CHAPTER R Review of Basic Topics

Exponents:

$\underbrace{a \cdot a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}} = a^n$

Order of Operations:

Please	**Parentheses** (start with the innermost grouping)	
Excuse	**Exponents**	
My	**Multiplication**	moving from left to right
Dear	**Division**	
Aunt	**Addition**	moving from left to right
Sally	**Subtraction**	

Tests for Divisibility:

An integer is divisible
By 2: if the units digits is 0, 2, 4, 6, or 8.
By 3: if the sum of the digits is divisible by 3.
By 5: if the units digit is 0 or 5.
By 6: if the number is divisible by both 2 and 3.
By 9: if the sum of the digits is divisible by 9.
By 10: if the units digit is 0.

Fractions:

Multiplication: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

Reciprocal: If $a \neq 0$ and $b \neq 0$, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$, and $\frac{a}{b} \cdot \frac{b}{a} = 1$.

Division: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$; $\frac{a}{0}$ is **undefined**

Addition: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$; If the fractions do not have a common denominator, first find the LCD, convert the two fractions to equivalent fractions with the LCD for the denominators, then add.

Subtraction: $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$; If the fractions do not have a common denominator, first find the LCD, convert the two fractions to equivalent fractions with the LCD for the denominators, then subtract.

Finding the LCM/LCD:

1. Find the prime factorization of each number.
2. List the prime factors that appear in any one of the prime factorizations.
3. Find the product of these primes using each prime the greatest number of times it appears in any one of the prime factorizations.

Decimals:

Addition/Subtraction:

1. Write the numbers in a vertical column.
2. Keep the decimal points aligned vertically.
3. Keep digits with the same position value aligned.
4. Add/subtract the numbers, keeping the decimal point in the sum/difference aligned with the other decimal points.

Multiplication:

1. Multiply the two numbers as if they were whole numbers.
2. Count the total number of places to the right of the decimal points in both numbers being multiplied.
3. Place the decimal point in the product so that the number of places to the right is the same as that found in step 2.

Division:

1. Move the decimal point in the divisor to the right so that the divisor is a whole number.
2. Move the decimal point in the dividend the same number of places to the right.
3. Place the decimal point in the quotient directly above the new decimal point in the dividend.
4. Divide just as with whole numbers.

Percent:

To Change a Decimal to a Percent:

1. Move the decimal point two places to the right.
2. Add the % symbol.

To Change a Percent to a Decimal:

1. Move the decimal point two places to the left.
2. Remove the % symbol.

CHAPTER 1 Integers and Real Numbers

Types of Numbers:

Natural Numbers (Counting Numbers): $\mathbb{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$
Whole Numbers: $\mathbb{W} = \{0, 1, 2, 3, 4, \ldots\}$
Integers: $\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$
Rational Numbers: A **rational number** is any number of the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.
Irrational Numbers: An **irrational number** is any number that can be written as an infinite nonrepeating decimal.
Real Numbers: Every rational number and irrational number is a **real number**.

Inequality Symbols:

$<$ "is less than"
$>$ "is greater than"
$\leq$ "is less than or equal to"
$\geq$ "is greater than or equal to"

Absolute Value:

$|a| = a$ if a is a positive number or 0.
$|a| = -a$ if a is a negative number.

Operations with Integers:

For real numbers a and b, $a - b = a + (-b)$.
For positive numbers a and b, $a(-b) = (-a)b = -ab$ and $(-a)(-b) = ab$.
For real numbers a and b $(b \neq 0)$, $\frac{a}{b} = x$ **if and only if** $a = b \cdot x$.

Properties of Real Numbers:

Addition	Name of property	Multiplication
$a + b = b + a$	Commutative	$a \cdot b = b \cdot a$
$(a + b) + c = a + (b + c)$	Associative	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
$a + 0 = 0 + a = a$	Identity	$a \cdot 1 = 1 \cdot a = a$
$a + (-a) = 0$	Inverse	$a \cdot \frac{1}{a} = 1 \ (a \neq 0)$

Zero Factor Law: $a \cdot 0 = 0 \cdot a = 0$
Distributive Property: $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$

CHAPTER 2 Fractions with Variables and Algebraic Expressions

Evaluating an expression:

1. Combine like terms, if possible.
2. Substitute the values given for any variables.
3. Follow the rules for order of operations and simplify.

