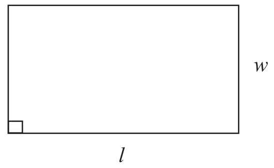
Geometry

Rectangle

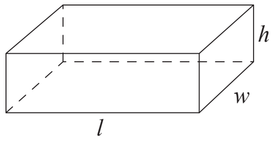
Area $= lw$

Perimeter $= 2l + 2w$



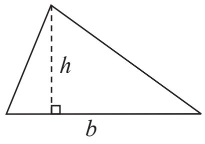
Rectangular Prism

Volume $= lwh$



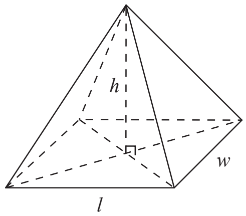
Triangle

Area $= \frac{1}{2}bh$



Rectangular Pyramid

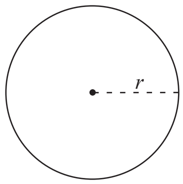
Volume $= \frac{1}{3}lwh$



Circle

Area $= \pi r^2$

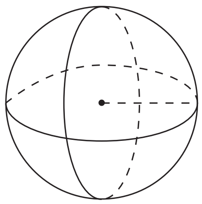
Circumference $= 2\pi r$



Sphere

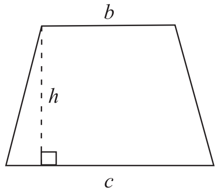
Volume $= \frac{4}{3}\pi r^3$

Surface Area $= 4\pi r^2$



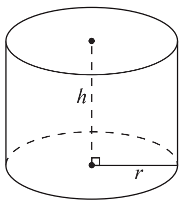
Trapezoid

Area $= \frac{1}{2}h(b + c)$



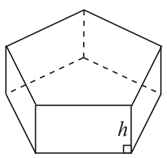
Right Circular Cylinder

Volume $= \pi r^2 h$



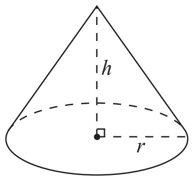
Right Cylinder

Volume $=$ (Area of Base)(h)



Circular Cone

Volume $= \frac{1}{3}\pi r^2 h$



Algebra

Properties of Exponents and Radicals

$$a^0 = 1 \qquad a^{-1} = \frac{1}{a}$$

$$a^m \cdot a^n = a^{m+n} \qquad \left(a^m\right)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad (ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n} \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$$

Special Product Formulas

$$X^2 - A^2 = (X + A)(X - A)$$

$$(X + A)^2 = X^2 + 2AX + A^2$$

$$(X - A)^2 = X^2 - 2AX + A^2$$

$$X^3 + A^3 = (X + A)\left(X^2 - AX + A^2\right)$$

$$X^3 - A^3 = (X - A)\left(X^2 + AX + A^2\right)$$

The Quadratic Formula

The solutions of the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The Distance Formula

The distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Slope of a Line

Let (x_1, y_1) and (x_2, y_2) be two points on a line.

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Horizontal lines have slope 0.

Vertical lines have an undefined slope.

Parallel and Perpendicular Lines

Given a line with slope m:

slope of parallel line $= m$,

slope of perpendicular line $= -\frac{1}{m}$.

Forms of Equations of Lines

Standard form: $Ax + By = C$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

The Pythagorean Theorem

Given a right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$.

Operations with Functions

Let f and g be functions.

$$f + g = f(x) + g(x)$$
$$f - g = f(x) - g(x)$$
$$f \cdot g = f(x) \cdot g(x)$$
$$\frac{f}{g} = \frac{f(x)}{g(x)}, \text{ when } g(x) \neq 0$$
$$(f \circ g)(x) = f(g(x))$$

Properties of Natural Logarithms

$e^y = x$ if and only if $y = \ln x$

Let M and N be positive real numbers and let r be any real number.

1. $\ln(MN) = \ln M + \ln N$
2. $\ln\left(\frac{M}{N}\right) = \ln M - \ln N$
3. $\ln(M^r) = r \cdot \ln M$
4. $\ln M = \ln N$ if and only if $M = N$
5. $\ln 1 = 0$
6. $\ln e = 1$
7. $e^{\ln x} = x$
8. $\ln(e^x) = x$

Limits

Properties of Limits as $x \to a$

Suppose that c is any constant, n is a positive integer, a is a real number, $\lim\limits_{x \to a} f(x) = L_1$, and $\lim\limits_{x \to a} g(x) = L_2$, where L_1 and L_2 are real numbers.

