Formulas and Tables from Discovering Business Statistics, Second Edition

Chapter 3

Number of Classes

Round $\sqrt{n}$ or $\sqrt[3]{2n}$ to the nearest whole number.

Class Width

$$\frac{\text{Largest Value} - \text{Smallest Value}}{\text{Number of Classes}}$$

Relative Frequency

$$\frac{\text{Number in Class}}{\text{Total Number of Observations}}$$

Chapter 4

Arithmetic Mean

$$\frac{1}{n}\left(x_1 + x_2 + \cdots + x_N\right)$$

Population Mean

$$\propto = \frac{1}{N}\left(x_1 + x_2 + \cdots + x_N\right) = \frac{\sum x_i}{N}$$

Sample Mean

$$\bar{x} = \frac{1}{n}\left(x_1 + x_2 + \cdots + x_n\right) = \frac{\sum x_i}{n}$$

Weighted Mean

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \cdots + w_nx_n}{w_1 + w_2 + \cdots + w_n} = \frac{\sum\left(w_ix_i\right)}{\sum w_i}$$

Median Location

$$L(m) = \frac{n+1}{2}$$

Range

largest value – smallest value

Mean Absolute Deviation

$$\text{MAD} = \frac{\sum\left|x_i - \bar{x}\right|}{n}$$

Population Variance

$$\sigma^2 = \frac{\sum\left(x_i - \mu\right)^2}{N}$$

Sample Variance

$$s^2 = \frac{\sum\left(x_i - \bar{x}\right)^2}{n-1}$$

Population Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

Sample Standard Deviation

$$s = \sqrt{s^2}$$

Empirical Rule

One Sigma Rule: $\mu \pm 1\sigma$ contains about 68% of the data

Two Sigma Rule: $\mu \pm 2\sigma$ contains about 95% of the data

Three Sigma Rule: $\mu \pm 3\sigma$ contains about 99.7% of the data

Chebyshev's Theorem

The proportion of any data set lying within k standard deviations of the mean is at least $1 - \frac{1}{k^2}$ for $k > 1$

Coefficient of Variation

$$\text{Population data: } CV = \left(\frac{\sigma}{\mu} \cdot 100\right)\%$$

$$\text{Sample data: } CV = \left(\frac{s}{\bar{x}} \cdot 100\right)\%$$

Location of the P^{th} Percentile

$$\ell = n\left(\frac{P}{100}\right)$$

Percentile of x

$$\frac{\text{number of data values less than or equal to } x}{\text{total number of data values}} \cdot 100$$

Quartiles

$$\text{Location of } Q_1\text{: } \ell = n\left(\frac{25}{100}\right)$$

$$\text{Location of } Q_2\text{: } \ell = n\left(\frac{50}{100}\right)$$

$$\text{Location of } Q_3\text{: } \ell = n\left(\frac{75}{100}\right)$$

Interquartile Range

$$\text{IQR} = Q_3 - Q_1$$

z-Score

$$z = \frac{x - \mu}{\sigma}$$

Mean of Grouped Data

$$\text{Population data: } \propto = \frac{\sum\left(f_iM_i\right)}{N}$$

$$\text{Sample data: } \bar{x} = \frac{\sum\left(f_iM_i\right)}{n}$$

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Variance of Grouped Data

$$\text{Population data: } \sigma^2 = \frac{\sum (M_i - \mu)^2 f_i}{N}$$

$$\text{Sample data: } s^2 = \frac{\sum (M_i - \bar{x})^2 f_i}{n-1}$$

Computational Formulas for the Variance of Grouped Data

$$\text{Population Data: } \sigma^2 = \frac{\sum (f_i M_i^2)}{N} - \left(\frac{\sum (f_i M_i)}{N} \right)^2$$

$$\text{Sample Data: } s^2 = \frac{\sum (f_i M_i^2) - \frac{\left(\sum (f_i M_i)\right)^2}{n}}{n-1}$$

Population Proportion

$$p = \frac{x}{N}$$

Sample Proportion

$$\hat{p} = \frac{x}{n}$$

Correlation Coefficient

$$r = \frac{1}{n-1} \left\{ \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right) \right\}$$

Computational Formula for the Correlation Coefficient

$$r = \frac{n \sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right)}{\sqrt{n \sum x_i^2 - \left(\sum x_i\right)^2} \sqrt{n \sum y_i^2 - \left(\sum y_i\right)^2}}$$

Chapter 5

Classical Probability

$$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes in the sample space}}$$