Fractions: See Chapter R

Placement of Negative Signs: $-\dfrac{a}{b} = \dfrac{-a}{b} = \dfrac{a}{-b}$

The Fundamental Principle of Fractions:

$\dfrac{a}{b} = \dfrac{a \cdot k}{b \cdot k}$ where $b, k \neq 0$

Decimals: See Chapter R

Order of Operations: See Chapter R

CHAPTER 3 Solving Equations and Inequalities

Linear Equation in *x* (first-degree equation in *x*):

$ax + b = c$, where $a \neq 0$

Addition Principle of Equality:

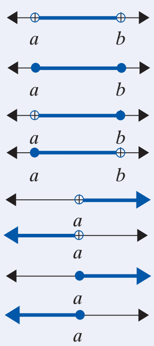
$A = B$ and $A + C = B + C$ have the same solutions (where A, B, and C are algebraic expressions).

Multiplication (or Division) Principle of Equality:

$A = B$ and $AC = BC$ and $\dfrac{A}{C} = \dfrac{B}{C}$

Intervals of Real Numbers:

Name	Symbolic Representation	Graph
Open Interval	$a < x < b$	
Closed Interval	$a \le x \le b$	
Half-Open Interval	$\begin{cases} a<x\le b \\ a\le x<b \end{cases}$	
Open Interval	$\begin{cases} x>a \\ x<a \end{cases}$	
Half-Open Interval	$\begin{cases} x\ge a \\ x\le a \end{cases}$	



Consecutive Integers:

$n, n+1, n+2, \ldots$

Consecutive Odd Integers:

$n, n+2, n+4, \ldots$ where n is an odd integer

Consecutive Even Integers:

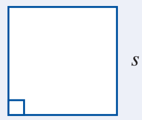
$n, n+2, n+4, \ldots$ where n is an even integer

Percent Formula:

$R \cdot B = A$

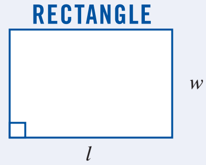
Geometric Formulas:

SQUARE



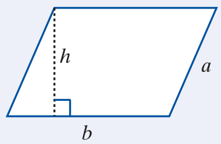
Perimeter: $P = 4s$
Area: $A = s^2$

RECTANGLE



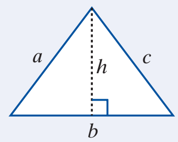
Perimeter: $P = 2l + 2w$
Area: $A = lw$

PARALLELOGRAM



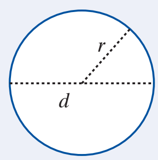
Perimeter: $P = 2a + 2b$
Area: $A = bh$

TRIANGLE



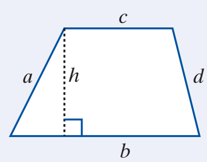
Perimeter: $P = a + b + c$
Area: $A = \dfrac{1}{2}bh$

CIRCLE



Circumference: $C = 2\pi r = \pi d$
Area: $A = \pi r^2$

TRAPEZOID



Perimeter: $P = a + b + c + d$
Area: $A = \dfrac{1}{2}h(b + c)$

Volume Formulas:

Rectangular Solid: $V = lwh$

Rectangular Pyramid: $V = \dfrac{1}{3}lwh$

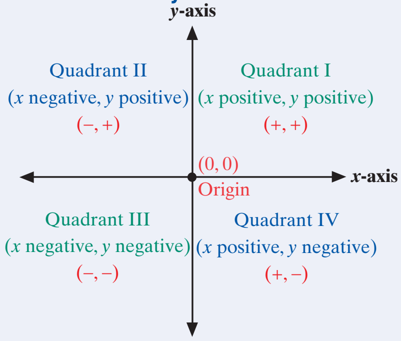
Right Circular Cylinder: $V = \pi r^2 h$

Right Circular Cone: $V = \dfrac{1}{3}\pi r^2 h$

Sphere: $V = \dfrac{4}{3}\pi r^3$

CHAPTER 4 Graphing Linear Equations and Inequalities in Two Variables

Cartesian Coordinate System:



Summary of Formulas and Properties of Straight Lines:

1. $Ax + By = C$, where A and B do not equal 0. — Standard form
2. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$. — Slope of a line
3. $y = mx + b$ — Slope-intercept form
4. $y - y_1 = m(x - x_1)$ — Point-slope form
5. $y = b$ — Horizontal line, $m = 0$
6. $x = a$ — Vertical line, m is undefined
7. Parallel lines have the same slope $(m_1 = m_2)$.
8. Perpendicular lines have slopes that are negative reciprocals of each other $\left(m_2 = \dfrac{-1}{m_1} \text{ or } m_1 m_2 = -1\right)$.

Relation, Domain, and Range:

A **relation** is a set of ordered pairs of real numbers.

The **domain**, ***D***, of a relation is the set of all first coordinates in the relation.

The **range**, ***R***, of a relation is the set of all second coordinates in the relation.

Function:

A **function** is a relation in which each domain element has a unique range element.