1. $\lim\limits_{x \to a} c = c$
2. $\lim\limits_{x \to a} x = a$
3. $\lim\limits_{x \to a} x^n = a^n$
4. $\lim\limits_{x \to a}\left[f(x) \pm g(x)\right] = \lim\limits_{x \to a} f(x) \pm \lim\limits_{x \to a} g(x) = L_1 \pm L_2$
5. $\lim\limits_{x \to a}\left[f(x) \cdot g(x)\right] = \left[\lim\limits_{x \to a} f(x)\right] \cdot \left[\lim\limits_{x \to a} g(x)\right] = L_1 \cdot L_2$
6. $\lim\limits_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} = \frac{L_1}{L_2}$, where $L_2 \neq 0$
7. $\lim\limits_{x \to a}\left[c \cdot f(x)\right] = c \cdot \lim\limits_{x \to a} f(x) = c \cdot L_1$
8. $\lim\limits_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x \to a} f(x)} = \sqrt[n]{L_1} = (L_1)^{\frac{1}{n}}$, where $(L_1)^{\frac{1}{n}}$ is defined.
9. If p is a polynomial function, then $\lim\limits_{x \to a} p(x) = p(a)$.

Summary of Limits for Rational Functions as $x \to +\infty$ (or $x \to -\infty$)

Consider the function $f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_0}$, where $a_n \neq 0$ and $b_m \neq 0$.

Case 1: For $m = n$, $\lim\limits_{x \to +\infty} f(x) = \frac{a_n}{b_m}$.

Case 2: For $m > n$, $\lim\limits_{x \to +\infty} f(x) = 0$.

Case 3: For $m < n$, $\lim\limits_{x \to +\infty} f(x) = +\infty$. (Or $-\infty$ depending on the signs of a_n and b_m.)

Continuity

A function $y = f(x)$ is **continuous** at $x = a$ if all of the following conditions are true:

1. $f(a)$ exists
2. $\lim\limits_{x \to a} f(x)$ exists
3. $\lim\limits_{x \to a} f(x) = f(a)$

Derivatives

Definition of the Derivative

For any given function $y = f(x)$, the **derivative** of f at x is defined to be

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

provided this limit exists.

Differentiation Rules

Suppose that c and r are real numbers and the functions $f(x)$ and $g(x)$ are differentiable.

1. $\frac{d}{dx}[c] = 0$ **Derivative of a constant**

2. $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$ **Sum and Difference Rule**

3. $\frac{d}{dx}[c \cdot f(x)] = c \cdot f'(x)$ **Constant Times a Function Rule**

4. $\frac{d}{dx}[c \cdot x^r] = c \cdot r \cdot x^{r-1}, \quad r \neq 0$ **Power Rule**

5. $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$ **Product Rule**

6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$ **Quotient Rule**

7. $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$ **Chain Rule**

8. $\frac{d}{dx}\left[(g(x))^r\right] = r[g(x)]^{r-1} \cdot g'(x)$ **General Power Rule**

9. $\frac{d}{dx}[\ln x] = \frac{1}{x}$

10. $\frac{d}{dx}[\log_B x] = \frac{1}{\ln B} \cdot \frac{1}{x}$

11. $\frac{d}{dx}[e^x] = e^x$

12. $\frac{d}{dx}[B^x] = (\ln B) \cdot B^x$

13. $\frac{d}{dx}[\sin x] = \cos x$

14. $\frac{d}{dx}[\cos x] = -\sin x$

15. $\frac{d}{dx}[\tan x] = \sec^2 x$

16. $\frac{d}{dx}[\cot x] = -\csc^2 x$

17. $\frac{d}{dx}[\sec x] = \sec x \tan x$

18. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

19. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$ for x in $(-1, 1)$

20. $\frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$ for x in $(-1, 1)$

21. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$ for x in $(-\infty, +\infty)$

Integrals

The Fundamental Theorem of Calculus

If a function $y = f(x)$ is continuous on the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a).$$

Properties of Definite Integrals

1. $\int_a^a f(x)\,dx = 0$
2. $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
3. $\int_a^b \left[f(x) \pm g(x)\right]dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
4. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
5. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ where c is any point with $a \le c \le b$.