Probability Law 3

$$0 \le P(A) \le 1$$

Probability Law 4

$$P(A_1) + P(A_2) + \cdots + P(A_n) = 1$$

Probability Law 5

$$P(A^c) = 1 - P(A)$$

Odds in Favor of

$$\frac{P(A)}{P(\text{not A})} = \frac{P(A)}{P(A^c)}$$

Odds Against

$$\frac{P(\text{not A})}{P(A)} = \frac{P(A^c)}{P(A)}$$

Union of Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B)$$

Intersection of Mutually Exclusive Events

$$P(A \cap B) = 0$$

The Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Independent Events

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B)$$

Multiplication Rule for Independent Events

$$P(A \cap B) = P(A)P(B)$$

Multiplication Rule for Dependent Events

$$P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$$

Intersection of Mutually Exclusive Events

$$P(A \cap B) = 0$$

Bayes' Theorem

$$P(B_i \mid A) = \frac{P(A \mid B_i)P(B_i)}{P(A \mid B_1)P(B_1) + \cdots + P(A \mid B_k)P(B_k)}$$

Fundamental Counting Principle

If E_1 is an event with n_1 possible outcomes and E_2 is an event with n_2 possible outcomes, the number of ways the events can occur in sequence is $n_1 \cdot n_2$.

n Factorial

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

Combination

$${}_nC_k = \frac{n!}{(n-k)!k!}$$

Permutation

$${}_nP_k = \frac{n!}{(n-k)!}$$

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Number of Distinguishable Permutations

$$\frac{n!}{(n_1!)(n_2!)(n_3!)\cdots(n_k!)}$$

Chapter 6

Expected Value of a Discrete Random Variable *X*

$$\propto = E(X) = \sum\left[xp(x)\right]$$

Variance of a Discrete Random Variable *X*

$$\sigma^2 = V(X) = \sum\left[(x-\mu)^2\, p(x)\right]$$

Computational Formula for Variance of a Discrete Random Variable *X*

$$\sigma^2 = V(X) = \sum\left[x_i^2\, p(x_i)\right] - \mu^2$$

Discrete Uniform Probability Distribution Function

$$P(X = x) = \frac{1}{n}$$

Binomial Probability Distribution Function

$$P(X = x) = {}_nC_x\, p^x (1-p)^{n-x}$$

Expected Value of a Binomial Random Variable

$$\propto = E(X) = np$$

Variance of a Binomial Random Variable

$$\sigma^2 = V(X) = np(1-p)$$

Poisson Probability Distribution Function

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

Expected Value of a Poisson Random Variable

$$\mu = E(X) = \lambda$$

Variance and Standard Deviation of a Poisson Random Variable

$$\sigma^2 = V(X) = \lambda$$

$$\sigma = \sqrt{V(X)} = \sqrt{\lambda}$$

Hypergeometric Probability Distribution Function

$$P(X = x) = \frac{{}_kC_x\, {}_{N-k}C_{n-x}}{{}_NC_n}$$

Expected Value of a Hypergeometric Random Variable

$$\propto = E(X) = n\left(\frac{k}{N}\right)$$

Variance of a Hypergeometric Random Variable

$$\sigma^2 = V(X) = n\left(\frac{k}{N}\right)\left(1-\frac{k}{N}\right)\frac{(N-n)}{(N-1)}$$

Chapter 7

Uniform Probability Density Function

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

Expected Value for a Continuous Uniform Random Variable

$$\propto = E(x) = \frac{a+b}{2}$$

Standard Deviation for a Continuous Uniform Random Variable

$$\sigma = \frac{b-a}{\sqrt{12}}$$

Normal Probability Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Creating a Normal Probability Plot by Hand

1. Arrange the data values in ascending order.
2. Calculate $f_i = \dfrac{(i-0.5)}{n}$, where i is the position of the data value in the ordered list and n is the number of observations in the data set.
3. Find the z-score for each value of f_i.
4. Plot the data values on the horizontal axis and the corresponding z-score on the vertical axis.

Standardizing a Normal Random Variable

$$z = \frac{x-\mu}{\sigma}$$

Normal Approximation for the Binomial Distribution

If X is a binomial random variable where $np \ge 5$ and $n(1-p) \ge 5$, then X can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.

Normal Approximation for the Poisson Distribution

If X has a Poisson distribution, X can be approximated by a normal distribution with $\mu = \lambda$ and $\sigma = \sqrt{\lambda}$.

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Chapter 8

Standard Deviation of the Sample Mean: Infinite Population

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard Deviation of the Sample Mean: Finite Population

$$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}} \cdot \frac{\sigma}{\sqrt{n}}$$

Characteristics of the Sample Mean

1. $E(\bar{x}) = \mu$
2. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Central Limit Theorem

If a sufficiently large random sample $(n \geq 30)$ is drawn from a population with mean μ and standard deviation σ, the distribution of the sample mean will have the following characteristics.

