

Solve each of the absolute value inequalities and graph the solution sets. Write each solution using interval notation. Assume that x is a real number.

62. $|x - 5| < 10$

63. $|2x + 3| \leq 4$

64. $|x + 6| > 2$

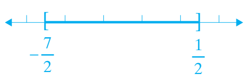
65. $|3x - 1| > 4$

57. $(-2.8, 1.9]$

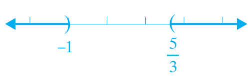
Half-open interval

58. $x = -10, 4$ **59.** $x = -\frac{5}{2}, \frac{7}{2}$ **60.** $x = -1, \frac{1}{5}$ **61.** $x = -2, 12$

62.  $(-5, 15)$

63.  $\left[-\frac{7}{2}, \frac{1}{2}\right]$

64.  $(-\infty, -8) \cup (-4, \infty)$

65.  $(-\infty, -1) \cup \left(\frac{5}{3}, \infty\right)$

R.4 Chapter 4 Review: Linear Equations and Functions

Section 4.1 The Cartesian Coordinate System

page 256

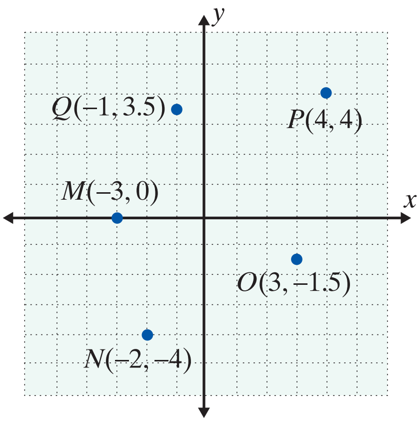
In the **Cartesian coordinate system**, the plane is separated into four **quadrants** by two perpendicular number lines called **axes**. The point of intersection of the two lines is designated by the **ordered pair** $(0, 0)$. The horizontal line is called the ***x*-axis**, the vertical line is called the ***y*-axis**, and ordered pairs of real numbers are represented in the form (x, y). In an ordered pair such as $(3, -5)$, 3 is called the **first coordinate** and –5 is called the **second coordinate**. Also, for an ordered pair (x, y), y is called the **dependent variable** and x is called the **independent variable**. The terms **ordered pair** and **point** are used interchangeably.

Example 1: Graphing Ordered Pairs

Graph the set of ordered pairs $\{M(-3, 0), N(-2, -4), O(3, -1.5), P(4, 4), Q(-1, 3.5)\}$.

Note: The listing of ordered pairs within the braces may be in any order.

Solution:



M is on the x-axis.
N is in quadrant III.
O is in quadrant IV.
P is in quadrant I.
Q is in quadrant II.

Example 2: Determining Ordered Pairs

Determine which, if any, of the ordered pairs $(0, -5)$, $\left(\frac{1}{2}, 4\right)$, and $(-2, -9)$ satisfy the equation $y = 2x - 5$.

Solution: Substitute 0, $\frac{1}{2}$, and –2 for x and see if the corresponding y-values match those in the given ordered pairs.

$x = 0;$ $\quad y = 2(0) - 5 = -5$ $\quad$ so, $(0, -5)$ satisfies the equation.

$x = \frac{1}{2};$ $\quad y = 2\left(\frac{1}{2}\right) - 5 = -4$ $\quad$ so, $\left(\frac{1}{2}, -4\right)$ satisfies the equation.

$x = -2;$ $\quad y = 2(-2) - 5 = -9$ $\quad$ so, $(-2, -9)$ satisfies the equation.

The point $\left(\frac{1}{2}, 4\right)$ does not satisfy the equation $y = 2x - 5$ because, as just illustrated, $y = -4$ when $x = \frac{1}{2}$.

Sections 4.2 page 273 Graphing Linear Equations in Two Variables: *Ax* + *By* = *C*

The **solution set** of an equation in two variables, such as $y = -3x + 6$, consists of ordered pairs that are said to **satisfy** the equation For example, the ordered pair $(2, 0)$ satisfies the equation $y = -3x + 6$ because $0 = -3 \cdot 2 + 6$ is a true statement. To find some of the solutions of the equation $y = -3x + 6$, we form a table by:

1. choosing arbitrary values for x, and
2. finding the corresponding values for y by substituting into the equation.

Figure 1 shows five points that satisfy the equation $y = -3x + 6$.

Choices	Substitution	Results
x	$-3x + 6 = y$	(x, y)
0	$-3(0) + 6 = y$	$(0, 6)$
1	$-3(1) + 6 = y$	$(1, 3)$
2	$-3(2) + 6 = y$	$(2, 0)$
3	$-3(3) + 6 = y$	$(3, -3)$
4	$-3(4) + 6 = y$	$(4, -6)$

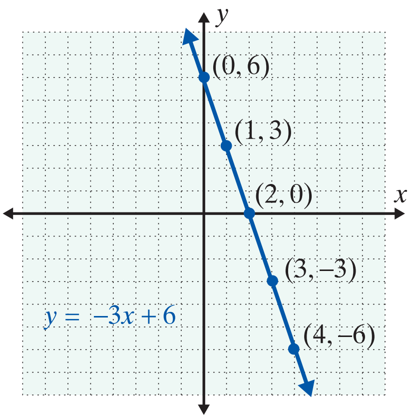


Figure 1

The five points in Figure 1 lie on a line and so will any ordered pair that satisfies the equation $y = -3x + 6$.

Just as the terms **ordered pair** and **point** are used interchangeably, the terms **equation** and **graph of an equation** are interchangeable. The equations

$$4x - y = 8, \qquad y = -7, \qquad x = 3.4, \qquad \text{and} \qquad y = -x + 5$$

are called **linear equations**, and their graphs are lines on the Cartesian plane.

Standard Form of a Linear Equation

Any equation of the form

$$\boldsymbol{Ax + By = C},$$

where A, B, and C are real numbers and A and B are not both equal to 0, is called the **standard form** of a **linear equation**.

From geometry we know that **two points determine a line**. Thus the graph of a linear equation can be found by locating any two points that satisfy the equation.

To Graph a Linear Equation in Two Variables

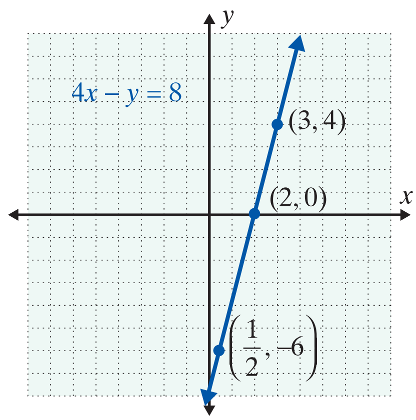
1. Locate any two points that satisfy the equation. (Choose values for x and y that lead to simple solutions. Remember that there are an infinite number of choices for either x or y. But, once a value for x or y is chosen, the corresponding value for the other variable is found by substituting into the equation.)
2. Plot these two points on a Cartesian coordinate system.
3. Draw a line through these two points. (**Note:** Every point on that line will satisfy the equation.)
4. **To check:** Locate a third point that satisfies the equation and check to see that it does indeed lie on the line.

Example 3: Graphing a Linear Equations in Two Variables

Graph the linear equation $4x - y = 8$.

Solution: Make a table with headings x and y and, whenever possible, choose values for x or y that lead to simple solutions for the other variable. In this example, we have found three points that satisfy the equation.

x	$4x - y = 8$	y
2	$4(2) - y = 8$	0
$\frac{1}{2}$	$4\left(\frac{1}{2}\right) - y = 8$	-6
3	$4x - (4) = 8$	4



The point where a line crosses the y-axis, if it exists, can be found by letting $x = 0$. This point is called the **y-intercept** and is of the form $(0, y)$. The **x-intercept**, the point where the line crosses the x-axis, is found by letting $y = 0$ and is of the form $(x, 0)$. These points, if they exist, are generally easy to locate and can be used as the two points for drawing the graph of a linear equation.

Example 4: x- and y-Intercepts

Graph the following linear equation by locating the y-intercept and the x-intercept:

$$x + 2y = -6$$

Solution: $x = 0 \longrightarrow (0) + 2y = -6$

$$2y = -6$$

$$y = -3$$

$(0, -3)$ is the y-intercept.

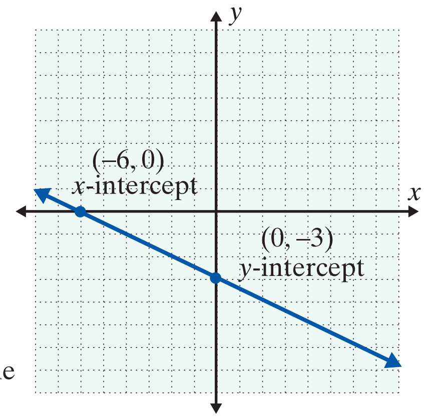
$y = 0 \longrightarrow x + 2(0) = -6$

$$x + 0 = -6$$

$$x = -6$$

$(-6, 0)$ is the x-intercept.

Plot the two integers and draw the line that contains them.



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The Slope-Intercept Form: $y = mx + b$

Slope is a measure of steepness. For a line the slope is the ratio of *rise* (vertical distance) over *run* (horizontal distance). In general, the ratio of a change in one variable (say y) to a change in another variable (say x) is called the **rate of change of y with respect to x**. For example, rate of change can be in feet per second or dollars per month.

To calculate the slope of a line, we need to find two points on the line and then use the following formula.

Slope

Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be two points on a line. The **slope** can be calculated as follows:

$$\textbf{slope} = m = \frac{\textbf{rise}}{\textbf{run}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Note: The letter m is standard notation for representing the slope of a line.

Example 5: Finding the Slope of a Line

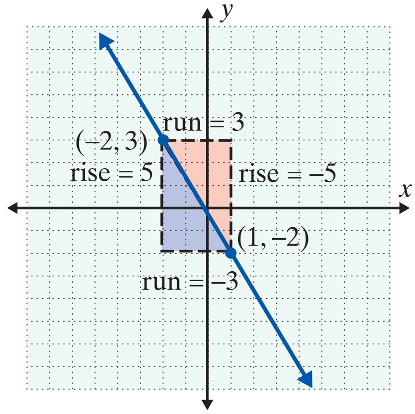
Find the slope of the line that contains the points $(-2, 3)$ and $(1, -2)$, and then graph the line.

Solution: Using $\underset{(x_1,\ y_1)}{(-2,\ 3)}$ and $\underset{(x_2,\ y_2)}{(1,\ -2)}$,

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{1-(-2)} = \frac{-5}{3}$$

Or, using $\underset{(x_1,\ y_1)}{(1,\ -2)}$ and $\underset{(x_2,\ y_2)}{(-2,\ 3)}$,

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{-2-1} = \frac{5}{-3}$$

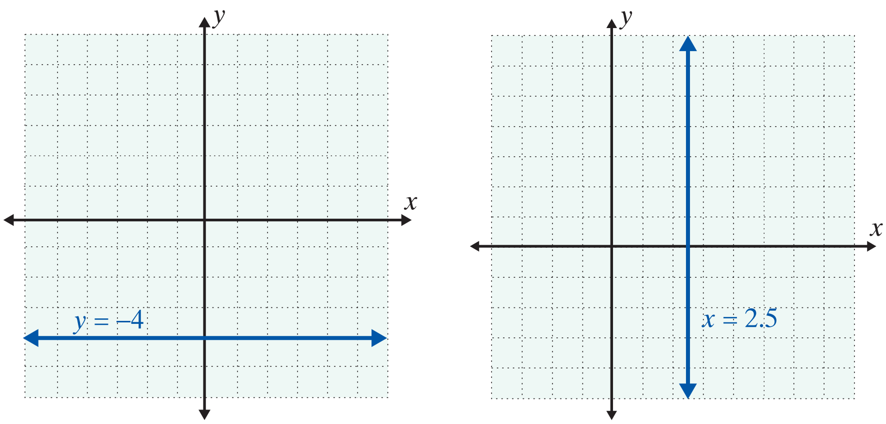


NOTES

Lines with **positive slope go up** as we move along the line from left to right.

Lines with **negative slope go down** as we move along the line from left to right.

For a horizontal line (an equation of the form $y = b$) all y-values will be the same and the **slope is 0**. For a vertical line (an equation of the form $x = a$) all the x-values will be the same and the **slope is undefined**. See Figure 2.



a. Horizontal line: Slope 0 **b. Vertical Line:** Undefined slope

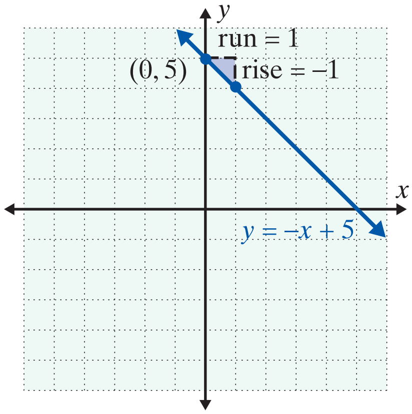
Figure 2

For the line $y = mx + b$, the point $(0, b)$ is the y-intercept and the slope is m. (A complete analysis of these ideas is in Section 4.3.)

Example 6: Using the Form $y = mx + b$

Find the slope and y-intercept of $y = -x + 5$ and graph the line.

Solution: The slope is $m = -1$ and the y-intercept is $(0, 5)$.



A linear equation in the standard form $Ax + By = C$ can be written in the slope-intercept form by solving for y. Example 7 illustrates how this can be done.

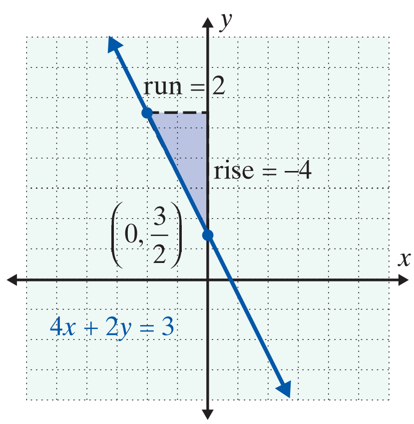
Example 7: Using the Form $y = mx + b$

Find the slope and y-intercept of $4x + 2y = 3$ and graph the line.

Solution: Solve for y.

$$4x + 2y = 3$$
$$2y = -4x + 3$$
$$y = \frac{-4x}{2} + \frac{3}{2}$$
$$y = -2x + \frac{3}{2}$$

Thus $m = -2$, which is the slope, and b is $\frac{3}{2}$ making the y-intercept equal $\left(0, \frac{3}{2}\right)$.



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The Point-Slope Form: $y - y_1 = m(x - x_1)$

We have discussed lines represented by equations in the **standard form** $Ax + By = C$ and in the **slope-intercept form** $y = mx + b$. We have also discussed graphing using the x- and y-intercepts. Now we would like to find the equation of a line if we are given a point (x_1, y_1) on a line and the slope, m. If (x, y) is any other point on the line, then we can apply the slope formula to get the following equation.

$$\frac{y - y_1}{x - x_1} = m$$

Multiplying both sides of this equation by the denominator (assuming the denominator is nonzero) gives

$$y - y_1 = m(x - x_1) \text{ which is called the } \textbf{point-slope form}.$$

Example 8: Using Two Points to Find the Equation of a Line

Find the equation of the line containing the two points $(-4, 1)$ and $(1, 6)$.

Solution: First, find the slope.

$$m = \frac{6 - 1}{1 - (-4)} = \frac{5}{5} = 1$$

Now use one of the given points and the point-slope form for the equation of a line. The point $(1, 6)$ is used in this example.
(You should check to see that the other point gives the same answer.)

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (6) &= 1(x - (1)) \\ y - 6 &= x - 1 \\ y &= x + 5 \end{aligned}$$

Parallel and Perpendicular Lines

Parallel lines are lines that never intersect (never cross each other) and these lines have the **same slope**. (**Note:** All vertical lines (undefined slopes) are parallel to one another.)

Perpendicular lines are lines that intersect at 90º (right) angles and whose slopes are **negative reciprocals** of each other. Horizontal lines are perpendicular to vertical lines.

Example 9: Parallel and Perpendicular Lines

Determine whether each pair of lines are parallel, perpendicular, or neither.

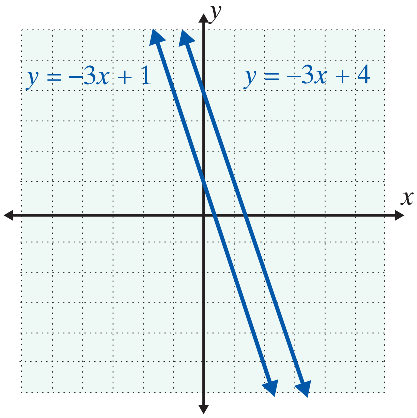
a. $3x + y = 4$

$y = -3x + 1$

Solution: The slope for $y = -3x + 1$ is -3. Solving the first equation for y gives:

$$3x + y = 4$$
$$y = -3x + 4$$

and we see that the slope is also -3. Thus the two lines are parallel.



b. $3x + y = 4$

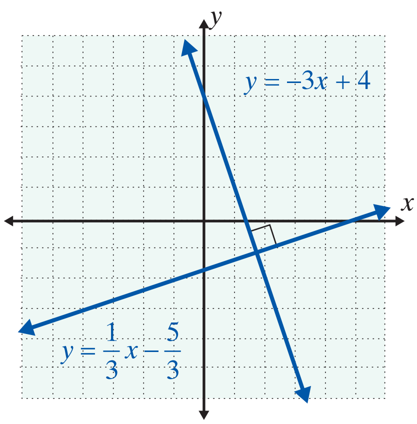
$x - 3y = 5$

Solution: Solving each equation for y gives:

$$3x + y = 4$$
$$y = -3x + 4$$

and

$$x - 3y = 5$$
$$-3y = -x + 5$$
$$y = \frac{1}{3}x - \frac{5}{3}$$



The slopes -3 and $\frac{1}{3}$ are negative reciprocals of each other $\left(-3 \cdot \frac{1}{3} = -1\right)$. Thus the lines are perpendicular.