Improper Integrals

The **improper integral** $\int_a^{+\infty} f(x)\,dx$ is defined as the following limit:

$$\int_a^{+\infty} f(x)\,dx = \lim_{b \to +\infty} \int_a^b f(x)\,dx.$$

If $\lim_{b \to +\infty} \int_a^b f(x)\,dx$ exists, then the improper integral is said to be **convergent**.

If $\lim_{b \to +\infty} \int_a^b f(x)\,dx$ does not exist, then the improper integral is said to be **divergent**.

Formulas of Integration

Suppose that a, b, c, d, and k are real numbers and the functions $f(x)$, $g(x)$, u, and v are integrable.

1. $\int k\,dx = kx + C$ — **Integrating a constant**
2. $\int x^r\,dx = \frac{1}{r+1}x^{r+1} + C \qquad (r \ne -1)$ — **Power Rule**
3. $\int e^x\,dx = e^x + C$
4. $\int \frac{1}{x}\,dx = \ln|x| + C$
5. $\int kf(x)\,dx = k\int f(x)\,dx$ — **Constant Multiple Rule**
6. $\int \left[f(x) \pm g(x)\right]dx = \int f(x)\,dx \pm \int g(x)\,dx$ — **Sum and Difference Rule**
7. $\int u\,dv = uv - \int v\,du$ — **Integration by parts**
8. $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$
9. $\int \frac{1}{(ax+b)^2}\,dx = -\frac{1}{a(ax+b)} + C$
10. $\int \frac{1}{x(ax+b)}\,dx = \frac{1}{b}\ln\left|\frac{x}{ax+b}\right| + C$
11. $\int \frac{x}{ax+b}\,dx = \frac{1}{a^2}\left(ax - b\ln|ax+b|\right) + C$

12. $\int \frac{1}{(ax+b)(cx+d)}\,dx = \frac{1}{ad-bc}\ln\left|\frac{ax+b}{cx+d}\right| + C$

13. $\int \frac{x}{(ax+b)(cx+d)}\,dx = \frac{1}{ad-bc}\left(\frac{d}{c}\ln|cx+d| - \frac{b}{a}\ln|ax+b|\right) + C$

14. $\int \sqrt{ax+b}\,dx = \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$

15. $\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)}{15a^2}(ax+b)^{\frac{3}{2}} + C$

16. $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C \qquad (x^2 > a^2)$

17. $\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

18. $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

19. $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

20. $\int x^n e^{kx}\,dx = \frac{x^n e^{kx}}{k} - \frac{n}{k}\int x^{n-1}e^{kx}\,dx$

21. $\int \frac{1}{a+be^{kx}}\,dx = \frac{x}{a} - \frac{1}{ak}\ln\left|a+be^{kx}\right| + C$

22. $\int \ln x\,dx = x\ln x - x + C$

23. $\int x^n \ln x\,dx = \frac{x^{n+1}\ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C$

24. $\int \cos x\,dx = \sin x + C$

25. $\int \sin x\,dx = -\cos x + C$

26. $\int \sec^2 x\,dx = \tan x + C$

27. $\int \tan x\,dx = -\ln|\cos x| + C$

28. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

29. $\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1} x + C$

30. $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$

Series

Convergence of a Geometric Series

When $-1 < r < 1$, the sum of a geometric series $a + ar + ar^2 + ar^3 + \cdots = \sum_{n=0}^{\infty} ar^n$ is finite and converges to $\frac{a}{1-r}$.

The Taylor Polynomial of Degree *k* Approximating *f*(*x*) near *x* = 0

$$f(x) \approx P_k(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \frac{1}{3!}f'''(0)x^3 + \cdots + \frac{1}{k!}f^{(k)}(0)x^k$$

where $k! = k(k-1)(k-2)\cdots(3)(2)(1)$.

The General Taylor Polynomial of Degree *k* Centered at *x* = *a*

$$f(x) \approx P_k(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots + \frac{f^{(k)}(a)(x-a)^k}{k!}$$

The Binomial Series

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}\cdot x^2 + \frac{p(p-1)(p-2)}{3!}\cdot x^3 + \cdots$$