1. An approximately normal distribution regardless of the distribution of the underlying population.
2. $\mu_{\bar{x}} = E(\bar{x}) = \mu$
3. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Sample Proportion

$$\hat{p} = \frac{x}{n}$$

Standard Deviation of the Sample Proportion: Infinite Population

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Standard Deviation of the Sample Proportion: Finite Population

$$\sigma_{\hat{p}} = \sqrt{\frac{N-n}{N-1}} \cdot \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{N-n}{N-1}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Chapter 9

Mean Squared Error for the Sample Mean

$$\text{MSE}(\bar{x}) = E(\bar{x} - \mu)^2$$

100(1 – α)% Confidence Interval for the Population Mean

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Student's t-Distribution

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Degrees of Freedom

$$df = n - 1$$

100(1 – α)% Confidence Interval for the Population Mean: σ Unknown

$$\bar{x} \pm t_{\alpha/2,df} \frac{s}{\sqrt{n}}$$

Margin of Error for the Population Mean (Maximum Error of Estimation)

$$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Determining the Sample Size for the Population Mean: σ Known

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$$

Determining the Sample Size for the Population Mean: σ Unknown

$$n = \left(\frac{z_{\alpha/2}s}{E}\right)^2$$

Point Estimate of the Population Proportion

$$\hat{p} = \frac{x}{n}$$

100(1 – α)% Confidence Interval for the Population Proportion

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}}$$

Margin of Error for the Population Proportion (Maximum Error of Estimation)

$$E = z_{\alpha/2}\sigma_{\hat{p}} = z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

Determining the Sample Size for the Population Proportion

$$n = \frac{z_{\alpha/2}^2 p(1-p)}{E^2} \approx \frac{z_{\alpha/2}^2 \hat{p}(1-\hat{p})}{E^2}$$

Determining the Sample Size for the Population Proportion: No Estimate $(\hat{p})$ Available

$$n = \frac{z_{\alpha/2}^2 (0.5)(1-0.5)}{E^2} \approx \frac{z_{\alpha/2}^2 (0.25)}{E^2}$$

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χ^2 Test Statistic

If we have a random sample of size n taken from a normal population, then the sampling distribution of the test statistic is given by

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

which has a chi-square distribution with $n - 1$ degrees of freedom.

100(1– α)% Confidence Interval for σ^2

A 100(1 – σ)% confidence interval for σ^2 is given by

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

where $\chi^2_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$ are values under the curve of the chi-square distribution with $n - 1$ degrees of freedom.

Chapter 10

z-Test Statistic for a Population Mean

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}, \text{ where } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

t-Test Statistic

$$t = \frac{\bar{x} - \propto_0}{s_{\bar{x}}}, \text{ where } s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

z-Test Statistic for a Population Proportion

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}}, \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}}$$

Test Statistic for a Population Variance

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \text{ with } n-1 \text{ degrees of freedom}$$

Chapter 11

Sampling Distribution of $\bar{x}_1 - \bar{x}_2$

$$\propto_{\bar{x}_1 - \bar{x}_2} = \propto_1 - \propto_2$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

100(1 – α)% Confidence Interval for $\mu_1 - \mu_2$

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Test Statistic for a Hypothesis Test about $\mu_1 - \mu_2$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \mu_{\bar{x}_1 - \bar{x}_2}}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

100(1 – α)% Confidence Interval for $\mu_1 - \mu_2$, σ_1 and σ_2 Unknown ($\sigma_1 = \sigma_2$)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2,df}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$\text{where } s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Test Statistic for a Hypothesis Test about $\mu_1 - \mu_2$, σ_1 and σ_2 Unknown ($\sigma_1 = \sigma_2$)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\propto_1 - \propto_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\text{where } s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

100(1 – α)% Confidence Interval for $\mu_1 - \mu_2$, σ_1 and σ_2 Unknown ($\sigma_1 \neq \sigma_2$)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2,df}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}},$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

Test Statistic for a Hypothesis Test about $\mu_1 - \mu_2$, σ_1 and σ_2 Unknown ($\sigma_1 \neq \sigma_2$)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\propto_1 - \propto_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}},$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

100(1 – α)% Confidence Interval for μ_d

$$\bar{x}_d \pm t_{\alpha/2,df}\frac{s_d}{\sqrt{n_d}}$$

Test Statistic for a Paired Difference Hypothesis Test

$$t = \frac{\bar{x}_d - \propto_d}{s_d / \sqrt{n_d}}$$

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100(1 − α)% Confidence Interval for $p_1 - p_2$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Sampling Distribution of $\hat{p}_1 - \hat{p}_2$

$$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$$

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$\text{where } \bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