Section 4.5 Introduction to Functions and Function Notation

page 307

The concept of a function is one of the most important and useful ideas in all of mathematics. For example, distance traveled is a function of time, area of a circle is a function of radius, and taxes are a function of income (taxable income, that is). Every equation of the form $y = mx + b$ represents a function, and we say that

$$y \text{ "is a function of" } x.$$

Relation, Domain, and Range

A **relation** is a set of ordered pairs of real numbers.

The **domain**, ***D***, of a relation is the set of all first coordinates in the relation.

The **range**, ***R***, of a relation is the set of all second coordinates in the relation.

In the graph of a relation, the horizontal axis (the x-axis) is called the **domain axis**, and the vertical axis (the y-axis) is called the **range axis**.

Function

A **function** is a relation in which each domain element has exactly one corresponding range element.

The definition can also be stated in the following ways.

1. A function is a relation in which each first coordinate appears only once.
2. A function is a relation in which no two ordered pairs have the same first coordinate.

Example 10: Functions, Domain and Range

Determine whether the relation $r = \{(-5, 3), (-2.2, 4), (0, 4), (1, 6)\}$ represents a function and state its domain and range.

Solution: r is a function. Each first coordinate appears only once.

$$D = \{-5, -2.2, 0, 1\} \text{ and } R = \{3, 4, 6\}$$

The vertical line test can be used to tell whether or not a graph represents a function.

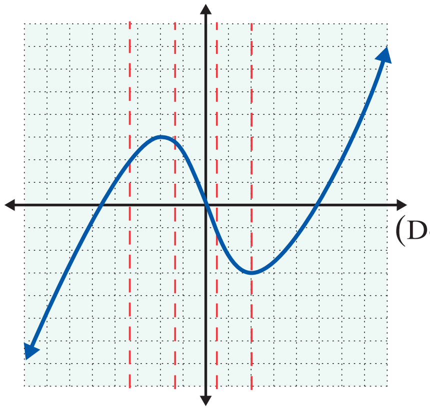
Vertical Line Test

If **any** vertical line intersects the graph of a relation at more than one point, then the relation graphed is **not** a function.

Example 11: The Vertical Line Test

Use the vertical line test to determine whether or not each graph represents a function. Then list the domain and range of each graph.

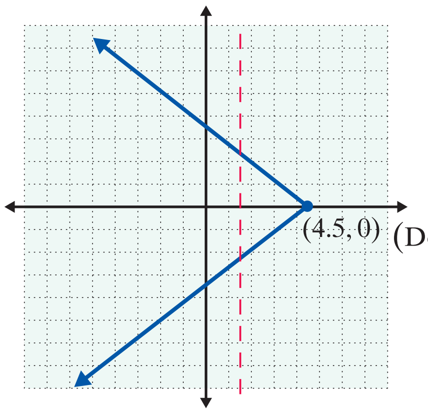
a.



Solution: The relation is a **function**. No vertical line will intersect the graph at more than one point. Several vertical lines are drawn to illustrate this.

For this function, we see from the graph that $D = (-\infty, \infty)$ and $R = (-\infty, \infty)$.

b.



Solution: The relation is **not** a function. At least one vertical line (drawn) intersects the graph at more than one point.

Here $D = (-\infty, 4.5]$ and $R = (-\infty, \infty)$.

All non-vertical lines represent functions. Thus we have the following definition for a linear function.

Linear Function

A **linear function** is a function represented by an equation of the form

$$\boldsymbol{y = mx + b}.$$

The domain of a linear function is the set of all real numbers: $D = (-\infty, \infty)$.

Function notation is convenient for indicating calculations with functions. In function notation,

instead of writing $\boldsymbol{y}$, write $\boldsymbol{f(x)}$, read "$\boldsymbol{f}$ of $\boldsymbol{x}$."

The letter f is the name of the function. Other letters commonly used to name functions are g, h, F, G, and H.

The linear equation $y = -5x + 6$ represents a linear function and we can replace y with $f(x)$ as follows:

$$f(x) = -5x + 6$$

In function notation, $f(3)$ means to replace x with 3 in the function.

$$f(3) = -5(3) + 6 = -15 + 6 = -9$$

Thus the ordered pair $(3, -9)$ can be written as $(3, f(3))$.

Not all functions are linear functions, just as not all graphs are lines. Example 3 illustrates function notation for two types of functions.

Example 12: Function Evaluation

a. For the function $g(x) = 5x - 10$, find $g(-2)$.

Solution: $g(-2) = 5(-2) - 10 = -10 - 10 = -20$

b. For the function $h(x) = x^2 - 4x - 15$, find $h(-3)$.

Solution: $h(-3) = (-3)^2 - 4(-3) - 15 = 9 + 12 - 15 = 6$

The point where a graph intersects the x-axis (where $y = 0$) is called a **zero of the function**. See Section 4.5 for detailed instructions on using a TI-84 Plus graphing calculator to graph functions and find their zeros.

Section 4.6 Graphing Linear Inequalities in Two Variables

page 326

A straight line separates a plane into two **half-planes**. The points on one side of the line are in one of the half-planes, and the points on the other side of the line are in the other half-plane. The line itself is called the **boundary line**. The half-plane is **closed** if the boundary line is included in the solution set, otherwise it is **open**.

Graphing Linear Inequalities

1. First, graph the boundary line (dashed if the inequality is < or >, solid if the inequality is ≤ or ≥).
2. Next, determine which side of the line to shade using one of the following methods.

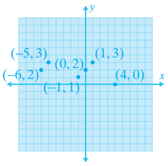
Method 1

a. Test any one point obviously on one side of the line.
b. If the test-point satisfies the inequality, shade the half-plane on that side of the line. Otherwise, shade the other half-plane.
(**Note:** The point $(0, 0)$, if it is not on the boundary line, is usually the easiest point to test.)

Method 2

a. Solve the inequality for y (assuming that the line is not vertical).
b. If the solution shows $y <$ or $y \le$, then shade the half-plane below the line.
c. If the solution shows $y >$ or $y \ge$, then shade the half-plane above the line.
Note: If the boundary line is vertical, then solve for x. If the solution shows $x >$ or $x \ge$, then shade the half-plane to the right. If the solution shows $x <$ or $x \le$, then shade the half-plane to the left.

3. The shaded half-plane (and the line if it is solid) is the solution to the inequality.

1.


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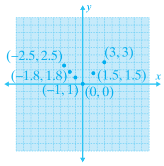

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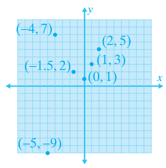

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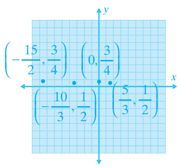


Example 13: Graphing a Linear Inequality

Graph the half-plane that satisfies the inequality $y - 2x \ge 1$.

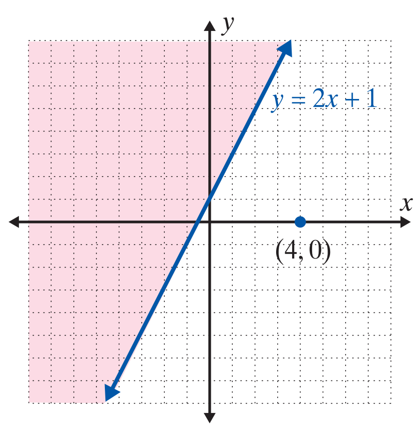
Solution: Solving the inequality for y gives $y \ge 2x + 1$.

Now Method 2 can be applied.

Step 1: Graph the line $y = 2x + 1$ as a solid line.

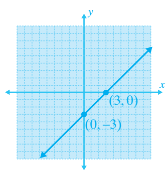
Step 2: By Method 2, the graph consists of those points above the line as well as the line itself. Shade the half-plane above the line.

Note: As a check, we see that the point $(4, 0)$ gives $(0) - 2(4) \ge 1$, a false statement.



5. $(0, -12), (4, 0),$ $(6, 6), (2, -6)$

6. $(-6, 4), (-2, 3),$ $(10, 0), (-2, 3)$

7.


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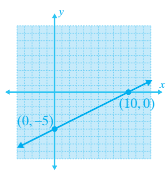


R.4 Exercises

Graph the sets of ordered pairs and label the points.

1. $\{(-6,2),(-5,3),(-1,1),(0,2),(1,3),(4,0)\}$

2. $\{(-2.5,2.5),(-1.8,1.8),(-1,1),(0,0),(1.5,1.5),(3,3)\}$

3. $\{(-5,-9),(-4,7),(-1.5,2),(0,1),(1,3),(2,5)\}$

4. $\left\{\left(-\frac{15}{2},\frac{3}{4}\right),\left(-\frac{10}{3},\frac{1}{2}\right),\left(0,\frac{3}{4}\right),\left(\frac{5}{3},\frac{1}{2}\right)\right\}$

Determine the missing coordinate in each of the ordered pairs so that the point will satisfy the equation given.

5. $3x - y = 12$
   $(0,\ ),(4,\ ),(\ ,6),(\ ,-6)$

6. $x + 4y = 10$
   $(-6,\ ),(-2,\ ),(\ ,0),(\ ,3)$

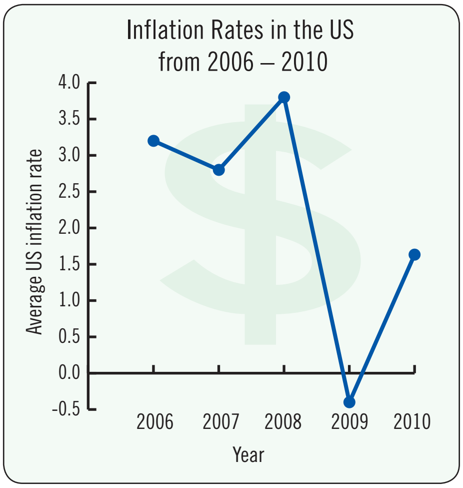
Graph the following linear equations by locating the x-intercept and the y-intercept.

7. $x - y = 3$
8. $x - 2y = 10$
9. $2x + 3y = -6$
10. $x + 4y = 2$

11. The line graph given below gives the average inflation rates in the US from 2006–2010.
    a. Find the slope of each line segment
    b. Interpret each slope as a rate of change.

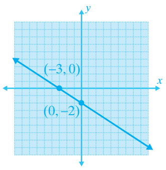
Year	Average US inflation rate
2006	3.2%
2007	2.8%
2008	3.8%
2009	–0.4%
2010	1.6%

Source: http://www.bls.gov/cpi/



Find the slope of the line determined by each pair of points.

12. $(-6,5)$ and $(0,1)$
13. $(-1,2)$ and $(5,4)$
14. $(-1.5,3)$ and $(2.5,3)$
15. $\left(-\frac{3}{4},\frac{1}{8}\right)$ and $\left(\frac{1}{2},\frac{5}{16}\right)$
16. $(5.2,-3)$ and $(5.2,3)$
17. $(7,-2)$ and $(4,-2)$

9.


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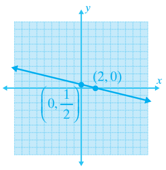


11. a. –0.4; 1; –4.2; 2
b. The average inflation rates in the US dec. 0.4% from '06-'07; inc. 1% from '07-'08; dec. 4.2% from '08-'09; inc. 2% from '09-'10

12. $m = -\frac{2}{3}$

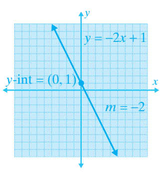
13. $m = \frac{1}{3}$

14. $m = 0$

15. $m = \frac{3}{20}$

16. $m =$ undefined

17. $m = 0$

18.


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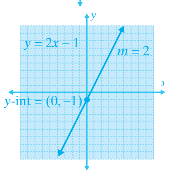

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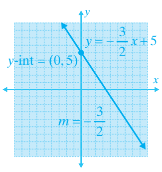

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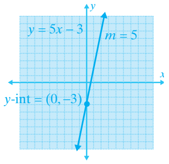

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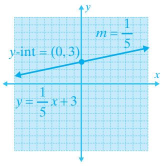


23. $3x - y = 3$
24. $3x - 4y = -1$
25. $y = 5$
26. $x = -7$
27. $2x + 3y = -2$
28. Neither
29. Perpendicular
30. Parallel
31. $y = -3x - 13$
32. $5x - 8y = -4$
33. $x + 2y = 12$
34. $y = -\frac{1}{5}x - 4$
35. $y = -3$
36. Function; $\text{Domain} = \{-9, -8, -7, -6\}$; $\text{Range} = \{2, 3\}$
37. Not a function; $\text{Domain} = \{4, 6, 7\}$; $\text{Range} = \{1, 5, 6, 7, 8\}$
38. Function; $\text{Domain} = \left\{\frac{5}{6}, \frac{9}{10}, \frac{11}{5}, \frac{15}{2}\right\}$; $\text{Range} = \left\{\frac{1}{6}, \frac{2}{3}, \frac{3}{5}\right\}$
39. Function; $\text{Domain} = \{-3.4, -1.5, -0.5, 0.5\}$; $\text{Range} = \{2.4\}$
40. Function; $\text{Domain} = \{-10, -8, -6, -2, 0, 5, 7\}$; $\text{Range} = \{-2, 1, 2, 3, 5\}$

Write each equation in slope-intercept form. Find the slope and the y-intercept, and then use them to draw the graph.

18. $2x + y = 1$ **19.** $2x - y = 1$ **20.** $3x + 2y = 10$

21. $y = 5x - 3$ **22.** $x = 5y - 15$

Find an equation in standard form for the line passing through the given point with the given slope.

23. $(1, 0)$; $m = 3$ **24.** $(-3, -2)$; $m = \frac{3}{4}$ **25.** $(0, 5)$; $m = 0$

26. $(-7, 5)$; m is undefined **27.** $(-4, 2)$; $m = -\frac{2}{3}$

Determine whether each pair of lines is parallel, perpendicular, or neither.

28. $y = -2x + 5$, $3x + 6y = 1$ **29.** $y = 5x + 2$, $x + 5y = 7$ **30.** $5x + y = 4$, $10x + 2y = -1$

31. Write an equation in slope-intercept form for the line that passes through the point $(-5, 2)$ and is parallel to the line $3x + y = 4$.

32. Write an equation in standard form for the line that contains the two points $(-4, -2)$ and $(4, 3)$.

33. Write an equation in standard form for the line that contains the point $(0, 6)$ and is perpendicular to the line $2x - y = 8$.

34. Write an equation in slope-intercept form for the line that is perpendicular to the line $5x - y = 3$ and has the same y-intercept as the line $3x - 2y = 8$.

35. Write an equation in standard form for the line that is parallel to the x-axis and has the same y-intercept as the line $3x - 4y = 12$.

Determine whether each relation is a function. State its domain and range.

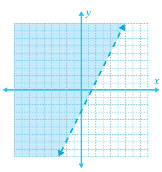
36. $\{(-9, 2), (-8, 3), (-7, 3), (-6, 2)\}$ **37.** $\{(4, 5), (6, 7), (4, 6), (7, 1), (4, 8)\}$

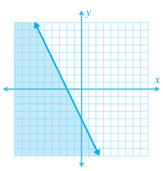
38. $\left\{\left(\frac{5}{6}, \frac{2}{3}\right), \left(\frac{9}{10}, \frac{3}{5}\right), \left(\frac{11}{5}, \frac{2}{3}\right), \left(\frac{15}{2}, \frac{1}{6}\right)\right\}$

39. $\{(-3.4, 2.4), (-1.5, 2.4), (-0.5, 2.4), (0.5, 2.4)\}$

40. $\{(-10, -2), (-8, 3), (-6, 1), (-2, -2), (0, 3), (5, 2), (7, 5)\}$

41. $f(2)=1$; $f(0)=-5$
42. $h(-3)=29$; $h(4)=-13$
43. $g(1.5)=6.25$; $g(-2)=15$
44. $f(-3)=33$; $f(1)=1$
45. $A(4)=50.24$; $A(6)=113.04$
46. Function; $D=(-\infty,\infty)$; $R=[0,\infty)$
47. Function; $D=[-6,6]$; $R=[-4,4]$
48. Not a function; $D=\{-3,0,2,3,4\}$; $R=\{-2,1,2,3,5\}$
49. Not a function; $D=[0,\infty)$; $R=(-\infty,\infty)$

50.