Vertical Line Test:

If **any** vertical line intersects the graph of a relation at more than one point, then the relation graphed is **not** a function.

Linear Inequality Terminology:

Half-plane: A straight line separates a plane into two **half-planes**.

Boundary line: The line itself is called the **boundary line**.

Closed half-plane: If the boundary line is included, then the half-plane is said to be **closed**.

Open half-plane: If the boundary line is not included, then the half-plane is said to be **open**.

Graphing Linear Inequalities:

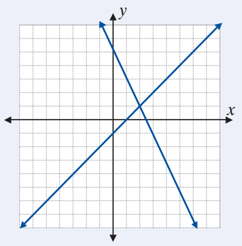
1. Graph the boundary line, dashed if the inequality is < or >, solid if the inequality is ≤ or ≥.

2. a. Method 1: Test any point obviously on one side of the line. If the test point satisfies the inequality, shade the half-plane on the same side of the line. Otherwise, shade the other half-plane.

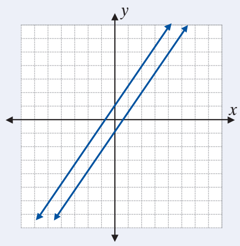
b. Method 2: Solve the inequality for y. If the solution shows $y <$ or $y \leq$, then shade the half-plane below the line. If the solution shows $y >$ or $y \geq$, then shade the half-plane above the line.

CHAPTER 5 Systems of Linear Equations

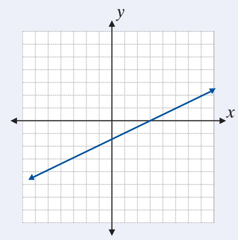
Solutions of Systems of Linear Equations:



Consistent
(One solution)



Inconsistent
(No solution)



Dependent
(Infinite number of solutions)

Solving a System of Two Linear Inequalities:

1. Graph both half planes.

2. Shade the region that is common to both of these half-planes, called the **intersection**. (If there is no intersection, then the system is inconsistent and has no solution.)

3. To check, pick one test-point in the intersection and verify that it satisfies *both* inequalities.

CHAPTER 6 Exponents and Polynomials

Properties of Exponents:

For nonzero real numbers a and b and integers m and n;

The Exponent 1: $a = a^1$ (a is any real number.)

The Exponent 0: $a^0 = 1 \quad (a \neq 0)$

Product Rule: $a^m \cdot a^n = a^{m+n}$

Quotient Rule: $\dfrac{a^m}{a^n} = a^{m-n}$

Negative Exponents Rule: $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$

Power Rule: $\left(a^m\right)^n = a^{mn}$

Power Rule for Products: $(ab)^n = a^n b^n$

Power Rule for Quotients: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Scientific Notation:

$N = a \times 10^n$ where N is a decimal number, $1 \leq a < 10$, and n is an integer.

Classification of Polynomials:

Monomial: polynomial with one term

Binomial: polynomial with two terms

Trinomial: polynomial with three terms

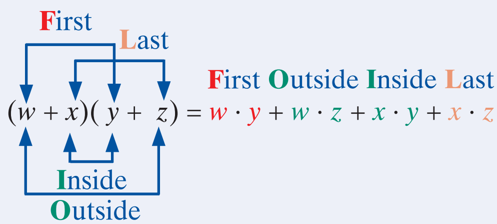
Degree: The **degree of a polynomial** is the largest of the degrees of its terms.

Leading Coefficient: The coefficient of the term with the largest degree.

Division Algorithm:

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}, \ (D(x) \neq 0)$$

FOIL Method:



First Outside Inside Last

$$(w + x)(y + z) = w \cdot y + w \cdot z + x \cdot y + x \cdot z$$

Special Products of Binomials:

1. $(X + A)(X - A) = X^2 - A^2$: Difference of two squares
2. $(X + A)^2 = X^2 + 2AX + A^2$: Perfect square trinomial
3. $(X - A)^2 = X^2 - 2AX + A^2$: Perfect square trinomial

CHAPTER 7 Factoring Polynomials and Solving Quadratic Equations

Finding the GCF:

1. Find the prime factorization of all integers and integer coefficients.
2. List all factors common to all terms, including variables.
3. Choose the greatest power of each factor common to all terms.
4. Multiply these powers to find the GCF. (If there are no common factors, the GCF is 1.)

Factoring Trinomials with Leading Coefficient 1 ($x^2 + bx + c$):

Find factors of c whose sum is b.

The *ac*-Method:

1. Multiply ac.
2. Find two integers whose product is ac and whose sum is b. (If this is not possible, the trinomial is not factorable.)
3. Rewrite the middle term, bx, using the two numbers found in Step 2 as coefficients.
4. Factor by grouping the first two terms and the last two terms.
5. Factor out the common binomial factor.