Test Statistic for $p_1 - p_2$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Confidence Interval for $\frac{\sigma_1^2}{\sigma_2^2}$

A 100(1 − α)% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$, the ratio of two population variances, is given by

$$\left(\left(\frac{s_1^2}{s_2^2}\right)\frac{1}{F_{\alpha/2, df_{num}, df_{den}}}, \left(\frac{s_1^2}{s_2^2}\right)\frac{1}{F_{1-\alpha/2, df_{num}, df_{den}}}\right)$$

where s_1^2 and s_2^2 are the two sample variances, and $df_{num} = n_1 - 1$ (the denominator of s_1^2) and $df_{den} = n_2 - 1$ (the denominator of s_2^2).

Inferences about $\frac{\sigma_1^2}{\sigma_2^2}$

Test Statistic:

$$F = \frac{s_1^2}{s_2^2}$$

where s_1^2 and s_2^2 are the sample variances from the two samples of sizes n_1 and n_2, respectively, collected from two independent normally distributed populations. The degrees of freedom for the F-test are $df_{num} = n_1 - 1$ and $df_{den} = n_2 - 1$.

Chapter 12

Grand Mean

$$\bar{\bar{x}} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} x_{ij}}{n_T}$$

Sample Variance

$$s^2 = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j}\left(x_{ij} - \bar{\bar{x}}\right)^2}{n_T - 1}$$

Sum of Squares for Treatments

$$\text{SST} = \sum_{j=1}^{k} n_j\left(\bar{x}_j - \bar{\bar{x}}\right)^2$$

Mean Square for Treatments

$$\text{MST} = \frac{\text{SST}}{k-1}$$

Sum of Squares for Error

$$\text{SSE} = \sum_{i=1}^{n_1}(x_{i1} - \bar{x}_1)^2 + \sum_{i=1}^{n_2}(x_{i2} - \bar{x}_2)^2 + \cdots + \sum_{i=1}^{n_k}(x_{ik} - \bar{x}_k)^2$$

Total Sum of Squares

$$\text{TSS} = \text{SST} + \text{SSE} = \sum_{j=1}^{k}\sum_{i=1}^{n_j}\left(x_{ij} - \bar{\bar{x}}\right)^2$$

Mean Square for Error

$$\text{MSE} = \frac{\text{SSE}}{n_T - k}$$

F-Statistic for One-Way ANOVA

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{\dfrac{\sum_{i=1}^{k} n_i\left(\bar{x}_i - \bar{\bar{x}}\right)^2}{k-1}}{\dfrac{\sum_{i=1}^{n_1}(x_{i1} - \bar{x}_1)^2 + \sum_{i=1}^{n_2}(x_{i2} - \bar{x}_2)^2 + \cdots + \sum_{i=1}^{n_k}(x_{ik} - \bar{x}_k)^2}{n_T - k}}$$

Computational Formula for MST

$$\text{MST} = \frac{\left[\dfrac{\left(\sum_{i=1}^{n_1} x_{i1}\right)^2}{n_1} + \dfrac{\left(\sum_{i=1}^{n_2} x_{i2}\right)^2}{n_2} + \cdots + \dfrac{\left(\sum_{i=1}^{n_k} x_{ik}\right)^2}{n_k}\right] - \dfrac{\left(\sum_{j=1}^{k}\sum_{i=1}^{n_j} x_{ij}\right)^2}{n_T}}{k-1}$$

Computational Formula for MSE

$$\text{MSE} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} x_{ij}^2 - \left[\dfrac{\left(\sum_{i=1}^{n_1} x_{i1}\right)^2}{n_1} + \dfrac{\left(\sum_{i=1}^{n_2} x_{i2}\right)^2}{n_2} + \cdots + \dfrac{\left(\sum_{i=1}^{n_k} x_{ik}\right)^2}{n_k}\right]}{n_T - k}$$