51.


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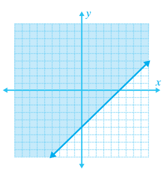


Find the values of the functions as indicated.

41. Find $f(2)$ and $f(0)$ for $f(x)=3x-5$.

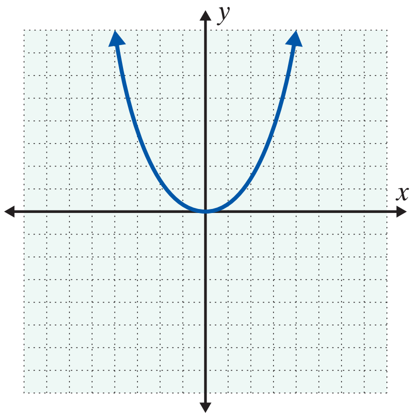
42. Find $h(-3)$ and $h(4)$ for $h(x)=-6x+11$.

43. Find $g(1.5)$ and $g(-2)$ for $g(x)=x^2-2x+7$.

44. If $f(x)=2x^2-4x+3$, find $f(-3)$ and $f(1)$.

45. If $A(r)=\pi r^2$, find $A(4)$ and $A(6)$. (Use $\pi=3.14$.)

Use the vertical line test to determine whether each graph represents a function. State the domain and range using interval notation.

46.


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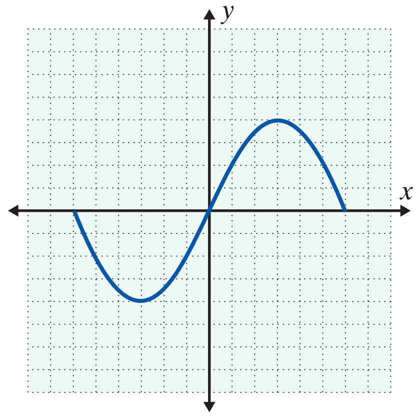

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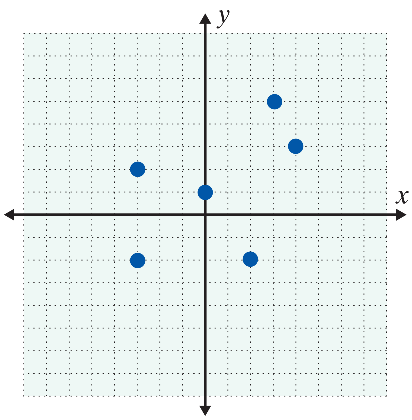

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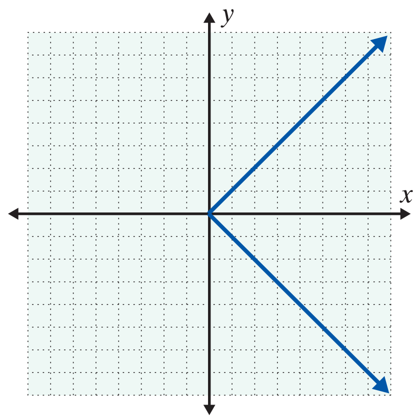


Graph the solution set of each of the linear inequalities.

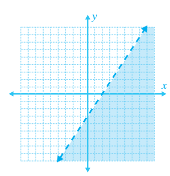
50. $y>2x-3$

51. $y\le -2x-4$

52. $x-y\le 5$

53. $3x-2y>6$

54. $4x-y\ge 2$

53.


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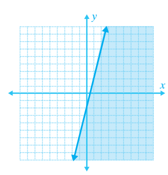

R.5 Chapter 5 Review: Exponents and Polynomials

Sections 5.1 and 5.2
pages 354 and 367

Exponents

We know that a positive integer exponent indicates repeated multiplication. For example,

3 is the exponent 4 is the exponent

$$2^3 = 2 \cdot 2 \cdot 2 \quad \text{and} \quad x^4 = x \cdot x \cdot x \cdot x$$

2 is the base x is the base

With this basic idea, we can develop the following rules for integer exponents.

Summary of the Rules for Exponents

If a and b are nonzero real numbers and m and n are integers:

1. The exponent 1: $\boldsymbol{a = a^1}$
2. The exponent 0: $\boldsymbol{a^0 = 1}$
3. The product rule: $\boldsymbol{a^m \cdot a^n = a^{m+n}}$
4. The quotient rule: $\boldsymbol{\dfrac{a^m}{a^n} = a^{m-n}}$
5. Negative exponents: $\boldsymbol{a^{-n} = \dfrac{1}{a^n}}$
6. Power rule: $\boldsymbol{\left(a^m\right)^n = a^{mn}}$
7. Power of a product: $\boldsymbol{(ab)^n = a^n b^n}$
8. Power of a quotient: $\boldsymbol{\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}}$

Example 1: Applying Combinations of Rules for Exponents

Use the appropriate rules for exponents to simplify each expresssion. The final result should contain only positive exponents.

a. $(-10)^0 = 1$

b. $5xy^3 \cdot 2xy^2 = 5 \cdot 2 \cdot x^{1+1}y^{3+2} = 10x^2y^5$

c. $\left(\dfrac{x}{3}\right)^5 = \dfrac{x^5}{3^5} = \dfrac{x^5}{243}$

d. $\dfrac{10^{-6}}{10^{-5}} = 10^{-6-(-5)} = 10^{-6+5} = 10^{-1} = \dfrac{1}{10}$

Continued on the next page...

e. $\left(-6x^2y^{-1}\right)^2 = (-6)^2\cdot\left(x^2\right)^2\cdot\left(y^{-1}\right)^2 = 36x^4y^{-2} = \dfrac{36x^4}{y^2}$

f. $\dfrac{28x^4y^5}{4xy^5} = 7x^{4-1}y^{5-5} = 7x^3y^0 = 7x^3$

g. $\dfrac{20y^8\cdot 3y^3}{15y^{15}} = \dfrac{60y^{8+3}}{15y^{15}} = \dfrac{4y^{11}}{y^{15}} = 4y^{11-15} = 4y^{-4} = \dfrac{4}{y^4}$

h. $\left(\dfrac{-5x}{y^3}\right)^3 = \dfrac{(-5x)^3}{\left(y^3\right)^3} = \dfrac{(-5)^3\cdot x^3}{y^{3(3)}} = \dfrac{-125x^3}{y^9}$

i. $\left(\dfrac{x^4}{y^3}\right)^{-5} = \dfrac{x^{4(-5)}}{y^{3(-5)}} = \dfrac{x^{-20}}{y^{-15}} = \dfrac{y^{15}}{x^{20}}$

Section 5.2 Scientific Notation

page 373

A basic application of integer exponents (called **scientific notation**) occurs when very large and very small decimal numbers are involved. **In scientific notation, decimal numbers are written as the product of a number greater than or equal to 1 and less than 10, and an integer power of 10.** In scientific notation there is just one digit to the left of the decimal point. For example,

$$84{,}000{,}000 = 8.4\times10^7 \qquad \text{and} \qquad 0.000000096 = 9.6\times10^{-8}$$

Scientific Notation

If N is a decimal number, then in **scientific notation**

$\boldsymbol{N = a\times 10^n}$ where $1 \le a < 10$ and n is an integer.

Example 2: Scientific Notation and the Properties of Exponents

Simplify the following expression by first writing the decimal numbers in scientific notation and then using the properties of exponents.

Solution:

$$\frac{0.00035\times960{,}000}{0.0006} = \frac{3.5\times10^{-4}\times9.6\times10^5}{6.0\times10^{-4}}$$

$$= \frac{3.5\times9.6}{6.0}\times\frac{10^{-4}\times10^5}{10^{-4}}$$

$$= 5.6\times10^{-4+5-(-4)} = 5.6\times10^5$$

Or, converting back to decimal notation: $5.6\times10^5 = 560{,}000$

Section 5.3 Introduction to Polynomials

page 380

A **term** is an expression that involves only multiplication and/or division with constants and/or variables. A term that consists of only a number, such as 13, is called a **constant** or a **constant term**.

Monomial

A **monomial in *x*** is a term of the form

$$kx^n$$

where ***k*** is a real number and ***n*** is a whole number.

n is called the **degree** of the term, and ***k*** is called the **coefficient**.

A nonzero constant is a monomial of **degree 0**. However, the constant 0 is a monomial of **no degree**.

A **polynomial** is a monomial or the indicated sums or differences of monomials. Examples of polynomials are

$$5x, \quad y-7, \quad 3x^2+10x+20, \quad \text{and} \quad a^9-4a^3+3a^2+8.$$

Polynomial

A **polynomial** is a monomial or the indicated sum or difference of monomials.

The **degree of a polynomial** is the largest of the degrees of its terms.

The coefficient of the term of the largest degree is called the **leading coefficient**.

Special Terminology for Some Polynomials

		Examples
Monomial:	polynomial with one term	$-4x$ and $3a^6$
Binomial:	polynomial with two terms	$2x+1$ and a^3+2a
Trinomial:	polynomial with three terms	x^2+4x-3 and a^3+6a-1

Note: Polynomials with four or more terms are simply referred to as **polynomials**.

The terms in a polynomial are written in **descending order** if the exponents on the variables decrease in order from left to right. If the exponents on the terms increase in order from left to right, the terms are written in **ascending order**. **As a general rule, the polynomials in this text will be written in descending order.**

Example 3: Simplifying Polynomials

Simplify each of the following polynomials by combining like terms. Write the polynomial in descending order and state the degree and type of the polynomial.

a. $15x^3 - 8x^3 = 7x^3$ Third-degree monomial

b. $5x^4 + 6x^2 - x^4 - x^2 = 4x^4 + 5x^2$ Fourth-degree binomial

To evaluate a polynomial for a given value of the variable, substitute the value for the variable wherever it occurs in the polynomial and follow the rules for order of operations. The function notation $p(x)$ [read "p of x"] is convenient for evaluating polynomials.

Example 4: Evaluating Polynomials

Given $p(x) = 7x^2 + 6x - 13$, find $x = 3$ and $x = -2$.

Solution:

$$\begin{aligned} p(3) &= 7(3)^2 + 6(3) - 13 \\ &= 7 \cdot 9 + 18 - 13 \\ &= 63 + 18 - 13 \\ &= 68 \end{aligned}$$

and

$$\begin{aligned} p(-2) &= 7 \cdot (-2)^2 + 6 \cdot (-2) - 13 \\ &= 7 \cdot 4 - 12 - 13 \\ &= 28 - 12 - 13 \\ &= 3 \end{aligned}$$

Section 5.4 Addition and Subtraction with Polynomials

page 387

The **sum** of two or more polynomials is found by combining **like terms**. Remember that like terms (or similar terms) are constants or terms that contain the same variables raised to the same powers.

Example 5: Addition with Polynomials

a. Add: $(4x^2 - 8x + 23) + (3x^2 - 2x - 10) = (4x^2 + 3x^2) + (-8x - 2x) + (23 - 10)$

$$= 7x^2 - 10x + 13$$

b. Add:

$$\begin{array}{rrrr} x^3 & - 4x^2 & & + 5 \\ 3x^3 & + x^2 & - 6x & + 7 \\ \hline 4x^3 & - 3x^2 & - 6x & + 12 \end{array}$$

A negative sign written in front of a polynomial in parentheses indicates the opposite of the entire polynomial. So, the **difference** between two polynomials can be found by changing the sign of each term of the second polynomial and then combining like terms.

Example 6: Subtraction with Polynomials

a. Subtract: $(6x^3 + 14x^2 - x + 30) - (2x^3 - 12x^2 + x + 30)$

$$= 6x^3 + 14x^2 - x + 30 - 2x^3 + 12x^2 - x - 30$$
$$= (6x^3 - 2x^3) + (14x^2 + 12x^2) + (-x - x) + (30 - 30)$$
$$= 4x^3 + 26x^2 - 2x$$

b. Subtract:

$$\begin{array}{r} 7x^3 + 4x^2 - 25 \\ \underline{-(-3x^3 + 2x^2 + 6x)} \end{array} \longrightarrow \begin{array}{r} 7x^3 + 4x^2 + 0x - 25 \\ \underline{3x^3 - 2x^2 - 6x + 0} \\ 10x^3 + 2x^2 - 6x - 25 \end{array}$$

Algebraic expressions may contain more than one pair of grouping symbols, such as parentheses, brackets, or braces. To simplify these expressions, work to remove the innermost pair of symbols first. Then **apply the rules for order of operations** just as if the variables were numbers and proceed to combine like terms.

Example 7: Simplifying an Algebraic Expression

Simplify: $14 - [5a + 2(6 - 4a) - 10] = 14 - [5a + 12 - 8a - 10]$

$$= 14 - [-3a + 2]$$
$$= 14 + 3a - 2$$
$$= 3a + 12$$

Section 5.5 Multiplication with Polynomials

page 394

The product of two polynomials can be found by applying the distributive property. If each of the polynomials has more than one term, then one can be written under the other and the distributive property applied.

Example 8: Multiplication with Polynomials

a. $-6x\left(x^2-8x+12\right) = -6x\left(x^2\right) - 6x(-8x) - 6x(12)$

$$= -6x^3 + 48x^2 - 72x$$

b. $(2y-5)(3y-4) = 2y(3y-4) - 5(3y-4)$

$$= 2y(3y) + 2y(-4) - 5(3y) - 5(-4)$$

$$= 6y^2 - 8y - 15y + 20$$

$$= 6y^2 - 23y + 20$$

c.

$$\begin{array}{rrrr}
 & 7x^2 & +\ 3x & -\ 2 \\
 & & 2x & -\ 4 \\
\hline
 & -\ 28x^2 & -\ 12x & +\ 8 \\
14x^3 & +\ 6x^2 & -\ 4x & \\
\hline
14x^3 & -\ 22x^2 & -\ 16x & +\ 8
\end{array}$$

Section 5.6 Special Products of Binomials

page 400

In Section 5.5 we multiplied polynomials using the distributive property. In the case of **two binomials** the product can be found more directly using the **F-O-I-L method** as shown below.

First Last

$$(3x+4)(2x+7)$$

Inside

Outside

$6x^2$ 28

$$(3x+4)(2x+7) = 6x^2 + 21x + 8x + 28$$

$+8x$

$+21x$

$$= 6x^2 + 29x + 28$$

Example 9: FOIL Method

Use the FOIL method to find the products of the given binomials.

a. $(x+9)(2x-3)$

Solution:

$2x^2$ -27

F O I L

$$(x+9)(2x-3) = 2x^2 - 3x + 18x - 27$$

$+18x$

$-3x$

$$= 2x^2 + 15x - 27$$

b. $(x-12)(x+12)$

Solution:

x^2 -144

F O I L

$$(x-12)(x+12) = x^2 + 12x - 12x - 144$$

$-12x$ $+12x$

$$= x^2 - 144$$

In Example 9b, the middle terms, $+12x$ and $-12x$, are opposites of each other and their sum is 0. Therefore, the product has only two terms and each term is a square. The two binomials are in the form of the sum and difference of the same two terms, and the product is called the **difference of two squares**.

Difference of Two Squares

$$\boldsymbol{(x+a)(x-a) = x^2 - a^2}$$

Example 10: Difference of Two Squares

a. $(2y+5)(2y-5) = (2y)^2 - 5^2 = 4y^2 - 25$ Difference of two squares

b. $(x^3-10)(x^3+10) = (x^3)^2 - 10^2 = x^6 - 100$ Difference of two squares

Now consider the case of the **square of a binomial**. As the following examples illustrate, there are two basic patterns, each with a different sign for the middle term.

$$\begin{aligned}(x+13)^2 &= x^2 + 13x + 13x + 169 \\ &= x^2 + 2 \cdot 13x + 169 \\ &= x^2 + 26x + 169\end{aligned}$$

$$\begin{aligned}(x-14)^2 &= x^2 - 14x - 14x + 196 \\ &= x^2 - 2 \cdot 14x + 196 \\ &= x^2 - 28x + 196\end{aligned}$$

Note that in each case **the result of squaring the binomial is a trinomial**. These trinomials are called **perfect square trinomials**.

Squares of Binomials (Perfect Square Trinomials)

$\boldsymbol{(x+a)^2 = x^2 + 2ax + a^2}$ **Square of a Binomial Sum**

$\boldsymbol{(x-a)^2 = x^2 - 2ax + a^2}$ **Square of a Binomial Difference**

Example 11: Squares of Binomials (Perfect Square Trinomials)

a. $(6y+1)^2 = (6y)^2 + 2 \cdot 6y \cdot 1 + 1^2 = 36y^2 + 12y + 1$ Perfect square trinomial

b. $(x^4-10)^2 = (x^4)^2 - 2 \cdot x^4 \cdot 10 + 10^2 = x^8 - 20x^4 + 100$ Perfect square trinomial

Section 5.7 Division with Polynomials

page 408

Fractions in which the numerator and denominator are polynomials are called **rational expressions**. (No denominator can be 0.) If the denominator in a rational expression is a monomial, we can perform the indicated division by dividing each term in the numerator by the monomial denominator and then simplifying each fraction. For example,

$$\frac{16x^3+10x^2-8x}{2x} = \frac{16x^3}{2x} + \frac{10x^2}{2x} - \frac{8x}{2x} = 8x^2 + 5x - 4.$$

Example 12: Division by a Monomial

$$\frac{15x^3-12x^2+9x}{3x^2} = \frac{15x^3}{3x^2} - \frac{12x^2}{3x^2} + \frac{9x}{3x^2} = 5x - 4 + \frac{3}{x}$$

In arithmetic, the **division algorithm** (called **long division**) is the process that we follow when dividing two numbers. In algebra, the **division algorithm with polynomials** is similar. In dividing one polynomial by another, with the degree of the divisor smaller than the degree of the dividend, the quotient will be another polynomial with a remainder.