Trial-and-Error Method Guidelines:

1. If the sign of the constant term is positive (+), the signs in both factors will be the same, either both positive or both negative.
2. If the sign of the constant term is negative (–), the signs of the factors will be different, one positive and one negative.

Quadratic Equation:

An equation that can be written in the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$ is called a **quadratic equation**.

Zero Factor Property:

If a and b are real numbers, and $a \cdot b = 0$, then $a = 0$ or $b = 0$ or both.

CHAPTER 8 Rational Expressions

Rational Expression:
A **rational expression** is an expression of the form $\frac{P}{Q}$ (or in function notation, $\frac{P(x)}{Q(x)}$) where P and Q are polynomials and $Q \neq 0$.

Fundamental Principle of Fractions:
If $\frac{P}{Q}$ is a rational expression and K is a polynomial and $K \neq 0$, then

$\frac{P}{Q} = \frac{P}{Q} \cdot \frac{K}{K} = \frac{P \cdot K}{Q \cdot K}$.

Negative Signs in Rational Expressions:
$-\frac{P}{Q} = \frac{P}{-Q} = \frac{-P}{Q}$ and $\frac{P}{Q} = \frac{-P}{-Q} = -\frac{-P}{Q} = -\frac{P}{-Q}$

Opposites in Rational Expressions:
For a ≠ b, $(a-b)$ and $(b-a)$ are opposites and $\frac{a-b}{b-a} = \frac{a-b}{-1(a-b)} = -1$

Multiplication of Rational Expressions:
$\frac{P}{Q} \cdot \frac{R}{S} = \frac{P \cdot R}{Q \cdot S}$ where $Q, S \neq 0$.

Division of Rational Expressions:
$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R}$ where $Q, R, S \neq 0$.

Addition and Subtraction of Rational Expressions:
$\frac{P}{Q} + \frac{R}{Q} = \frac{P+R}{Q}$ and $\frac{P}{Q} - \frac{R}{Q} = \frac{P-R}{Q}$ where $Q \neq 0$.

Complex Fraction:
A **complex fraction** is a fraction in which the numerator or denominator is a fraction or the sum or difference of fractions.

Proportions:
A proportion is an equation stating that two ratios are equal.

Direct Variation:
$y = kx$ (y is directly proportional to x)

Inverse Variation:
$y = \frac{k}{x}$ (y is inversely proportional to x)

CHAPTER 9 Real Numbers and Radicals

Properties of Square Roots:
1. If $a^2 = b$, then a is called the **square root** of b, $a = \sqrt{b}$ $(b \geq 0)$.

2. $\sqrt{ab} = \sqrt{a}\sqrt{b}$, where a and b are positive real numbers.

3. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where a and b are positive real numbers.

4. $\sqrt{x^{2m}} = \left|x^m\right|$, where m is a positive integer.

5. $\sqrt{x^{2m+1}} = \left|x^m\right|\sqrt{x}$, where m is a positive integer.

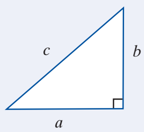
Properties of Radicals:
1. If $b^n = a$, then $b = \sqrt[n]{a} = a^{\frac{1}{n}}$, n is a positive integer.

2. $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$ or $a^{\frac{m}{n}} = \left(a^m\right)^{\frac{1}{n}}$, where n is a positive integer and m is any integer.

Cube Roots:
If $a^3 = b$, then a is called the **cube root** of b, $a = \sqrt[3]{b}$.

The Pythagorean Theorem:
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

$c^2 = a^2 + b^2$



Distance Between Two Points:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

CHAPTER 10 Quadratic Equations

Square Root Property:
If $x^2 = c$, then $x = \pm\sqrt{c}$.

If $(x-a)^2 = c$, then $x - a = \pm\sqrt{c}$ $\left(\text{or } x = a \pm \sqrt{c}\right)$.

Quadratic Formula:
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

Discriminant: $b^2 - 4ac$
$b^2 - 4ac > 0 \rightarrow$ Two real solutions

$b^2 - 4ac = 0 \rightarrow$ One real solution

$b^2 - 4ac < 0 \rightarrow$ No real solutions

Quadratic Functions:
A **quadratic function** is a function of the form $y = ax^2 + bx + c$, where $a \neq 0$.

The shape of a quadratic function is a **parabola**.

If $a > 0$, the parabola "opens upward".

If $a < 0$, the parabola "opens downward"

Line of symmetry: $x = -\frac{b}{2a}$

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

The vertex is the minimum (lowest point) of the curve if the parabola opens upward.

The vertex is the maximum (highest point) of the curve if the parabola opens downward.