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Fisher's Least Significant Difference Method

$$\left|\bar{x}_i - \bar{x}_j\right| \ge t_{\alpha/2, n_T - k}\sqrt{\text{MSE}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Fisher's LSD Method, Confidence Interval Approach

$$\left(\bar{x}_i - \bar{x}_j\right) \pm t_{\alpha/2, n_T - k}\sqrt{\text{MSE}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Tukey's Honest Significant Difference Method

$$\left|\bar{x}_i - \bar{x}_j\right| \ge q_{\alpha, k, n_T - k}\sqrt{\frac{\text{MSE}}{2}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Tukey's HSD Method, Confidence Interval Approach

$$\left(\bar{x}_i - \bar{x}_j\right) \pm q_{\alpha, k, n_T - k}\sqrt{\frac{\text{MSE}}{n}} \quad \text{for } \textbf{balanced data } (n = n_i = n_j)$$

or

$$\left(\bar{x}_i - \bar{x}_j\right) \pm q_{\alpha, k, n_T - k}\sqrt{\frac{\text{MSE}}{2}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} \quad \text{for } \textbf{unbalanced data } (n_i \ne n_j)$$

F-Statistic for Two-Way ANOVA (Randomized Block Design)

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{\dfrac{\text{SST}}{k-1}}{\dfrac{\text{SSE}}{(k-1)(b-1)}}$$

$$\text{where } \text{SSE} = \text{TSS} - \text{SST} - \text{SSBL}$$

Test Statistic for Interaction between Factors

$$F = \frac{\dfrac{\text{SSAB}}{(a-1)(b-1)}}{\dfrac{\text{SSE}}{ab(r-1)}} = \frac{\text{MSAB}}{\text{MSE}}$$

Test Statistic for Main Effects for Factor A

$$F = \frac{\dfrac{\text{SSA}}{(a-1)}}{\dfrac{\text{SSE}}{ab(r-1)}} = \frac{\text{MSA}}{\text{MSE}}$$

Test Statistic for Main Effects for Factor B

$$F = \frac{\dfrac{\text{SSB}}{(b-1)}}{\dfrac{\text{SSE}}{ab(r-1)}} = \frac{\text{MSB}}{\text{MSE}}$$

Chapter 13

Simple Linear Regression Model

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

Estimated Simple Linear Regression Equation

$$\hat{y}_i = b_0 + b_1 x_i$$

Sum of Squared Errors

$$\text{SSE} = \sum\left(y_i - \hat{y}_i\right)^2 = \sum\left(y_i - \left(b_0 + b_1 x_i\right)\right)^2$$

Slope of the Least Squares Line

$$b_1 = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

y-Intercept of the Least Squares Line

$$b_0 = \bar{y} - b_1\bar{x} = \frac{1}{n}\left(\sum y_i - b_1\sum x_i\right)$$

Mean Square Error

$$s_e^2 = \frac{\sum\left(y_i - \hat{y}_i\right)^2}{n-2} = \frac{\text{SSE}}{n-2}$$

Standard Error

$$s_e = \sqrt{\frac{\sum\left(y_i - \hat{y}_i\right)^2}{n-2}} = \sqrt{\frac{\text{SSE}}{n-2}}$$

Residual

$$\text{Residual} = e_i = \text{Observed } y - \text{Predicted } y = y_i - \hat{y}_i$$

Total Sum of Squares

$$\text{TSS} = \sum\left(y_i - \bar{y}\right)^2$$

$$\text{TSS} = \text{SSE} + \text{SSR}$$

Sum of Squares of Regression

$$\text{SSR} = \text{TSS} - \text{SSE}$$

Coefficient of Determination

$$R^2 = \frac{\text{SSR}}{\text{TSS}} = 1 - \frac{\text{SSE}}{\text{TSS}}$$

or

$$R^2 = \left(\frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{\left(n\sum x_i^2 - \left(\sum x_i\right)^2\right)\left(n\sum y_i^2 - \left(\sum y_i\right)^2\right)}}\right)^2$$

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Correlation Coefficient

$$r=\frac{n\sum x_iy_i-\sum x_i\sum y_i}{\sqrt{\left(n\sum x_i^2-\left(\sum x_i\right)^2\right)\left(n\sum y_i^2-\left(\sum y_i\right)^2\right)}}$$

Sample Estimate of the Variance of b_1

$$s_{b_1}^2=\frac{s_e^2}{\sum\left(x_i-\bar{x}\right)^2}$$

Sample Estimate of the Standard Deviation (Standard Error) of b_1

$$s_{b_1}=\sqrt{\frac{s_e^2}{\sum\left(x_i-\bar{x}\right)^2}}$$

100(1 – α)% Confidence Interval for β_1

$$b_1\pm t_{\alpha/2,df}s_{b_1}$$

Test Statistic for Testing the Hypothesis $\beta_1\neq 0$

$$t=\frac{b_1-0}{s_{b_1}}=\frac{b_1}{s_{b_1}}$$

100(1 – α)% Confidence Interval for the Mean Value of *y* Given *x*

$$\hat{y}_p\pm t_{\alpha/2,df}s_e\sqrt{\frac{1}{n}+\frac{\left(x_p-\bar{x}\right)^2}{\sum\left(x_i-\bar{x}\right)^2}}$$