The remainder must be of smaller degree than the divisor. If the remainder is 0, then the divisor and quotient are factors of the dividend.

The Division Algorithm

For polynomials P and D, the division algorithm gives

$$\frac{P}{D} = Q + \frac{R}{D}, \quad D \neq 0$$

where Q and R are polynomials and the **degree of R < degree of D**.

The actual process of long division is not clear from this abstract definition. Example 2 illustrates the procedure in a step-by-step format. Study it carefully. (Also, see more examples and a more detailed discussion in Section 5.7.)

Example 13: The Division Algorithm

Simplify $\dfrac{8x^2-12x+5}{2x+1}$ using the division algorithm.

Solution:

	Calculation	Explanation
Step 1:	$2x+1\overline{)8x^2-12x+5}$	Write both polynomials in order of descending powers. **If any powers are missing, fill in with 0's.**
Step 2:	$\begin{array}{r} 4x \\ 2x+1\overline{)8x^2-12x+5} \end{array}$	Mentally divide $8x^2$ by $2x$: $\dfrac{8x^2}{2x}=4x$. Write $4x$ above $8x^2$.
Step 3:	$\begin{array}{r} 4x \\ 2x+1\overline{)8x^2-12x+5} \\ \underline{-(8x^2+4x)} \end{array}$	Multiply $4x$ times $(2x+1)$ and write the terms of the product, $8x^2+4x$, under the like terms in the dividend. Use a minus sign to indicate that the product is to be subtracted.
Step 4:	$\begin{array}{r} 4x \\ 2x+1\overline{)8x^2-12x+5} \\ \underline{-8x^2-4x} \\ -16x \end{array}$	Subtract $8x^2+4x$ by changing signs and adding.
Step 5:	$\begin{array}{r} 4x \\ 2x+1\overline{)8x^2-12x+5} \\ \underline{-8x^2-4x} \\ -16x+5 \end{array}$	Bring down the $+5$.
Step 6:	$\begin{array}{r} 4x-8 \\ 2x+1\overline{)8x^2-12x+5} \\ \underline{-8x^2-4x} \\ -16x+5 \end{array}$	Mentally divide $-16x$ by $2x$: $\dfrac{-16x}{2x}=-8$. Write -8 in the quotient.
Step 7:	$\begin{array}{r} 4x-8 \\ 2x+1\overline{)8x^2-12x+5} \\ \underline{-8x^2-4x} \\ -16x+5 \\ \underline{-(-16x-8)} \end{array}$	Multiply -8 times $(2x+1)$ and write the product of the terms, $-16x-8$, under the like terms in the expression $-16x+5$. Use a minus sign to indicate that the product is to be subtracted.

Continued on the next page...

Step 8:

$$\begin{array}{r} 4x - 8 \\ 2x+1 \overline{\smash{)}\,8x^2 - 12x + 5} \\ \underline{-8x^2 - 4x} \\ -16x + 5 \\ \underline{+16x + 8} \\ +13 \end{array}$$

Subtract $-16x-8$ by changing signs and adding.

Thus the quotient is $4x-8$ and the remainder is 13.

In the form $Q+\dfrac{R}{D}$ we can write $\dfrac{8x^2-12x+5}{2x+1}=4x-8+\dfrac{13}{2x+1}$.

R.5 Exercises

1. 1
2. 1
3. $\dfrac{1}{16}$
4. $\dfrac{1}{32}$
5. $-15a^6$
6. $3x^8$
7. $\dfrac{x^3}{2}$
8. $\dfrac{2}{a^2}$
9. x^5
10. $\dfrac{y^6}{x^6}$
11. $9x^6$
12. $\dfrac{125a^6}{b^3}$
13. $\dfrac{x^4}{25y^4}$
14. $\dfrac{a^{10}}{32}$
15. $49x^6y^6$
16. $\dfrac{50}{a^7b^7}$
17. $\dfrac{72}{y^2}$
18. $20ab^{15}$
19. 5600
20. 94,100,000
21. 0.000 064 4
22. 0.000 003 792
23. 583,400,000
24. 5.37×10^7
25. 7.2×10^{-7}
26. 6×10^{-3}
27. 1.5×10^4
28. 2.24×10^{-2}

Use the rules for exponents to simplify each of the expressions. Each answer should have only positive exponents. Assume that all variables represent nonzero numbers.

1. $(-10)^0$ **2.** 3.56^0 **3.** 4^{-2} **4.** 2^{-5}

5. $(3a^2)(-5a^4)$ **6.** $(-1.5x^3)(-2x^5)$ **7.** $\dfrac{5x^5}{10x^2}$ **8.** $\dfrac{6a^{-4}}{3a^{-2}}$

9. $\dfrac{x}{x^{-4}}$ **10.** $\dfrac{x^2y^8}{x^8y^2}$ **11.** $(-3x^3)^2$ **12.** $(5a^2b^{-1})^3$

13. $\left(\dfrac{5xy^2}{x^3}\right)^{-2}$ **14.** $\left(\dfrac{2}{a^2}\right)^{-5}$ **15.** $\left(\dfrac{7x^0y^4}{x^{-3}y}\right)^2$

16. $\left(\dfrac{5a^{-2}b}{ab^4}\right)^2\left(\dfrac{ab^2}{2b}\right)^{-1}$ **17.** $\dfrac{(6^{-2}x^{-3}y^2)^{-1}}{(x^{-2}y)^{-2}(2xy^{-2})^{-1}}$ **18.** $\dfrac{(5a^2b)(2a^{-1}b^3)^2}{(ab^2)^{-3}(ab^{-1})^2}$

Write the following numbers in decimal form.

19. 5.6×10^3 **20.** 9.41×10^7 **21.** 6.44×10^{-5}

22. 3.792×10^{-6} **23.** 5.834×10^8

First write each of the numbers in scientific notation. Then perform the indicated operations and leave your answer in scientific notation.

24. $53,700\times1000$ **25.** 0.006×0.00012 **26.** $\dfrac{1800\times0.0045}{1350}$

27. $\dfrac{460\times0.09}{0.023\times0.12}$ **28.** $\dfrac{1520\times168\times0.063}{9000\times0.38\times210}$

29. **Astronomy:** One light-year is about 9.46×10^{12} meters. The distance to a certain star is about 5.1 light-years. How many meters is this?

30. **Atomic weight:** An atom of gold weighs approximately 3.25×10^{-22} grams. What would be the weight of 5000 atoms of gold?

31. State the degree and type of the polynomial $5x^3-3x+17$.

32. State the degree and type of the polynomial $-7x^5+3x$.

33. Give two examples that show how the sum of two trinomials might not be a trinomial.

Evaluate the given polynomials as indicated.

34. Given $p(x)=x^3+5x^2-x-15$, find **a.** $p(-1)$ and **b.** $p(3)$.

35. Given $p(y)=4y^2+15y-10$, find **a.** $p(5)$ and **b.** $p(-2)$.

Perform the indicated operations and simplify each expression.

36. $(2x^2+4x+9)+(x^2+3x+4)$

37. $(x^2+4x-4)+(-3x^2+2x+1)$

38. $(a^2+5a+10)+(3a+4)+(a^2-6)$

39. $(y^3-5y-11)+(3y^3-5y+10)+(y^2-8y+16)$

40. Add: $\begin{array}{r} 6x^2-3x+14 \\ \underline{-x^2-5x-6} \end{array}$

41. Add: $\begin{array}{r} 10x^3-4x^2-6x-9 \\ \underline{12x^3\qquad\quad +5x-8} \end{array}$

42. $(3a^2+6a+7)-(a^2+2a-5)$

43. $(2y^3-5y+4)-(-2y^3+3y+5)$

44. $(5t^2+7t-10)-(2t^2+4t+5)$

45. $(5x^3-10x+20)-(x^3+5x^2-x-6)$

46. $\begin{array}{r} 4x^4+10x^3-9x^2-8x-7 \\ \underline{-(2x^4+\ \ x^3\qquad\quad +5x+2)} \end{array}$

47. $\begin{array}{r} 2x^4-6x^3-5x^2+6x+6 \\ \underline{-(x^4\qquad\quad +3x^2+4x+6)} \end{array}$

48. Subtract $3x^2-4x+5$ from the sum of $4x^2+x+2$ and $11x-5$.

49. Subtract $-2x^2+6x+8$ from the sum of x^2+6x+5 and x^2+3x+4.

50. Add $5a^3+7a+3$ to the difference between $2a^3+13a-3$ and $a^2+10a+14$.

29. 4.8246×10^{13} meters
30. 1.625×10^{-18} grams
31. Third-degree, trinomial
32. Fifth-degree, binomial
33. Example 1: $(x^3+x^2-1)+(2x^3-3x^2+1)=3x^3-2x^2$ which is a binomial
Example 2: $(x^2+x+1)+(x^2-x-1)=2x^2$ which is a monomial; Answers will vary.
34. a. -10 **b.** 54
35. a. 165 **b.** -24
36. $3x^2+7x+13$
37. $-2x^2+6x-3$
38. $2a^2+8a+8$
39. $4y^3+y^2-18y+15$
40. $5x^2-8x+8$
41. $22x^3-4x^2-x-17$
42. $2a^2+4a+12$
43. $4y^3-8y-1$
44. $3t^2+3t-15$
45. $4x^3-5x^2-9x+26$
46. $2x^4+9x^3-9x^2-13x-9$
47. $x^4-6x^3-8x^2+2x$
48. $x^2+16x-8$
49. $4x^2+3x+1$
50. $7a^3-a^2+10a-14$

Simplify the following algebraic expressions.

51. $15-\left[6x+2(10-4x)-22\right]$ **52.** $10^2+2\left[3a-5(a+6)+4(a-7)\right]$

Multiply as indicated and simplify if possible.

53. $4x^3\left(5x^2+7x+6\right)$ **54.** $-3y^2\left(2y^2-7y+9\right)$ **55.** $(x+3)\left(x^3-2x^2+5\right)$

56. $(2a-7)\left(3a^2+6a+4\right)$ **57.** $(2y+5)(y+2)$ **58.** $(3x+4)(2x-5)$

59. $(7x+1)(7x-1)$ **60.** $\left(3x^2+8\right)\left(3x^2-8\right)$ **61.** $(5x+6)^2$

62. $(3t-4)^2$ **63.** $(x-2y)\left(x^2+2xy+4y^2\right)$ **64.** $(a+3)\left(a^2-3a+9\right)$

65. $\left(y^2+3\right)(y-5)$ **66.** $(6x+5)(x-7)$

67. $$\begin{array}{r} 2x^2-4x+5 \\ \underline{\qquad 2x+3} \end{array}$$

68. $$\begin{array}{r} x^2+2x+1 \\ \underline{x^2+2x+1} \end{array}$$

Simplify the following algebraic expressions.

69. $(y+3)(y-3)+(y+4)(y-4)$ **70.** $(x+5)(x-2)+(2x+1)(x-3)$

71. $(4t-1)(t+3)-(t+5)(t-5)$ **72.** $(3a+2)(a+1)-(a+6)(a-6)$

Find the indicated quotients. If the divisor is not a monomial, use the division algorithm and write the answers in the form $Q+\frac{R}{D}$, *where the degree of R < the degree of D.*

73. $\frac{8x^2-20x+16}{4}$ **74.** $\frac{27y^3-9y^2+3y}{3y}$ **75.** $\frac{20x^2y^2+35xy+3y^2}{5xy}$

76. $\frac{24a^2b^2+12ab-6b^2}{3ab}$ **77.** $\left(6x^2+x-4\right)\div(2x+1)$ **78.** $\left(4x^2-20x+2\right)\div(x+2)$

79. $\frac{16x^3+7x+12}{4x-1}$ **80.** $\frac{x^4+x^3-4x+1}{x^2+1}$ **81.** $\left(a^3-64\right)\div(a-4)$

82. $\frac{4x^3+6x^2-3x+1}{x+2}$ **83.** $\frac{y^4+2y^2-3y-18}{y-2}$ **84.** $\frac{2x^3-7x+5}{x+4}$

51. $2x+17$
52. $4a-16$
53. $20x^5+28x^4+24x^3$
54. $-6y^4+21y^3-27y^2$
55. $x^4+x^3-6x^2+5x+15$
56. $6a^3-9a^2-34a-28$
57. $2y^2+9y+10$
58. $6x^2-7x-20$
59. $49x^2-1$
60. $9x^4-64$
61. $25x^2+60x+36$
62. $9t^2-24t+16$
63. x^3-8y^3
64. a^3+27
65. $y^3-5y^2+3y-15$
66. $6x^2-37x-35$
67. $4x^3-2x^2-2x+15$
68. $x^4+4x^3+6x^2+4x+1$
69. $2y^2-25$
70. $3x^2-2x-13$
71. $3t^2+11t+22$
72. $2a^2+5a+38$
73. $2x^2-5x+4$
74. $9y^2-3y+1$
75. $4xy+7+\frac{3y}{5x}$
76. $8ab+4-\frac{2b}{a}$
77. $3x-1-\frac{3}{2x+1}$
78. $4x-28+\frac{58}{x+2}$
79. $4x^2+x+2+\frac{14}{4x-1}$ **80.** $x^2+x-1+\frac{-5x+2}{x^2+1}$ **81.** $a^2+4a+16$ **82.** $4x^2-2x+1-\frac{1}{x+2}$ **83.** y^3+2y^2+6y+9
84. $2x^2-8x+25-\frac{95}{x+4}$

R.6 Chapter 6 Review: Factoring Polynomials and Solving Quadratic Equations

Section 6.1 Greatest Common Factor and Factoring by Grouping

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The result of multiplication is called the **product** and the numbers or expressions being multiplied are called **factors** of the product. The reverse of multiplication is called **factoring**. That is, given a product, factoring means to find the factors.

The **greatest common factor** (**GCF**) of two or more integers is the largest integer that is a factor (or divisor) of all of the integers. For example, the GCF of 60 and 72 is 12. While 3 is a common factor of 60 and 72, it is not the **greatest** common factor. The following procedure can be used to find the GCF for any set of integers or algebraic terms with integer exponents.

Procedure for Finding the GCF of a Set of Terms

1. Find the prime factorization of all integers and integer coefficients.
2. List all the factors that are common to all terms, including variables.
3. Choose the greatest power of each factor that is common to all terms.
4. Multiply these powers to find the GCF.

   Note: If there is no common prime factor or variable, then the GCF is 1.

Example 1: Finding the GCF

Find the GCF for the following set of algebraic terms: $\{36x^3y, 60x^2y, 108xy^2\}$

Solution: Writing each integer coefficient in prime factored form gives:

$$36x^3y = 2^2\cdot 3^2\cdot x^3\cdot y$$

$$60x^2y = 2^2\cdot 3\cdot 5\cdot x^2\cdot y$$

$$108xy^2 = 2^2\cdot 3^3\cdot x\cdot y^2$$

Thus $\text{GCF} = 2^2\cdot 3\cdot x\cdot y = 12xy.$

The first step in factoring polynomials is to factor out the monomial that is the **greatest common factor (GCF)**.

Factoring Out the GCF

To find a monomial that is the **greatest common factor (GCF)** of a polynomial:

1. Find the variable(s) of highest degree and the largest integer coefficient that is a factor of each term of the polynomial. (This is one factor.)
2. Divide this monomial factor into each term of the polynomial resulting in another polynomial factor.

By definition, the GCF of a polynomial will have a positive coefficient. **However, if the leading coefficient is negative, we may choose to factor out the negative of the GCF (or –1 · GCF).** This technique will leave a positive coefficient for the first term of the other polynomial factor. For example,

$$-15a^3 + 10a^2 = 5a^2(-3a + 2)$$

or $$-15a^3 + 10a^2 = -5a^2(3a - 2)$$

Both answers are correct. However, we will see later that having a positive leading coefficient for the polynomial in parentheses may make that polynomial easier to factor. Therefore, the second answer is generally preferred.

Example 2: Factoring out the GCF of a Polynomial

Factor each polynomial by finding the GCF (or –1 · GCF).

a. $20x^4 + 12x^3 = 4x^3(5x + 3)$

b. $-14y^4 + 21y^3 - 14y^2 = -7y^2(2y^2 - 3y + 2)$

c. $8x^2y^2 - 12xy^2 = 4xy^2(2x - 3)$

Now consider the expression

$$(2y + 5)(x + 3) = 2y(x + 3) + 5(x + 3) = 2xy + 6y + 5x + 15.$$

We see that there are **four terms** in the product and no common monomial factor. Yet we know that the product has two factors, namely $(x + 3)$ and $(2y + 5)$. Factoring polynomials with four or more terms can sometimes be accomplished by **grouping the terms**. This means looking for common factors in each group and then looking for common binomial factors. This process is illustrated in Example 3.

Example 3: Factoring by Grouping

Factor each polynomial by grouping.

a. $$xy+5x+2y+10=(xy+5x)+(2y+10)$$
$$=x(y+5)+2(y+5)$$
$$=(y+5)(x+2)$$

b. $$xy+16-8x-2y=xy-8x+16-2y$$
$$=x(y-8)+2(8-y)$$
$$=x(y-8)-2(y-8) \quad \text{Note that } (8-y) \text{ is the opposite of } (y-8).$$
$$=(y-8)(x-2)$$

c. $$x^2+6x+ax+ay=x(x+6)+a(x+y)$$

But $x+6 \neq x+y$ and there is no common factor.

So $x^2+6x+ax+ay$ is **not factorable**.

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Factoring Trinomials: $x^2 + bx + c$

Consider the following general product of two binomials:

F O I L

$$(x+a)(x+b)=x^2+bx+ax+ab$$
$$=x^2+(b+a)x+ab$$

To factor a trinomial with leading coefficient 1, find two factors of the constant term whose sum is the coefficient of the middle term. (If these factors do not exist, the polynomial is **not factorable.**)

For example, to factor $x^2+18x+45$ we need two factors of +45 whose sum is +18.

Positive Factors of 45			Sums of These Factors
1	45	$\longrightarrow$	1 + 45 = 46
3	15	$\longrightarrow$	3 + 15 = 18
5	9	$\longrightarrow$	5 + 9 = 14

Now because 3 · 15 = 45 and 3 + 15 = 18, we have

$$x^2+18x+45=(x+3)(x+15)$$

Example 4: Factoring Trinomials with Leading Coefficient 1

Factor the following trinomials.

a. $y^2 - 9y + 20$

Solution: We want a pair of integer factors of 20 whose sum is –9. In this case, both factors must be negative.

Negative Factors of +20	
–1	–20
–2	–10
–4	–5

$\longrightarrow -4 + (-5) = -9$

Thus $y^2 - 9y + 20 = (y-4)(y-5)$.

b. $x^2 - 5x - 14$

Solution: We want a pair of integer factors of –14 whose sum is –5. In this case, one factor must be positive and the other negative.

Factors of –14	
–1	+14
+1	–14
+2	–7
–2	+7

$\longrightarrow +2 + (-7) = -5$

Thus $x^2 - 5x - 14 = (x+2)(x-7)$.

The exercises in Example 4 illustrate the **trial-and-error method of factoring**. That is, in each case, you can **try** different pairs of integer factors (mentally or by making a list) until you find the correct pair. If such a pair does not exist, the polynomial is **not factorable.**

An expression is factored completely if none of its factors can be factored.

Example 5: Finding a Common Monomial Factor

Completely factor the trinomial $10y^4 - 40y^3 - 120y^2$ by first factoring out the GCF.

Solution: First factor out the GCF, $10y^2$.

$$10y^4 - 40y^3 - 120y^2 = 10y^2\left(y^2 - 4y - 12\right)$$

Now factoring the trinomial, we have

$$\begin{aligned} 10y^4 - 40y^3 - 120y^2 &= 10y^2\left(y^2 - 4y - 12\right) \\ &= 10y^2(y-6)(y+2). \end{aligned}$$

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Factoring Trinomials: $ax^2 + bx + c$

To factor a trinomial with a leading coefficient other than 1, we can use the **FOIL** method with a **trial-and-error** approach. For example, consider the problem of factoring

$$4x^2 + 24x + 35$$

as the product of two binomials.

$$\mathbf{F} = 4x^2 \qquad \mathbf{L} = +35$$

$$4x^2 + 24x + 35 = (\qquad)(\qquad)$$

For $\mathbf{F} = 4x^2$, we know that $4x^2 = 4x \cdot x$ and $4x^2 = 2x \cdot 2x$.

For $\mathbf{L} = +35$, we know that $35 = 1 \cdot 35$ and $35 = 5 \cdot 7$.
(Only positive signs are considered here because 35 is positive and the middle term is positive. If the middle term was negative, then negative signs would be considered.)

Now we use various combinations for **F** and **L** in the trial-and-error method as follows:

1. List all the possible combinations of factors of $4x^2$ and $+35$ in their respective **F** and **L** positions.
2. Check the sum of the products in the **O** and **I** positions until you find the sum to be $+24x$.
3. If none of these sums is $+24x$, the trinomial is not factorable.

$$4x^2 \qquad 35$$

a. $(4x+5)(x+7)$ $\qquad$ $5x$ $\qquad$ $28x$ $\qquad$ $\mathbf{O} + \mathbf{I} = 28x + 5x = 33x$

b. $(4x+7)(x+5)$ $\qquad$ $7x$ $\qquad$ $20x$ $\qquad$ $\mathbf{O} + \mathbf{I} = 20x + 7x = 27x$

c. $(2x+7)(2x+5)$ $\qquad$ $14x$ $\qquad$ $10x$ $\qquad$ $\mathbf{O} + \mathbf{I} = 10x + 14x = 24x$

So, $4x^2 + 24x + 35 = (2x+7)(2x+5)$.

The following guidelines will help limit the trial-and-error search.

Guidelines for the Trial-and-Error Method

1. If the sign of the constant term is positive (+), the signs in both factors will be the same, either both positive or both negative.
2. If the sign of the constant term is negative (–), the signs in the factors will be different, one positive and one negative.

Example 6: Using the Trial-and-Error Method: Leading Coefficient Not 1

a. Factor $6x^2 - 17x + 5$ by using the trial-and-error method.

Solution: Since the middle term is –17x and the constant is +5, we know that the two factors of 5 must both be negative.

Trials: $(6x-1)(x-5)$ $\quad$ $\mathbf{O}+\mathbf{I} = -30x - x = -31x$

$(6x-5)(x-1)$ $\quad$ $\mathbf{O}+\mathbf{I} = -6x - 5x = -11x$

$(3x-1)(2x-5)$ $\quad$ $\mathbf{O}+\mathbf{I} = -15x - 2x = -17x$

Thus $6x^2 - 17x + 5 = (3x-1)(2x-5)$.

b. Factor $9x^2 + 2x - 7$ by using the trial-and-error method.

Solution: **Trials:** $(3x-7)(3x+1)$ $\quad$ $\mathbf{O}+\mathbf{I} = 3x - 21x = -18x$

$(9x+7)(x-1)$ $\quad$ $\mathbf{O}+\mathbf{I} = -9x + 7x = -2x$

Wrong sign only, so just switch signs in the binomials.

This gives the correct factors: $9x^2 + 2x - 7 = (9x-7)(x+1)$.

The ***ac*-method** of factoring trinomials is very systematic and involves the method of factoring by grouping. See Section 6.3 (page 453) to review a detailed analysis of this method.

Example 7: Using the *ac*-Method

Factor $6x^2 - 23x - 4$ using the *ac*-method.

Solution: $a = 6$, $b = -23$, and $c = -4$.

Step 1: Find the product $a \cdot c$: $6(-4) = -24$.

Step 2: Find two integers whose product is –24 and whose sum is –23.

$$-24 \cdot 1 = -24 \text{ and } -24 + 1 = -23$$

Step 3: Rewrite $-23x$ as $-24x + x$ to obtain

$$6x^2 - 23x - 4 = 6x^2 - 24x + x - 4$$

Step 4: Factor by grouping.

$$6x^2 - 24x + x - 4 = 6x(x-4) + 1(x-4)$$

Step 5: Factor out the common binomial factor $(x-4)$.

$$\begin{aligned} 6x^2 - 23x - 4 &= 6x^2 - 24x + x - 4 \\ &= 6x(x-4) + 1(x-4) \\ &= (x-4)(6x+1) \end{aligned}$$

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Special Factoring Techniques

The following special products were discussed in Section 5.6.

I. $(x+a)(x-a) = x^2 - a^2$ — Difference of Two Squares

II. $(x+a)^2 = x^2 + 2ax + a^2$ — Square of a Binomial Sum

III. $(x-a)^2 = x^2 - 2ax + a^2$ — Square of a Binomial Difference

Memorize the products and their names as listed above. By recognizing a polynomial as one of these three special forms you can go directly to the factors.

Example 8: Factoring Special Products

Completely factor each of the following polynomials

a. $3x^2 - 75 = 3(x^2 - 25) = 3(x+5)(x-5)$

b. $y^2 + 28y + 196 = (y+14)^2$

c. $20a^3 - 60a^2 + 45a = 5a(4a^2 - 12a + 9) = 5a(2a-3)^2$

d. $y^2 + 8y + 16 - x^2 = (y+4)^2 - x^2 = (y+4+x)(y+4-x)$

Sum of Two Squares

The **sum of two squares** is an expression of the form $\boldsymbol{x^2 + a^2}$ and is **not factorable**. For example, $x^2 + 36$ is the sum of two squares and is not factorable. There are no factors with integer coefficients whose product is $x^2 + 36$. To understand this situation, write

$$\boldsymbol{x^2 + 36 = x^2 + 0x + 36}$$

and note that there are no factors of +36 that will add to 0.

Two more special formulas that should be memorized relate to the **sum and difference of two cubes.**

IV. $x^3 + a^3 = (x+a)(x^2 - ax + a^2)$ Sum of Two Cubes

V. $x^3 - a^3 = (x-a)(x^2 + ax + a^2)$ Difference of Two Cubes

Example 9: Factoring Sums and Differences of Two Cubes

Factor completely.

a. $x^6 + 1000 = (x^2)^3 + 10^3 = (x^2 + 10)(x^4 - 10x^2 + 100)$

b. $16y^3 - 54 = 2(8y^3 - 27)$

$$= 2\left[(2y)^3 - 3^3\right]$$

$$= 2(2y-3)(4y^2 + 6y + 9)$$

Section 6.6 Solving Quadratic Equations by Factoring

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Second-degree polynomials are called **quadratic polynomials** (or simply **quadratics**). The form $ax^2 + bx + c = 0$ is called the **standard form** of a **quadratic equation**.

Quadratic Equations

Quadratic equations are equations that can be written in the form

$\boldsymbol{ax^2 + bx + c = 0}$ where a, b, and c are real numbers and $a \neq 0$.

Note that in the standard form one side of the equation is a quadratic polynomial and the other side is 0. When solving quadratic equations by factoring, having 0 on one side before we factor is necessary because of the **zero-factor property**.

Zero-Factor Property

If the product of two (or more) factors is 0, then at least one of the factors must be 0. That is, if a and b are real numbers, then

if $a \cdot b = 0$, then $a = 0$ or $b = 0$ or both.

In general, a quadratic equation has two solutions (or roots). In the special cases where the two factors are the same, there is only one solution and it is called a **double solution** (or **double root**).

Example 10: Solving Quadratic Equations by Factoring

Solve the following equations by writing the equation in standard form with one side 0 and factoring the polynomial. Then set each factor equal to 0 and solve.

a. $(x+3)(2x-9)=0$

Solution: The quadratic is already factored and the product is 0. So we set each factor equal to 0 and solve.

$$\begin{aligned} x+3&=0 \quad\text{or}\quad & 2x-9&=0\\ x&=-3 & 2x&=9\\ & & x&=\frac{9}{2}\end{aligned}$$

The two solutions (or roots) are $x=-3$ and $x=\frac{9}{2}$. Or, we can say that the solution set is $\left\{-3, \frac{9}{2}\right\}$.

b. $y^2+6y=27$

Solution:
$$\begin{aligned} y^2+6y&=27\\ y^2+6y-27&=0\\ (y+9)(y-3)&=0\end{aligned}$$

$$\begin{aligned} y+9&=0 \quad\text{or}\quad & y-3&=0\\ y&=-9 & y&=3\end{aligned}$$

The solutions are -9 and 3.

c. $4x^2-8x=-4$

Solution:
$$\begin{aligned} 4x^2-8x&=-4\\ 4x^2-8x+4&=0\\ 4(x^2-2x+1)&=0\\ 4(x-1)^2&=0\\ x-1&=0 \qquad \text{Both binomial factors are the same.}\\ x&=1\end{aligned}$$

The solution 1 is a double root.

Continued on the next page...

d. $3x(x-2)=5(x+4)$

Solution: $3x(x-2)=5(x+4)$

$$3x^2-6x=5x+20$$
$$3x^2-11x-20=0$$
$$(3x+4)(x-5)=0$$

$3x+4=0$ or $x-5=0$

$3x=-4$ $\quad$ $x=5$

$x=-\dfrac{4}{3}$

The solution set is $\left\{-\dfrac{4}{3}, 5\right\}$.

Example 11 illustrates how factoring can be used to solve equations of higher degree. There are more than two factors, but as with quadratics, if the product is 0, the solutions are found by setting each factor equal to 0.

Example 11: Solving Higher Degree Equations

Solve the third-degree (cubic) equation: $144x=4x^3$.

Solution: $144x=4x^3$

$$0=4x^3-144x$$
$$0=4x(x^2-36)$$
$$0=4x(x+6)(x-6)$$

$4x=0$ or $x+6=0$ or $x-6=0$

$x=0$ $\quad$ $x=-6$ $\quad$ $x=6$

The solutions are –6, 0, and 6.

We use the following theorem, called the **factor theorem**, as a basis for finding an equation given the roots of the equation.

Factor Theorem

If $x=c$ is a root of a polynomial equation in the form $P(x)=0$, then $x-c$ is a factor of the polynomial $P(x)$.

Example 12: Using the Factor Theorem to Find an Equation with Given Roots

Find a polynomial equation with integer coefficients that has the given roots:

$$x = 5 \text{ and } x = -\frac{3}{4}.$$

Solution: Form the linear equations and then the product of the factors.

$$\begin{aligned} x &= 5 \qquad & x &= -\frac{3}{4} \\ x - 5 &= 0 & 4x &= -3 \\ & & 4x + 3 &= 0 \end{aligned}$$

Form the equation by setting the product of the factors equal to 0 and multiplying.

$(x-5)(4x+3) = 0$

$4x^2 - 17x - 15 = 0$ This quadratic equation has the two given roots.

Section 6.7 Applications of Quadratic Equations

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Several types of problems lead to quadratic equations. The problems in this section are set up so that the equations can be solved by factoring. More general problems and approaches to solving quadratic equations are discussed in Chapter 11.

Example 13: Numbers

Twice the square of an integer is equal to 10 times the integer. What is the integer?

Solution: Let $n =$ the integer.

Set up the equation and solve:

$$2n^2 = 10n$$

$$2n^2 - 10n = 0$$

$$2n(n-5) = 0$$

$$2n = 0 \quad \text{or} \quad n - 5 = 0$$

$$n = 0 \qquad\qquad n = 5$$

There are two possible values for the integer, 0 and 5.

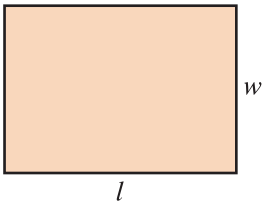
Example 14: Geometry

A rectangle has an area of 700 square yards and a perimeter of 110 yards. What are the dimensions of the rectangle?

Solution: The area of a rectangle is the product of its length and width ($A = lw$).
The perimeter of a rectangle is given by $P = 2l + 2w$.
Let w = width
Then $55 - w$ = length (55 is half of the perimeter)

Set up the equation and solve.

$$w(55-w) = 700$$
$$55w - w^2 = 700$$
$$0 = w^2 - 55w + 700$$
$$0 = (w-20)(w-35)$$



$$w - 20 = 0 \quad \text{or} \quad w - 35 = 0$$
$$w = 20 \qquad\qquad w = 35$$
$$55 - 20 = 35 \qquad 55 - 35 = 20$$

The width is 20 yards and the length is 35 yards.

Example 15: Consecutive Integers

Find three consecutive odd integers such that the product of the first and third is 4 less than 7 times the second.

Solution: Let n = first odd integer
and $n + 2$ = second consecutive odd integer
and $n + 4$ = third consecutive odd integer

Set up and solve the related equation.

$$n(n+4) = 7(n+2) - 4$$
$$n^2 + 4n = 7n + 14 - 4$$
$$n^2 - 3n - 10 = 0$$
$$(n-5)(n+2) = 0$$

$$n - 5 = 0 \quad \text{or} \quad n + 2 = 0$$
$$n = 5 \qquad\qquad n = -2$$
$$n + 2 = 7 \qquad n + 2 = 0$$
$$n + 4 = 9 \qquad n + 4 = 2$$

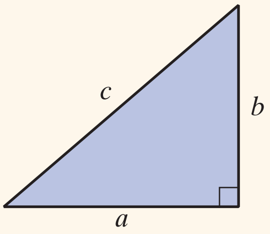
The three consecutive **odd** integers are 5, 7, and 9.

In a **right triangle**, one of the angles is a right angle (measures 90°), and the side opposite this angle (the longest side) is called the **hypotenuse**. The other two sides are called **legs**. The following **Pythagorean theorem** is used to solve problems related to right triangles.