100(1 – α)% Prediction Interval for the Value of *y* Given *x*

$$\hat{y}_p\pm t_{\alpha/2,df}s_e\sqrt{1+\frac{1}{n}+\frac{\left(x_p-\bar{x}\right)^2}{\sum\left(x_i-\bar{x}\right)^2}}$$

Chapter 14

Multiple Regression Model

$$y_i=\beta_0+\beta_1x_{1i}+\beta_2x_{2i}+\cdots+\beta_kx_{ki}+\varepsilon_i$$

Estimated Multiple Regression Equation

$$\hat{y}_i=b_0+b_1x_{1i}+b_2x_{2i}+\cdots+b_kx_{ki}$$

Sum of Squared Errors

$$\text{SSE}=\sum\left(y_i-\hat{y}_i\right)^2$$

Adjusted R^2

$$R_a^2=1-\left(\frac{n-1}{n-k-1}\right)\frac{\text{SSE}}{\text{TSS}}$$

F-Statistic

$$F=\frac{\dfrac{\text{Sum of Squares of Regression}}{k}}{\dfrac{\text{Sum of Squared Errors}}{n-(k+1)}}$$

$$=\frac{\dfrac{\text{SSR}}{k}}{\dfrac{\text{SSE}}{n-(k+1)}}=\frac{\text{Mean Square Regression}}{\text{Mean Square Error}}$$

Test Statistic for Testing the Hypothesis $\beta_i\neq 0$

$$t=\frac{b_i-0}{s_{b_i}}=\frac{b_i}{s_{b_i}}$$

100(1 – α)% Confidence Interval for an Individual Coefficient, β_i

$$b_i\pm t_{\alpha/2,df}s_{b_i}$$

Chapter 15

Simple Moving Average

$$\text{MA}_n=\frac{\sum_{i=1}^{n}D_i}{n}$$

Weighted Moving Average

$$\text{WMA}_n=\sum_{i=1}^{n}W_iD_i$$

Simple Exponential Smoothing

$$F_{t+1}=\alpha D_t+(1-\alpha)F_t$$

Adjusted Exponential Smoothing

$$AF_{t+1}=F_{t+1}+T_{t+1}$$

where F_{t+1} = the forecast component of the period computed as a simple exponential smoothed forecast

$$\begin{aligned}F_{t+1}&=\alpha D_t+(1-\alpha)AF_t\\&=AF_t+\alpha\left(D_t-AF_t\right)\\&=\text{Previous Adjusted Forecast}+\\&\quad\alpha\left(\text{Forecast Error in the previous period}\right),\end{aligned}$$

and T_{t+1} = the forecast for the trend factor.

Mean Absolute Deviation (MAD)

$$\text{MAD}=\frac{\sum\left|D_t-F_t\right|}{n}$$

Mean Absolute Percentage Error (MAPE)

$$\text{MAPE}=\frac{100}{n}\sum_{t=1}^{n}\frac{\left|D_t-F_t\right|}{D_t}$$

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Mean Squared Error (MSE)

$$\text{MSE} = \frac{\sum\left(D_t - F_t\right)^2}{n}$$

Chapter 16

Chi-Square Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Test Statistic for the Chi-Square Test for Goodness of Fit

$$\chi^2 = \sum_{i=1}^{k} \frac{\left[n_i - E\left(n_i\right)\right]^2}{E\left(n_i\right)}$$

where n_i is the actual number of observations for each category, and $E\left(n_i\right)$ is the expected number of observations for each category.

Multiplication Rule for Independent Events

$$P(A \cap B) = P(A)P(B)$$

Test Statistic for the Chi-Square Test for Association between Two Qualitative Variables

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{\left[n_{ij} - E\left(n_{ij}\right)\right]^2}{E\left(n_{ij}\right)}$$

Cumulative Error (*E*)

$$E = \sum(D_t - F_t)$$

Tracking Signal (*TS*)

$$TS = \frac{E}{\text{MAD}}$$

Multiplicative Seasonal Factor

$$S_i = \frac{D_i}{\sum D_i}$$

Chapter 17

Test Statistic for the Sign Test, $n \le 25$

X = the number of times the less frequent sign occurs

Test Statistic for the Sign Test, $n > 25$

$$z = \frac{X + 0.5 - \left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}}$$

Test Statistic for the Wilcoxon Signed-Rank Test, $n \le 25$

If H_a is > One-Tailed:

$T = T_+$ = the sum of the ranks associated with positive differences.