The Pythagorean Theorem

In a right triangle, if c is the length of the hypotenuse and a and b are the lengths of the legs, then

$$\boldsymbol{c^2 = a^2 + b^2}$$



Example 16: Pythagorean Theorem

A flagpole is to have a support cable attached to its top and anchored to the ground at a point that is 17 meters less than the height of the flagpole. If the cable is 1 meter longer than the height of the flagpole, what is the height of the flagpole?

Solution: Let x = height of the flagpole

Then $x - 17$ = distance from base of flagpole to attachment point on ground

And $x + 1$ = length of cable

By the Pythagorean theorem, we have:

$$x^2 + (x-17)^2 = (x+1)^2$$

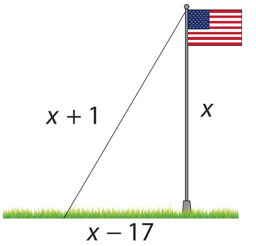
$$x^2 + x^2 - 34x + 289 = x^2 + 2x + 1$$

$$x^2 - 36x + 288 = 0$$

$$(x-12)(x-24) = 0$$

$$x - 12 = 0 \quad \text{or} \quad x - 24 = 0$$

$$x = 12 \qquad\qquad x = 24$$



The height of the flagpole is 24 meters.

(12 meters would lead to a negative value for the distance from the base of the pole.)

Exercises R.6

1. 15 2. 6
3. $6x^2y$ 4. $12a^2b^2$
5. $10a^2bc$ 6. $6xyz^2$
7. $-2x^2(x-1)$
8. $-5a(a^2+25)$
9. $6x^2y(5+8y^2+9y^3)$
10. $12ab^2(2a+3b-4ab)$
11. $(x-3)(x+4)$
12. $(x-5)(x+3)$
13. Not factorable
14. $(m-4)(m+8)$
15. $(a+3)^2$
16. $(x+1)(x+13)$
17. $2(x^2+9x+10)$
18. $3(3x-8)(x+3)$
19. $-(x+5)^2$
20. $5(a^2+25)$
21. $(y+5)(y+1)$
22. $(2x+1)(2x+3)$
23. $(3x-2)(y+7)$
24. $(2x-3)(x+y)$
25. $(x+4)(x^2-4x+16)$
26. $(a-5b^2)(a^2+5ab^2+25b^4)$
27. $-(3x-2)(2x-3)$
28. $-2x(x^2-2x+2)$
29. $(6x^2+5)(6x^2-5)$
30. $(y-3+7x)(y-3-7x)$
31. $(x-2+9y^2)(x-2-9y^2)$
32. Not factorable
33. $a^2(11-12a)(11+12a)$
34. $3x^3(1-4x)(1+4x)$
35. $y(y-6)^2$

Find the GCF for each set of terms.

1. $\{15,\ 45,\ 75\}$ **2.** $\{30,\ 60,\ 96\}$

3. $\{30x^2y,\ 48x^2y^3,\ 54x^2y^4\}$ **4.** $\{24a^3b^2,\ 36a^2b^3,\ 48a^3b^3\}$

5. $\{20a^5bc,\ 30a^3b^2c,\ 40a^2b^2c\}$ **6.** $\{36xyz^2,\ 42x^2yz^3,\ 60xy^2z^2\}$

Completely factor each of the given polynomials. If a polynomial cannot be factored, write "Not factorable."

7. $-2x^3+2x^2$ **8.** $-5a^3-125a$ **9.** $30x^2y+48x^2y^3+54x^2y^4$

10. $24a^2b^2+36ab^3-48a^2b^3$ **11.** x^2+x-12 **12.** $x^2-2x-15$

13. y^2+3y+1 **14.** $m^2+4m-32$ **15.** a^2+6a+9

16. $x^2+14x+13$ **17.** $2x^2+18x+20$ **18.** $9x^2+3x-72$

19. $-x^2-10x-25$ **20.** $5a^2+125$ **21.** $(y+2)^2+2(y+2)-3$

22. $(2x-1)^2+6(2x-1)+8$ **23.** $3xy-2y+21x-14$ **24.** $2x^2+2xy-3x-3y$

25. x^3+64 **26.** a^3-125b^6 **27.** $-6x^2+13x-6$

28. $-2x^3+4x^2-4x$ **29.** $36x^4-25$ **30.** $(y^2-6y+9)-49x^2$

31. $(x^2-4x+4)-81y^4$ **32.** $36x^2+25$ **33.** $121a^2-144a^4$

34. $3x^3-48x^5$ **35.** y^3-12y^2+36y **36.** $8x^3-2x^2-15x$

Solve the equations.

37. $-5x^2=30x$ **38.** $\frac{2x^2}{3}-10=\frac{4x}{3}$ **39.** $(4x+1)(2x-3)=0$

40. $(y-7)(5y-1)=0$ **41.** $\frac{6x^2}{5}=2-\frac{11x}{5}$ **42.** $1.5y^2=4.5y$

43. $8x^2+10x=3$ **44.** $3x^3+27x^2+42x=0$ **45.** $2x^3-14x^2+20x=0$

46. $a^2+2a=-1$ **47.** $(x-5)(x+1)=-5$ **48.** $\frac{1}{6}x^2-\frac{5}{3}x+\frac{25}{6}=0$

49. $(3x+1)(x+1)=18x-7$ **50.** $35x^2=x+6$ **51.** $144x^3=10x^2+50x$

Write a polynomial equation with integer coefficients that has the given roots.

52. $x=6,\ x=3$ **53.** $x=-3,\ x=0,\ x=5$ **54.** $y=-\frac{1}{2},\ y=\frac{2}{5}$

36. $x(4x+5)(2x-3)$
37. $x=-6, 0$
38. $x=-3, 5$
39. $x=-\frac{1}{4}, \frac{3}{2}$
40. $y=\frac{1}{5}, 7$
41. $x=-\frac{5}{2}, \frac{2}{3}$
42. $y=0, 3$
43. $x=-\frac{3}{2}, \frac{1}{4}$
44. $x=-7, -2, 0$
45. $x=0, 2, 5$
46. $a=-1$
47. $x=0, 4$
48. $x=5$
49. $x=\frac{2}{3}, 4$
50. $x=-\frac{2}{5}, \frac{3}{7}$
51. $x=-\frac{5}{9}, 0, \frac{5}{8}$
52. $x^2-9x+18=0$
53. $x^3-2x^2-15x=0$
54. $10y^2+y-2=0$
55. 10, 25 **56.** 11, 12
57. 23, 25 **58.** 12, 18
59. 30 cm, 40 cm
60. 16 feet
61. **a.** 5 seconds
b. 3.5 seconds
c. 2 seconds
62. 20 yards by 25 yards
63. 60 feet
64. Length = 12 in.; Width = 10 in.

55. One number is 15 less than four times another number. Their product is 250. Find the positive numbers.

56. The product of two consecutive positive integers is 132. Find the two integers.

57. The product of two consecutive odd positive integers is 575. What are the integers?

58. The difference between two positive numbers is 6. If twice the smaller is added to the square of the larger, the result is 348. What are the numbers?

59. **Triangles:** In a right triangle the hypotenuse is known to be 50 cm and one of the legs is 10 cm longer than the other. What are the lengths of the legs?

60. **Triangles:** The base of a triangle is 10 feet greater than the height. If the area of the triangle is 48 ft^2, find the length of the base.

61. **Gravity on a ball:** A ball is dropped from the edge of a cliff that is known to be 400 feet above the beach. The formula for finding the height of the ball above the beach in t seconds after it is dropped is $h=400-16t^2$.

a. In how many seconds will the ball hit the beach?

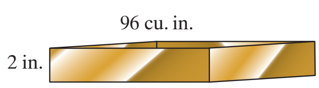
b. How many seconds did it take for the ball to reach a height of 204 feet above the beach?

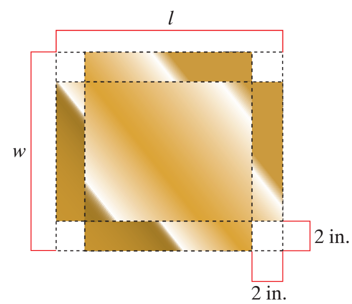
c. How many seconds did it take for the ball to fall 64 feet?

62. **Rectangles:** A rectangle has an area of 500 square yards and a perimeter of 90 yards. What are the dimensions of the rectangle?

63. **Telephone poles:** A telephone pole is to have a guy wire attached at the top and anchored to a point on the ground at a distance of 19 feet less than half the height of the pole. If the wire is to be 1 foot longer than the height of the pole, what is the height of the pole?

64. **Building a box:** A box is formed by cutting squares (2 in. on a side) from each corner of a rectangular piece of metal and folding up the edges. If the volume of the box is 96 in.^3, what were the dimensions of the original rectangle if the length is 2 in. more than the width?





R.7 Chapter 7 Review: Rational Expressions

Section 7.1 Multiplication and Division with Rational Expressions

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The term **rational expression** is the technical name for a fraction in which both the numerator and denominator are polynomials.

Rational Expressions

A **rational expression** is an algebraic expression that can be written in the form

$$\frac{P}{Q} \quad \text{where } P \text{ and } Q \text{ are polynomials and } Q \neq 0.$$

Examples of rational expressions are

$$\frac{5x^3}{4}, \quad \frac{y^2-36}{y^2+36}, \quad \text{and} \quad \frac{x^2+10x+9}{x^2+5x-14}.$$

Remember, the denominator of a rational expression can never be 0. Division by 0 is **undefined**.

The rules for operating with rational expressions are essentially the same as those for operating with fractions in arithmetic. That is, simplifying, multiplying, and dividing rational expressions involve factoring and reducing. Addition and subtraction of rational expressions require common denominators. In particular, the fundamental principle for fractions can be restated as follows.

The Fundamental Principle of Rational Expressions

If $\frac{P}{Q}$ is a rational expression and P, Q, and K are polynomials where $Q, K \neq 0$, then

$$\frac{P}{Q} = \frac{P \cdot K}{Q \cdot K}.$$

Example 1: Reducing Rational Expressions

Use the fundamental principle to reduce each expression to lowest terms. State any restrictions on the variable by using the fact that no denominator can be 0.

a. $\dfrac{4x-12}{5x-15} = \dfrac{4\cancel{(x-3)}}{5\cancel{(x-3)}} = \dfrac{4}{5}$ $\quad (x \neq 3)$

b. $\dfrac{y-15}{15-y} = \dfrac{y-15}{-y+15} = \dfrac{\cancel{y-15}}{-1\cancel{(y-15)}} = \dfrac{1}{-1} = -1$ $\quad (y \neq 15)$

c. $\dfrac{x^3+27}{x^2-9} = \dfrac{\cancel{(x+3)}(x^2-3x+9)}{\cancel{(x+3)}(x-3)} = \dfrac{x^2-3x+9}{x-3}$ $\quad (x \neq 3, -3)$

The following procedure can be used to **multiply** two (or more) rational expressions.

To Multiply Rational Expressions

To multiply any two (or more) rational expressions,

1. Completely factor each numerator and denominator.
2. Multiply the numerators and multiply the denominators, keeping the expressions in factored form.
3. "Divide out" any common factors from the numerators and denominators. Remember that no denominator can have a value of 0.

Example 2: Multiplication with Rational Expressions

Multiply and reduce, if possible. State any restrictions on the variable(s).

a. $\dfrac{y}{y-4} \cdot \dfrac{y^2-16}{y^3} = \dfrac{\cancel{y}(y+4)\cancel{(y-4)}}{\cancel{(y-4)}\underbrace{\cancel{y^3}}_{y^2}} = \dfrac{y+4}{y^2}$ $\quad (y \neq 0, 4)$

b. $\dfrac{5x+5}{x^2-x} \cdot \dfrac{x^2-2x+1}{5x^2+10x+5} = \dfrac{\cancel{5}\cancel{(x+1)}\overbrace{\cancel{(x-1)^2}}^{(x-1)}}{x\cancel{(x-1)} \cdot \cancel{5}\underbrace{\cancel{(x+1)^2}}_{(x+1)}} = \dfrac{x-1}{x(x+1)}$ $\quad (x \neq 0, -1, 1)$

To **divide** any two rational expressions, multiply by the **reciprocal** of the divisor.

Division with Rational Expressions

If $P, Q, R,$ and S are polynomials with $Q, R, S \neq 0$, then

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R}.$$

Note that $\frac{S}{R}$ is the reciprocal of $\frac{R}{S}$.

Example 3: Division with Rational Expressions

Divide and reduce, if possible. Assume that no denominator has a value of 0.

a. $$\frac{20x^2y}{9xy^4} \div \frac{25x^3y^2}{21x^5} = \frac{20x^2y}{9xy^4} \cdot \frac{21x^5}{25x^3y^2}$$
$$= \frac{2 \cdot 2 \cdot \cancel{5} \cdot \cancel{3} \cdot 7 \cdot x^7 \cdot y}{\cancel{3} \cdot 3 \cdot \cancel{5} \cdot 5 \cdot x^4 y^6}$$
$$= \frac{28x^{7-4}y^{1-6}}{15} = \frac{28x^3y^{-5}}{15} = \frac{28x^3}{15y^5}$$

b. $$\frac{x^2 - 8x + 12}{2x^2 - 5x - 3} \div \frac{x^2 - 5x - 6}{2x^2 + 3x + 1} = \frac{x^2 - 8x + 12}{2x^2 - 5x - 3} \cdot \frac{2x^2 + 3x + 1}{x^2 - 5x - 6}$$
$$= \frac{\cancel{(x-6)}(x-2)\cancel{(2x+1)}\cancel{(x+1)}}{\cancel{(2x+1)}(x-3)\cancel{(x-6)}\cancel{(x+1)}}$$
$$= \frac{x-2}{x-3}$$

Section 7.2 Addition and Subtraction with Rational Expressions

page 519

To **add** rational expressions with a common denominator, proceed just as with fractions: add the numerators and keep the common denominator. Reduce if possible.

Addition with Rational Expressions

For polynomials $P, Q,$ and R, with $Q \neq 0$,

$$\frac{P}{Q} + \frac{R}{Q} = \frac{P+R}{Q}.$$

Example 4: Adding Rational Expressions with a Common Denominator

Find the sum and reduce if possible. Assume that no denominator has a value of 0.

$$\frac{1}{x^2+11x+10}+\frac{2x+19}{x^2+11x+10}$$

Solution:

$$\frac{1}{x^2+11x+10}+\frac{2x+19}{x^2+11x+10}=\frac{1+2x+19}{x^2+11x+10}$$

$$=\frac{2x+20}{x^2+11x+10}$$

$$=\frac{2\cancel{(x+10)}}{\cancel{(x+10)}(x+1)}=\frac{2}{x+1} \qquad (x\neq -1,-10)$$

To add rational expressions with different denominators, we need to find the **least common multiple (LCM)** of the denominators. The procedure is stated here for polynomials.

To Find the LCM for a Set of Polynomials

1. Completely factor each polynomial (including prime factors for numerical factors).
2. Form the product of all factors that appear, using each factor the most number of times it appears in any one polynomial.

The LCM of a set of denominators is called the **least common denominator (LCD)**. To add fractions with different denominators, use the fundamental principle to **build each fraction to higher terms** so that they have the LCD as their denominator.

Procedure for Adding Rational Expressions with Different Denominators

1. Find the LCD (the LCM of the denominators).
2. Rewrite each fraction in an equivalent form with the LCD as the denominator.
3. Add the numerators and keep the common denominator.
4. Reduce if possible.

Example 5: Adding Rational Expressions with Different Denominators

Find each sum and reduce if possible. Assume that no denominator has a value of 0.

$$\frac{1}{2x+6}+\frac{1}{x^2+7x+12}$$

Solution: First, find the LCD.

Step 1: Factor each expression completely:

$$2x+6=2(x+3)$$
$$x^2+7x+12=(x+3)(x+4)$$

Step 2: Find the LCM of these factors:

$$\text{LCM} = 2(x+3)(x+4)$$

Now use the LCM and add as follows:

$$\frac{1}{2x+6}+\frac{1}{x^2+7x+12}=\frac{1}{2(x+3)}\cdot\frac{x+4}{x+4}+\frac{1}{(x+3)(x+4)}\cdot\frac{2}{2}$$
$$=\frac{x+4+2}{2(x+3)(x+4)}=\frac{x+6}{2(x+3)(x+4)}$$

To **subtract** rational expressions with a common denominator, proceed just as with fractions: subtract the numerators and keep the common denominator.

Subtracting Rational Expressions

For polynomials P, Q, and R, with $Q \neq 0$,

$$\frac{\boldsymbol{P}}{\boldsymbol{Q}}-\frac{\boldsymbol{R}}{\boldsymbol{Q}}=\frac{\boldsymbol{P}-\boldsymbol{R}}{\boldsymbol{Q}}.$$

Example 6: Subtracting Rational Expressions with a Common Denominator

Find each difference and reduce if possible. Assume that no denominator has a value of 0.

a. $$\frac{y^2}{y^2-4}-\frac{2y+8}{y^2-4}=\frac{y^2-(2y+8)}{y^2-4}=\frac{y^2-2y-8}{y^2-4}=\frac{(y-4)\cancel{(y+2)}}{\cancel{(y+2)}(y-2)}=\frac{y-4}{y-2}$$

b. $$\frac{2x}{x-8}-\frac{16}{8-x}=\frac{2x}{x-8}-\frac{16}{8-x}\cdot\frac{(-1)}{(-1)}=\frac{2x}{x-8}-\frac{-16}{x-8}=\frac{2x-(-16)}{x-8}=\frac{2x+16}{x-8}$$

(Note that $x-8$ and $8-x$ are opposites of each other.)

In subtraction, as with addition, if the rational expressions have different denominators, find the LCM of the denominators (the LCD) and use the fundamental principle to **build each fraction to higher terms** so that each has the LCD as the denominator.

Example 7: Subtracting Rational Expressions with Different Denominators

Find the indicated difference and reduce if possible. Assume that no denominator has a value of 0.

$$\frac{2x}{x^2-9}-\frac{1}{x^2+7x+12}$$

Solution:

$$\left.\begin{aligned} x^2-9&=(x+3)(x-3)\\ x^2+7x+12&=(x+3)(x+4)\end{aligned}\right\}\ \text{LCM} = (x+3)(x-3)(x+4)$$

$$\begin{aligned}\frac{2x}{x^2-9}-\frac{1}{x^2+7x+12}&=\frac{2x(x+4)}{(x+3)(x-3)(x+4)}+\frac{-1(x-3)}{(x+3)(x+4)(x-3)}\\&=\frac{2x^2+8x-x+3}{(x+3)(x-3)(x+4)}\\&=\frac{2x^2+7x+3}{(x+3)(x-3)(x+4)}\\&=\frac{(2x+1)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)(x+4)}\\&=\frac{2x+1}{(x-3)(x+4)}\end{aligned}$$

Section 7.3 Complex Fractions

page 529

A **complex fraction** is a fraction in which the numerator and/or denominator are themselves fractions or the sum or difference of fractions. There are two basic methods for simplifying complex fractions. We discuss only the first method here. The second method can be reviewed in Section 7.3 on page 531.

A complex fraction indicates that the numerator is to be divided by the denominator. This fact is the basis of the method described here.

Simplifying Complex Fractions (First Method)

1. Simplify the numerator so that it is a single rational expression.
2. Simplify the denominator so that it is a single rational expression.
3. Divide the numerator by the denominator and reduce to lowest terms.

Example 8: Simplifying a Complex Fraction

Simplify the complex fraction: $\dfrac{\dfrac{1}{x-2}-\dfrac{1}{x}}{1-\dfrac{2}{x}}$

Solution: $\dfrac{\dfrac{1}{x-2}-\dfrac{1}{x}}{1-\dfrac{2}{x}}=\dfrac{\dfrac{1\cdot x}{(x-2)\cdot x}-\dfrac{1(x-2)}{x(x-2)}}{\dfrac{x}{x}-\dfrac{2}{x}}$

$$=\frac{\dfrac{x-(x-2)}{x(x-2)}}{\dfrac{x-2}{x}}=\frac{\dfrac{x-x+2}{x(x-2)}}{\dfrac{x-2}{x}}$$

$$=\frac{2}{\cancel{x}(x-2)}\cdot\frac{\cancel{x}}{x-2}=\frac{2}{(x-2)^2}$$

A complex algebraic expression is an expression that involves rational expressions and more than one operation. The rules for order of operations apply. The objective is to simplify the expression so that it is written in the form of a **single reduced rational expression**.

Example 9: Simplifying Complex Algebraic Expressions

Simplify the following expression: $\dfrac{7}{3x}-\dfrac{4}{x-5}\div\dfrac{x}{x-5}$

Solution: The rules for order of operations indicate that the division is to be done first.

$$\frac{7}{3x}-\frac{4}{x-5}\div\frac{x}{x-5}=\frac{7}{3x}-\frac{4}{\cancel{x-5}}\cdot\frac{\cancel{x-5}}{x}=\frac{7}{3x}-\frac{4}{x}\cdot\frac{3}{3}=\frac{7-12}{3x}=-\frac{5}{3x}$$

Section 7.4 Solving Equations with Rational Expressions

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A **ratio** is a comparison of two numbers by division. Ratios are written in the form

$$a:b \qquad \text{or} \qquad \frac{a}{b} \qquad \text{or} \qquad a \text{ to } b$$

A **proportion** is a special type of equation stating that two ratios are equal.

Proportion

A **proportion** is an equation stating that two ratios are equal.

In symbols, $\frac{a}{b}=\frac{c}{d}$ is a proportion.

One method of solving proportions with variables is to multiply both sides of the equation by the LCM of the denominators. This method is illustrated in Example 10.

Example 10: Proportions

Solve the proportion: $\frac{4}{x+3}=\frac{2}{x-2}$

Solution:

$$\frac{4}{x+3}=\frac{2}{x-2}$$

$$(x-2)\cancel{(x+3)}\frac{4}{\cancel{x+3}}=\frac{2}{\cancel{x-2}}\cancel{(x-2)}(x+3)$$

$$4(x-2)=2(x+3)$$

$$4x-8=2x+6$$

$$2x=14$$

$$x=7$$

When using proportions to solve word problems, one of the following must be true:

1. The numerators agree in type and the denominators agree in type.
2. The numerators correspond and the denominators correspond.

Example 11: An Application of Proportions

On a cartographer's map 2.5 inches is used to represent 75 miles. What distance is represented by 3.5 inches?

Solution: Let x = the unknown distance

Next, set up a proportion representing the information and solve.

$$\frac{2.5 \text{ inches}}{75 \text{ miles}}=\frac{3.5 \text{ inches}}{x \text{ miles}}$$

$$\cancel{75}x\cdot\frac{2.5}{\cancel{75}}=\frac{3.5}{\cancel{x}}\cdot 75\cancel{x}$$

$$2.5x=262.5$$

$$x=105$$

On this map, 3.5 inches represents 105 miles.

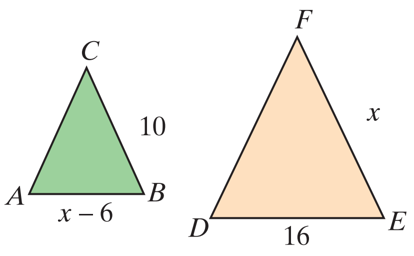
Proportions are used when working with similar geometric figures. **Similar figures** are figures that meet the following two conditions:

1. The measures of the corresponding angles are equal.
2. The lengths of corresponding sides are proportional.

In **similar triangles**, corresponding sides are those sides opposite equal angles. As illustrated in Example 12, proportions can be used to find the length of an unknown side (or unknown sides) when similar triangles are involved.

Example 12: Proportions and Similar Triangles

In the figure shown, $\Delta ABC \sim \Delta DEF$. (The symbol $\sim$ is read "is similar to".) Find the lengths of the sides $\overline{AB}$ and $\overline{EF}$.



Solution: Set up a proportion involving corresponding sides and solve for x.

$$\frac{10}{x} = \frac{x-6}{16}$$

$$16\cancel{x} \cdot \frac{10}{\cancel{x}} = \frac{x-6}{\cancel{16}} \cdot \cancel{16}x$$

$$16 \cdot 10 = x(x-6)$$

$$160 = x^2 - 6x$$

$$0 = x^2 - 6x - 160$$

$$0 = (x-16)(x+10)$$

$$x - 16 = 0 \qquad x + 10 = 0$$

$$x = 16 \qquad x = -10$$

Because the length of a side must be positive,

$$x = EF = 16 \text{ and } x - 6 = AB = 10.$$

A general approach to solving equations that contain rational expressions is as follows.

To Solve an Equation Containing Rational Expressions

1. Find the LCD of the fractions.
2. Multiply both sides of the equation by this LCD and simplify.
3. Solve the resulting equation. (This equation will have only polynomials on both sides.)
4. Check each solution in the **original equation**. (Remember that no denominator can be 0 and any solution that gives a 0 denominator is to be discarded.)

Multiplying by the LCD may introduce solutions that are not solutions to the original equation. Such solutions are called **extraneous solutions** or **extraneous roots** and occur because multiplication by a variable expression may, in effect, be multiplying the original equation by 0.

Example 13: Solving Equations Involving Rational Expressions

State any restrictions on the variable, and then solve the equation.

$$\frac{1}{x^2-4x}=\frac{3}{x^2-16}$$

Solution: First find the LCM of the denominators, and then multiply both sides of the equation by the LCM.

$$\left.\begin{aligned} x^2-4x&=x(x-4)\\ x-16&=(x+4)(x-4)\end{aligned}\right\rangle \text{LCM}=x(x+4)(x-4)$$

$$\frac{1}{x^2-4x}=\frac{3}{x^2-16} \qquad (x\neq -4,0,4)$$

$$\cancel{x}(x+4)\cancel{(x-4)}\cdot\frac{1}{\cancel{x}\cancel{(x-4)}}=\frac{3}{\cancel{(x+4)}\cancel{(x-4)}}\cdot x\cancel{(x+4)}\cancel{(x-4)}$$

$$x+4=3x$$

$$4=2x$$

$$2=x$$

Since 2 is not a restriction, 2 is the solution.

Section 7.5 Applications

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The first type of application discussed in this section is related to fractions represented by rational expressions.

Example 14: Fractions

The numerator of a fraction is 6 less than the denominator. If both the numerator and denominator are increased by 4, the new fraction is equal to $\frac{4}{7}$. Find the original fraction.

Solution: Let n = original denominator
Then $n-6$ = original numerator

$$\frac{n-6}{n} = \text{original fraction}$$

$$\frac{n-6+4}{n+4} = \frac{4}{7}$$

$$\frac{n-2}{n+4} = \frac{4}{7}$$

$$7\cancel{(n+4)} \cdot \frac{n-2}{\cancel{n+4}} = \frac{4}{\cancel{7}} \cdot \cancel{7}(n+4)$$

$$7(n-2) = 4(n+4)$$

$$7n - 14 = 4n + 16$$

$$3n = 30$$

$$n = 10$$

The original fraction is $\frac{n-6}{n} = \frac{10-6}{10} = \frac{4}{10}$.

Problems involving work usually translate into equations involving rational expressions. As illustrated in Example 15, the basic idea is **to represent what part of the work is done in one unit of time**.

Example 15: Work

Ryan found that he could do the landscaping maintenance for a group of neighborhood homes in 4 hours. One of his helpers did the same work in 6 hours. Ryan decided that they could save time by working together. How long would it take them to do this job by working together?

Solution: Let x = number of hours to do the job working together.

$$\underbrace{\frac{1}{4}}_{\substack{\text{Part done in}\\ \text{1 hour by Ryan}}} + \underbrace{\frac{1}{6}}_{\substack{\text{Part done in}\\ \text{1 hour by helper}}} = \underbrace{\frac{1}{x}}_{\substack{\text{Part done in}\\ \text{1 hour together}}}$$

$$\overset{3}{\cancel{12}}\,x\cdot\frac{1}{\cancel{4}}+\overset{2}{\cancel{12}}\,x\cdot\frac{1}{\cancel{6}}=12\cancel{x}\cdot\frac{1}{\cancel{x}}$$

$$3x+2x=12$$

$$5x=12$$

$$x=\frac{12}{5}$$

Working together, Ryan and his helper would take $\frac{12}{5}$ hours (or $2\frac{2}{5}$ hours) to do the maintenance work.

The basic formula involving distance, rate, and time is $d = rt$. This relationship can also be stated in the forms $t=\frac{d}{r}$ and $r=\frac{d}{t}$.

Example 16: Distance-Rate-Time

A small jet plane travels twice as fast as a twin engine plane. If the twin engine plane takes 2 hours longer to fly 600 miles, what is the speed of each plane?

Solution: Let r = rate of speed of twin engine plane
$2r$ = rate of jet plane

$$\underbrace{\frac{600}{r}}_{\substack{\text{Time for}\\ \text{twin engine}}} - \underbrace{\frac{600}{2r}}_{\substack{\text{Time for}\\ \text{jet}}} = \underbrace{2}_{\substack{\text{Difference in}\\ \text{time}}}$$

$$2\cancel{r}\cdot\frac{600}{\cancel{r}}-\cancel{2r}\cdot\frac{600}{\cancel{2r}}=2r\cdot 2$$

$$1200-600=4r$$

$$600=4r$$

$$150=r$$

The twin engine plane flies at 150 mph and the jet flies at 300 mph.

Section 7.6 Variation

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If a car is driven at a steady rate of 55 mph, then the distance the car would travel could be indicated using the formula $d = 55t$ where d is the distance traveled in miles and t is the time in hours. We say that distance and time **vary directly** (or are in **direct variation** or are **directly proportional**). In this example, 55 is constant and is called the **constant of variation**. When two variables vary directly, an increase in the value of one variable indicates an increase in the other, and the ratio of the two quantities is constant.

Direct Variation

A variable quantity y **varies directly as** (or is **directly proportional to**) a variable x if there is a constant k such that

$$\frac{\boldsymbol{y}}{\boldsymbol{x}} = \boldsymbol{k} \text{ or } \boldsymbol{y} = \boldsymbol{kx}.$$

The constant k is called the **constant of variation**.

Example 17: Direct Variation

The length that a hanging spring stretches varies directly as the weight placed on the end of the spring. If a weight of 20 mg stretches a certain spring 14 cm, how far will the spring stretch with a weight of 25 mg?

Solution: $d = kw$ This is an application of Hooke's Law.

$$14 = k \cdot 20$$

$$\frac{14}{20} = k$$

$$\frac{7}{10} = k$$

So, $d = \frac{7}{10}w$.

Thus if $w = 25$ mg, then $d = \frac{7}{10} \cdot 25 = \frac{175}{10} = 17.5$ cm.

When two variables vary in such a way that their product is constant, we say that the two variables **vary inversely** (or are **inversely proportional**). Note that if a product of two variables is to remain constant, then an increase in the value of one variable must be accompanied by a decrease in the other.

Inverse Variation

A variable quantity y **varies inversely as** (or is **inversely proportional to**) a variable x if there is a constant k such that

$$\boldsymbol{x \cdot y = k} \text{ or } \boldsymbol{y = \frac{k}{x}}.$$

The constant k is called the **constant of variation**.

Example 18: Inverse Variation

If y varies inversely as the square of x, and $y = 200$ when $x = 10$, find y when $x = 5$.

Solution:

$$y = \frac{k}{x^2} \quad \text{General formula for inverse variation}$$

$$200 = \frac{k}{10^2}$$

$$200 = \frac{k}{100}$$

$$20{,}000 = k$$

So, $y = \frac{20{,}000}{x^2}$. Thus if $x = 5$, then $y = \frac{20{,}000}{5^2} = \frac{20{,}000}{25} = 800$.

If a variable varies either directly or inversely with more than one other variable, the variation is said to be a **combined variation**. If the combined variation is all direct variation, then it is called **joint variation**.

Example 19: Variation

If z varies jointly as the square of x and the cube of y, and $z = 200$ when $x = 10$ and $z = 2$, find z when $x = 5$ and $y = 4$.

Solution:

$$z = kx^2y^3$$

$$200 = k \cdot 10^2 \cdot 2^3$$

$$200 = 800k$$

$$\frac{1}{4} = k$$

So, $z = \frac{1}{4}x^2y^3$. Thus if $x = 5$ and $y = 4$, then $z = \frac{1}{4} \cdot 5^2 \cdot 4^3 = \frac{1}{4} \cdot 25 \cdot 64 = 400$.

R.7 Exercises

Reduce to lowest terms. State any restrictions on the variable(s).

1. $\dfrac{y^2+y-2}{y^2+2y}$ **2.** $\dfrac{3x+9}{5x^2-45}$ **3.** $\dfrac{a^3+3a^2-4a}{2a^2+12a+16}$

4. $\dfrac{2x^3-128}{6x^2-96}$ **5.** $\dfrac{5y^3+625}{10y^2+20y-150}$

Perform the indicated operations and reduce to lowest terms. Assume that no denominator has a value of zero.

6. $\dfrac{5x}{x+y}\cdot\dfrac{3x+3y}{10x^2}$ **7.** $\dfrac{2x+12}{16x}\cdot\dfrac{8x^3}{x^2-36}$ **8.** $\dfrac{2x+5}{x-3}\div\dfrac{2x^2+3x-5}{3x-3}$

9. $\dfrac{y^3-8}{10y}\div\dfrac{y^2-4y+4}{20y}$ **10.** $\dfrac{2x+3}{x-4}\div\dfrac{2x^2-7x-15}{3x-12}$ **11.** $\dfrac{y}{y+5}+\dfrac{15-y}{y+5}$

12. $\dfrac{x}{x-5}-\dfrac{15-2x}{x-5}$ **13.** $\dfrac{3}{x}+\dfrac{5}{x^2}-\dfrac{2}{x}$ **14.** $\dfrac{x-4}{2x-1}-1$

15. $\dfrac{x-7}{2x+1}+1$ **16.** $\dfrac{4}{a+4}+\dfrac{3}{a+1}-\dfrac{1}{a^2+5a+4}$ **17.** $\dfrac{6}{2a+1}-\dfrac{5}{2a-1}-\dfrac{2a}{4a^2-1}$

Simplify the following complex fractions.