If H_a is < One-Tailed:

$T = T_-$ = the sum of the ranks associated with negative differences.

If H_a is ≠ Two-Tailed:

$T = \text{Min}\ (T_+, T_-)$.

Test Statistic for the Wilcoxon Signed-Rank Test, $n > 25$

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

where $T = \text{Min}\left(T_+,\ T_-\right)$

Test Statistic for the Wilcoxon Rank-Sum Test, $n_1 \le 10$

If H_a is > One-Tailed:

$T = T_x$ = the rank sum of the sample with the fewest members.

If H_a is < One-Tailed:

$T = T_x$ = the rank sum of the sample with the fewest members.

If H_a is ≠ Two-Tailed:

$T = T_x$ = the rank sum of the sample with the fewest members.

Test Statistic for the Wilcoxon Rank-Sum Test, $n_1 > 10$

$$z = \frac{T - \frac{n_1\left(n_1 + n_2 + 1\right)}{2}}{\sqrt{\frac{n_1 n_2\left(n_1 + n_2 + 1\right)}{12}}}$$

Spearman Rank Correlation Coefficient (Spearman's Rho)

$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n\left(n^2 - 1\right)}$$

where $d_i = R\left(x_i\right) - R\left(y_i\right)$

Test Statistic for the Runs Test for Randomness, $m > 20$ or $n > 20$

$$z = \frac{R - \mu_R}{\sigma_R}$$

where $\mu_R = 1 + \frac{2mn}{N}$ and $\sigma_R = \sqrt{\frac{2mn(2mn - N)}{N^2(N-1)}}$

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Test Statistic for the Kruskal-Wallis Test

$$H = \frac{12}{N(N+1)}\sum_{i=1}^{k}\frac{R_i^2}{n_i} - 3(N+1)$$

$$\text{where } R_i = \sum_{j=1}^{n_i} r_{ij} \text{ and } N = \sum_{i=1}^{k} n_i$$

Chapter 18

Control Limits for $\bar{x}$ When the Process Mean and Standard Deviation are Known

$$\text{UCL} = \mu + 3\sigma_{\bar{x}} = \mu + \frac{3\sigma}{\sqrt{n}}$$

$$\text{LCL} = \mu - 3\sigma_{\bar{x}} = \mu - \frac{3\sigma}{\sqrt{n}}$$

$$\text{Centerline} = \mu$$

Control Limits for $\bar{x}$ When the Process Mean and Standard Deviation are Unknown

$$\text{UCL} = \bar{\bar{x}} + A\bar{R}$$

$$\text{LCL} = \bar{\bar{x}} - A\bar{R}$$

$$\text{Centerline} = \bar{\bar{x}}$$

Control Limits for the Process Range

$$\text{Upper Control Limit (UCL)} = \bar{R}D_4$$

$$\text{Lower Control Limit (LCL)} = \bar{R}D_3$$

$$\text{Centerline} = \bar{R}$$

Control Limits for *p* When the Process Proportion is Known

$$\text{UCL} = p + 3\sigma_p = p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$\text{LCL} = p - 3\sigma_p = p - 3\sqrt{\frac{p(1-p)}{n}}$$

$$\text{Centerline} = p$$

Control Limits for *p* When the Process Proportion Is Unknown

$$\text{Upper Control Limit (UCL)} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{Lower Control Limit (LCL)} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{Centerline} = \bar{p}$$

where

$$\bar{p} = \frac{\text{Total number of defective units in all of the samples}}{\text{Total number of units sampled}}$$

and n = the sample size.

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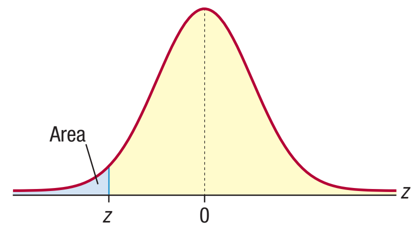
A Standard Normal Distribution

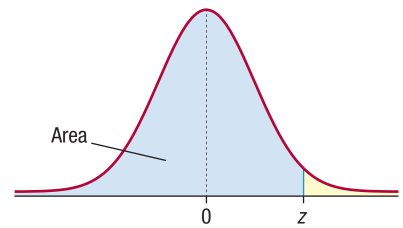
Numerical entries represent the probability that a standard normal random variable is between $-\infty$ and z where $z = \dfrac{x-\mu}{\sigma}$.



z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
–3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
–3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
–3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
–3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
–3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
–2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
–2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
–2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
–2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
–2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
–2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
–2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
–2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
–2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
–2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
–1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
–1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
–1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
–1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
–1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
–1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
–1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
–1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
–1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
–1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
–0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
–0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
–0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
–0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
–0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
–0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
–0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
–0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
–0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
–0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

B Standard Normal Distribution

Numerical entries represent the probability that a standard normal random variable is between $-\infty$ and z where $z = \dfrac{x - \mu}{\sigma}$.