18. $\dfrac{2-\frac{5}{x}}{3+\frac{5}{x}}$ **19.** $\dfrac{\frac{1}{3x}+\frac{1}{6x}}{\frac{1}{x^2}-\frac{1}{x}}$ **20.** $\dfrac{x^{-1}+x^{-2}}{x^{-1}-x^{-2}}$ **21.** $\dfrac{\frac{1}{a}-\frac{1}{3}}{\frac{a-3}{a}}$ **22.** $\dfrac{1-\frac{y}{x}}{\frac{x^2}{y^2-x^2}}$

23. $\dfrac{1}{x+2}-\dfrac{5}{3x}\cdot\dfrac{6x}{x+2}$ **24.** $\dfrac{7}{x}+\dfrac{2}{x^2-3x}\cdot\dfrac{x-3}{4}$ **25.** $\dfrac{x+1}{x-2}+\dfrac{x}{x-2}\div\dfrac{x^2}{x+5}$

Solve each of the equations. Assume no denominator has a value of 0.

26. $\dfrac{6}{a}=\dfrac{39}{42}$ **27.** $\dfrac{3}{10}=\dfrac{y}{200}$ **28.** $\dfrac{24}{x+3}=\dfrac{6}{7}$ **29.** $\dfrac{x+13}{38}=\dfrac{4}{19}$

30. $\dfrac{5}{x+3}=\dfrac{4}{x-5}$ **31.** $\dfrac{8}{a+1}=\dfrac{4}{a-2}$ **32.** $\dfrac{3y}{y+2}=\dfrac{2}{y-3}$ **33.** $\dfrac{4x}{x-5}=\dfrac{-20}{x-5}$

Answers

1. $\dfrac{y-1}{y}$, $y\neq-2, 0$
2. $\dfrac{3}{5(x-3)}$, $x\neq-3, 3$
3. $\dfrac{a(a-1)}{2(a+2)}$, $a\neq-4, -2$
4. $\dfrac{x^2+4x+16}{3(x+4)}$, $x\neq-4, 4$
5. $\dfrac{y^2-5y+25}{2(y-3)}$, $x\neq-5, 3$
6. $\dfrac{3}{2x}$
7. $\dfrac{x^2}{x-6}$
8. $\dfrac{3}{x-3}$
9. $\dfrac{2y^2+4y+8}{y-2}$
10. $\dfrac{3}{x-5}$
11. $\dfrac{15}{y+5}$
12. 3
13. $\dfrac{x+5}{x^2}$
14. $\dfrac{-x-3}{2x-1}$
15. $\dfrac{3x-6}{2x+1}$
16. $\dfrac{7a+15}{(a+1)(a+4)}$
17. $\dfrac{-11}{4a^2-1}$
18. $\dfrac{2x-5}{3x+5}$
19. $\dfrac{x}{2(1-x)}$
20. $\dfrac{x+1}{x-1}$
21. $-\dfrac{1}{3}$

22. $\dfrac{-x^3+x^2y+xy^2-y^3}{x^3}$

23. $\dfrac{-9}{x+2}$ **24.** $\dfrac{15}{2x}$

25. $\dfrac{x^2+2x+5}{x(x-2)}$

26. $a=\dfrac{84}{13}$

27. $y=60$

28. $x=25$

29. $x=-5$

30. $x=37$

31. $a=5$

32. $y=-\dfrac{1}{3}, 4$

33. $x=-5$

34. $t=\dfrac{12}{7}$

35. $t=3$

36. $x=-6, 7$

37. $x=0, 8$

38. $x=-6, 6$

39. $x=0$

40. $AB=6, QR=16$

41. $AB=15, CB=17.5$

42. 3.75 inches

43. 75 feet

44. Woman = 4.5 hours
Daughter = 9 hours

45. $\dfrac{3}{10}$

46. $\dfrac{40}{13}$ or 3.08 hours

47. 2 mph

34. $\dfrac{1}{3}+\dfrac{1}{4}=\dfrac{1}{t}$

35. $\dfrac{1}{2}-\dfrac{1}{6}=\dfrac{1}{t}$

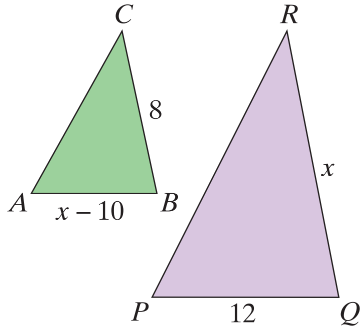
36. $\dfrac{3x}{x+2}+\dfrac{5}{x-4}=4$

37. $\dfrac{2x}{x+3}-\dfrac{5}{2x-5}=1$

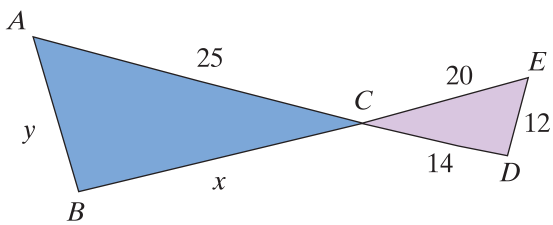
38. $\dfrac{1}{x-5}-1=\dfrac{x-6}{x^2-25}$

39. $\dfrac{6}{x+7}-\dfrac{6}{7-x}=0$

40. In the figure shown, $\Delta ABC \sim \Delta PQR$. Find the lengths of the sides $\overline{AB}$ and $\overline{QR}$. (Remember that in similar triangles, corresponding sides are proportional.)



41. In the figure shown, $\Delta ABC \sim \Delta EDC$. Find the lengths of the sides $\overline{AB}$ and $\overline{CB}$.



42. **Architecture:** An architect plans to make a drawing of a building that uses a scale of 1.5 inches to 20 feet. If two points on the building are known to be 50 feet apart, how many inches apart on the drawing should these two points be?

43. **Construction:** To estimate the height of an office building, an engineer (6 feet tall) notices that his shadow is 2 feet long. At that same time, he measures the length of the shadow of the building to be 25 feet long. What is his estimate of the height of the building?

44. **Waxing a car:** A woman can wax her car twice as fast as her daughter can. Together they can complete the job in 3 hours. How long would it take each of them working alone?

45. The denominator of a fraction is four more than twice the numerator. If eleven is added to both the numerator and the denominator, the resulting fraction is equal to $\dfrac{2}{3}$. Find the original fraction.

46. **Swimming pools:** A swimming pool has two inlet pipes. One of the pipes alone can fill the pool in 5 hours and the other, alone, can fill the pool in 8 hours. How long would it take to fill the pool if both pipes were open?

47. **Canoeing:** A woman can paddle her canoe 6 mph on a lake. On a river, it takes her the same time to paddle 8 miles downstream as it does to paddle 4 miles upstream. What is the speed of the current of the river in miles per hour?

48. Speed of private plane is 150 mph; Speed of commercial airliner is 350 mph

49. $\frac{5}{3}$ hours or $1\frac{2}{3}$ hours

50. 81

51. 15

52. 11.25 fc

53. $\frac{60}{13}$ or 4.62 cm

54. 144 feet

48. Airlines: A commercial airliner can travel 700 miles in the same time that it takes a private plane to travel 300 miles. The speed of the airliner is 50 mph faster than twice the speed of the private plane. Find the speed of each aircraft.

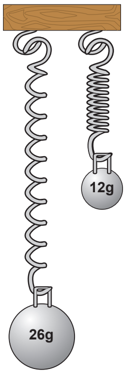
49. Boating: Greg travels 10 miles downriver and returns. It takes 2.5 hours to make the round trip. His rate in still water is three times the rate of the current. How long does the return trip take?

50. If y varies directly as the cube of x, and $y = 375$ when $x = 5$, find y when $x = 3$.

51. If z varies directly as x and inversely as the square of y, and $z = 72$ when $x = 6$ and $y = 2$, find z when $x = 5$ and $y = 4$.

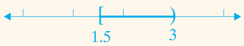
52. Illumination: The illumination (in foot-candles, fc) of a light source varies inversely as the square of the distance from the source. If a certain light source provides an illumination of 20 fc at a distance of 30 ft, what is the illumination at a distance of 40 ft?

53. Springs: The distance a spring stretches is directly proportional to the weight on the end of the spring. If a hanging spring stretches 10 cm when a weight of 26 g is placed at its end, how far will the spring stretch if a weight of 12 g is placed at its end?

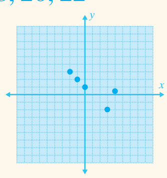


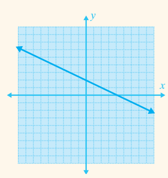
54. Falling objects: The distance an object falls (in feet) varies directly as the square of the time (in seconds) that it falls. If an object falls 64 feet in two seconds, how far will it fall in 3 seconds?

Chapter R Test

1. $x = -4$
2. $y = \frac{15}{2}$
3. $x = -16, 0$
4. $y = -\frac{1}{2}$
5. $x = -9, -\frac{1}{3}$
6. Contradiction
7. Identity
8. 

9. $[-1.1, 7.4]$


10. $r = \frac{A-P}{Pt}$
11. a. 36.56 cm
b. 89.12 cm^2
12. 18, 20, 22
13. a.


b. Domain $= \{-2, -1, 0, 3, 4\}$
c. Range $= \left\{-2, \frac{1}{2}, 1, 2, 3\right\}$
d. Yes, because it passes the vertical line test
14. a. $y = -\frac{1}{2}x + 2$
b.


Solve the following equations.

1. $4.4 + 0.6x = 1.2 - 0.2x$
2. $\frac{1}{2} + \frac{1}{5}y = \frac{2}{15}y + 1$
3. $|0.5x + 4| = 4$
4. $|6y + 3| = 0$
5. $|4 - x| = |5 + 2x|$

Determine whether each of the following equations is conditional, an identity, or a contradiction.

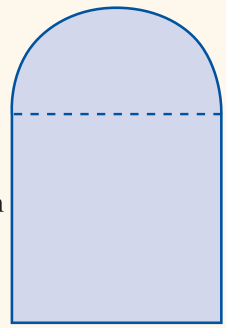
6. $3(x - 7) = 4x - (3 + x)$
7. $5x + 14 - 6 = 2(x + 1) + 3(x + 2)$

8. Graph the following set of real numbers on a real number line:
$\{x \mid x < 3 \text{ and } x \geq 1.5\}$

9. Solve the inequality and graph the solution set on a real number line:
$-6.2 \leq 2x - 4 \leq 10.8$

10. Solve the formula for r: $A = P + Prt$.

11. Find **a.** the perimeter and **b.** the area of the figure shown here. (Use $\pi = 3.14$.)


8 cm
8 cm

12. Find three consecutive even integers such that the sum of the first and twice the second is equal to 14 more than twice the third.

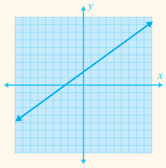
13. a. Graph the following set of ordered pairs: $\left\{(0, 1), (3, -2), (-2, 3), (-1, 2), \left(4, \frac{1}{2}\right)\right\}$
b. State the domain of the relation.
c. State the range of the relation.
d. Is the relation a function? Why or why not?

14. a. Find the equation in slope-intercept form of the line that passes through the point $(-2, 3)$ with slope $m = -\frac{1}{2}$.
b. Graph the line.

15. a. Find the equation in standard form of the line that passes through the two points $(3, 4)$ and $(-5, -2)$.
b. Graph the line.

15. a. $3x - 4y = -7$

b.



16. a. 52 **b.** 4

17. $AB = 6, EF = 15$

18. Length = 14 m; Width = 8 m

19. Joe = 1.5 hours; Bruce = 3 hours

20. 15 ohms

21. $\dfrac{(1.4\times10^{-2})(9.22\times10^{2})}{(3.5\times10^{3})(2\times10^{5})}$; 1.844×10^{-8}

22. $-2x^2 - x - 17$

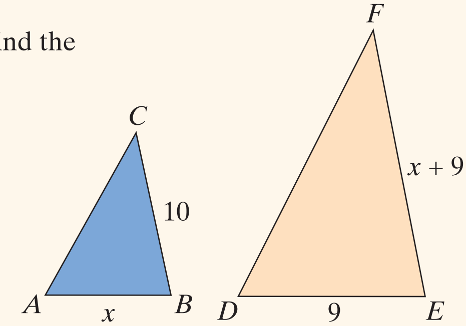
23. $5x^2 + 33x + 18$

24. $4x^2 + 20x + 25$

25. $2x^2 - x + 3 - \dfrac{2}{x+3}$

16. For the function $f(x) = 3x^2 + 2x - 4$, find **a.** $f(4)$ **b.** $f(-2)$.

17. In the figure shown, $\Delta ABC \sim \Delta DEF$. Find the lengths of $\overline{AB}$ and $\overline{EF}$.



18. Rectangles: The length of a rectangle is 10 meters more than one-half the width. If the perimeter is 44 meters what are the length and width?

19. Mowing the lawn: Joe can mow his lawn in half the time it takes his son Bruce. Together (using two mowers) they can do the job in 1 hour. How long does it take each of them working alone to mow the lawn?

20. Resistance in a wire: The resistance R (in ohms) in a wire is directly proportional to the length L of a wire and inversely proportional to the square of the diameter d of the wire. A wire is 240 ft long with a diameter of 0.01 in. has a resistance of 12 ohms. What is the resistance of a piece of the same type of wire with a length of 675 ft and a diameter of 0.015 in.?

21. Write each number in scientific notation, simplify the expression, and write the answer in scientific notation.

$$\frac{0.014(922)}{3500(200{,}000)}$$

Perform the indicated operations and simplify.

22. $(2x^2 - 5x - 7) - (5x^2 - 4x + 1) + (x^2 - 9)$

23. $(x+6)(5x+3)$

24. $(2x+5)^2$

25. Divide using long division and write the answer in the form $Q + \dfrac{R}{D}$.

$$\frac{2x^3 + 5x^2 + 7}{x+3}$$

26. $(5x)(x-1)(x-2)$
27. $(2x-5)(2x+3)$
28. $3(x+2)^2$
29. $2(3y-5)(9y^2+15y+25)$
30. $(x^2+5)(x^4-5x^2+25)$
31. $5x^2+18x-8=0$
32. $4x^3-7x^2-2x=0$
33. $x=-\frac{5}{3}, 2$
34. $x=-5, 5$
35. 5, 7
36. $\frac{5x}{2}$
37. 1
38. $\frac{5x^2+18x}{(x-3)(x+3)(x+4)}$
39. $x=-9, 2$
40. $x=0$
41. John = 60 mph; Bill = 45 mph
42. 352.8 m
43. a. – d. Answers will vary.
44. 4 hours and 12 hours
45. $\frac{35}{12}$ hours or $2\frac{11}{12}$ hours
46. 90 minutes

Completely factor each of the following expressions.

26. $5x^3-15x^2+10x$ **27.** $4x^2-4x-15$ **28.** $3x^2+12x+12$

29. $54y^3-250$ **30.** x^6+125

Find a polynomial equation with integer coefficients that has the given roots.

31. $x=-4, x=\frac{2}{5}$ **32.** $x=0, x=-\frac{1}{4}, x=2$

Solve the following equations.

33. $3x(x-1)=2(5-x)$ **34.** $3x^2-75=0$

35. One positive number is three less than twice another. The sum of their squares is 74. Find the numbers.

Perform the indicated operations and simplify.

36. $\frac{x^2+5x+6}{2x+4}\cdot\frac{5x}{x+3}$ **37.** $\frac{x^2-4}{x^2-5x+6}\div\frac{x^2+3x+2}{x^2-2x-3}$ **38.** $\frac{2x}{x^2+x-12}+\frac{3x}{x^2-9}$

Solve the following equations.

39. $\frac{12}{x+1}+1=\frac{5}{x-1}$ **40.** $\frac{2x}{x-4}+\frac{2}{x+1}=2$

41. Traveling: John travels 15 mph faster than Bill. John can travel 180 miles in the same amount of time that Bill can travel 135 miles. Find the two rates.

42. Falling objects: The distance a free falling object falls is directly proportional to the square of the time it falls (before it hits the ground). If an object falls 156.8 m in 4 seconds, how far will it have fallen by the end of 6 seconds?

43. Discuss, in your own words, the following formulas and ideas related to factoring
a. the difference of squares,
b. the sum of squares,
c. the difference of cubes, and
d. the sum of cubes.

44. Filling a tank: Two hoses, one of which has a flow-rate 3 times the other, can together fill a tank in 3 hours. How long does it take each of the hoses to individually fill the tank?

45. Painting: If Reneé were painting the living room alone, it would take 5 hours. Her sister Phyllis could do the job in 7 hours. How long would it take them working together?

46. Swimming pools: A swimming pool has one inlet pipe and one outlet pipe. The outlet pipe that can empty the pool in 45 minutes and inlet pipe can fill the pool in 30 minutes. If both are left open, how long does it take for the pool to fill?