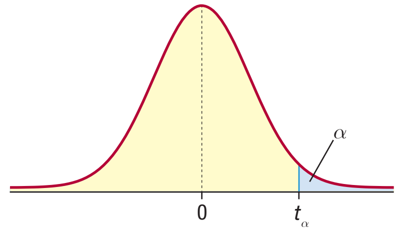


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

D Critical Values of t

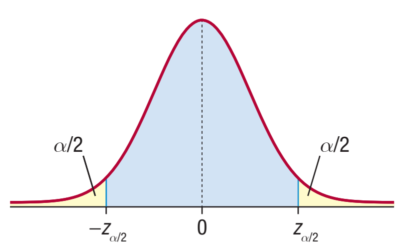
Numerical entries represent the value of t such that the area to the right of the t is equal to α.

Degrees of Freedom	Area to the Right of the Critical Value					
	$t_{0.200}$	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$
1	1.376	3.078	6.314	12.706	31.821	63.657
2	1.061	1.886	2.920	4.303	6.965	9.925
3	0.978	1.638	2.353	3.182	4.541	5.841
4	0.941	1.533	2.132	2.776	3.747	4.604
5	0.920	1.476	2.015	2.571	3.365	4.032
6	0.906	1.440	1.943	2.447	3.143	3.707
7	0.896	1.415	1.895	2.365	2.998	3.499
8	0.889	1.397	1.860	2.306	2.896	3.355
9	0.883	1.383	1.833	2.262	2.821	3.250
10	0.879	1.372	1.812	2.228	2.764	3.169
11	0.876	1.363	1.796	2.201	2.718	3.106
12	0.873	1.356	1.782	2.179	2.681	3.055
13	0.870	1.350	1.771	2.160	2.650	3.012
14	0.868	1.345	1.761	2.145	2.624	2.977
15	0.866	1.341	1.753	2.131	2.602	2.947
16	0.865	1.337	1.746	2.120	2.583	2.921
17	0.863	1.333	1.740	2.110	2.567	2.898
18	0.862	1.330	1.734	2.101	2.552	2.878
19	0.861	1.328	1.729	2.093	2.539	2.861
20	0.860	1.325	1.725	2.086	2.528	2.845
21	0.859	1.323	1.721	2.080	2.518	2.831
22	0.858	1.321	1.717	2.074	2.508	2.819
23	0.858	1.319	1.714	2.069	2.500	2.807
24	0.857	1.318	1.711	2.064	2.492	2.797
25	0.856	1.316	1.708	2.060	2.485	2.787
26	0.856	1.315	1.706	2.056	2.479	2.779
27	0.855	1.314	1.703	2.052	2.473	2.771
28	0.855	1.313	1.701	2.048	2.467	2.763
29	0.854	1.311	1.699	2.045	2.462	2.756
30	0.854	1.310	1.697	2.042	2.457	2.750
40	0.851	1.303	1.684	2.021	2.423	2.704
50	0.849	1.299	1.676	2.009	2.403	2.678
60	0.848	1.296	1.671	2.000	2.390	2.660
70	0.847	1.294	1.667	1.994	2.381	2.648
80	0.846	1.292	1.664	1.990	2.374	2.639
90	0.846	1.291	1.662	1.987	2.368	2.632
100	0.845	1.290	1.660	1.984	2.364	2.626
120	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.842	1.282	1.645	1.96	2.326	2.576



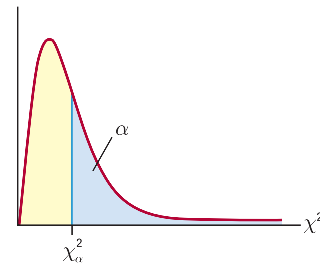
Critical Values of z

Level of Confidence	$z_{\alpha/2}$
0.80	1.28
0.90	1.645
0.95	1.96
0.99	2.575



G Critical Values of χ^2

Numerical entries represent the value of χ^2_α.



Area to the Right of the Critical Value

df	$\chi^2_{0.995}$	$\chi^2_{0.990}$	$\chi^2_{0.975}$	$\chi^2_{0.950}$	$\chi^2_{0.900}$	$\chi^2_{0.100}$	$\chi^2_{0.050}$	$\chi^2_{0.025}$	$\chi^2_{0.010}$	$\chi^2_{0.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169