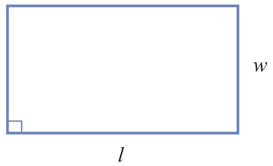
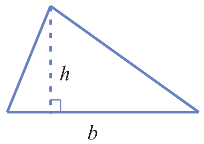
Formulas in Geometry

Area:

Rectangle
$A = lw$

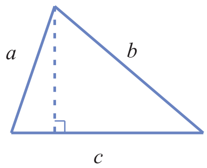


Triangle
$A = \frac{1}{2}bh$

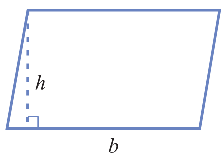


Heron's Formula:
$A = \sqrt{s(s-a)(s-b)(s-c)}$,

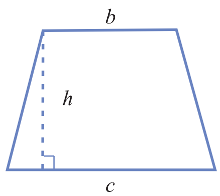
where $s = \frac{a+b+c}{2}$



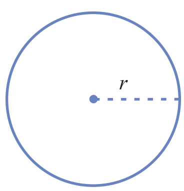
Parallelogram
$A = bh$



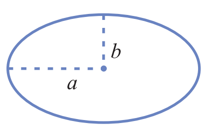
Trapezoid
$A = \frac{1}{2}h(b+c)$



Circle
$A = \pi r^2$

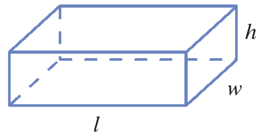


Ellipse
$A = \pi ab$

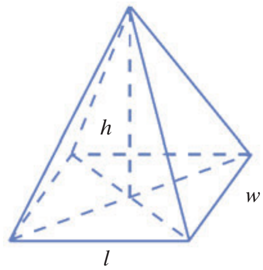


Volume/ Surface Area:

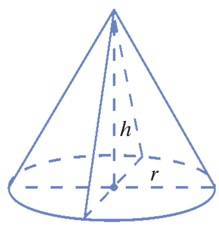
Rectangular Box
$V = lwh$
$SA = 2(lh) + 2(wh) + 2(lw)$



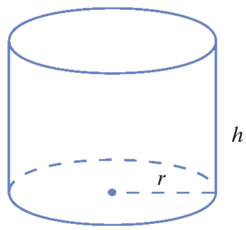
Pyramid
$V = \frac{1}{3}lwh$



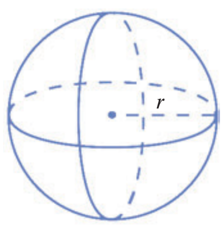
Cone
$V = \frac{1}{3}\pi r^2 h$



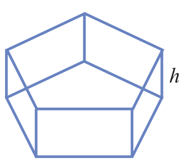
Right Circular Cylinder
$V = \pi r^2 h$
$SA = 2\pi r^2 + 2\pi rh$



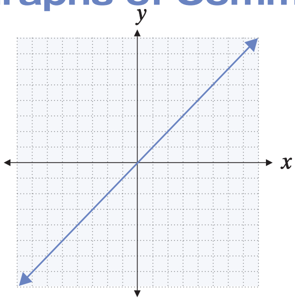
Sphere
$V = \frac{4}{3}\pi r^3$
$SA = 4\pi r^2$



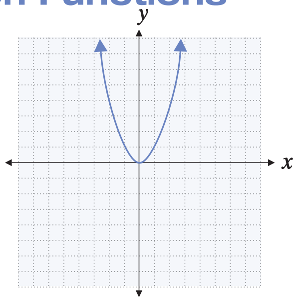
Right Cylinder
$V = (\text{Area of Base})h$



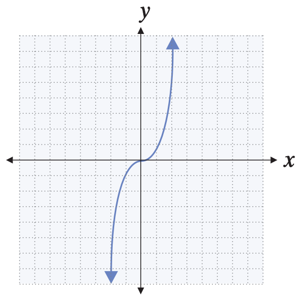
Graphs of Common Functions



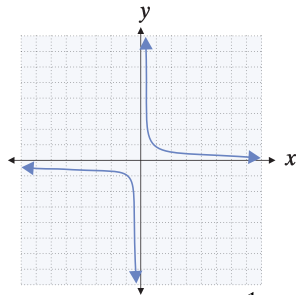
The function $f(x) = x$



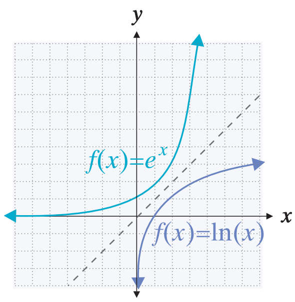
The function $f(x) = x^2$



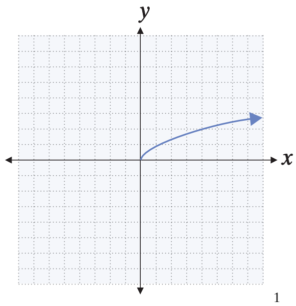
The function $f(x) = x^3$



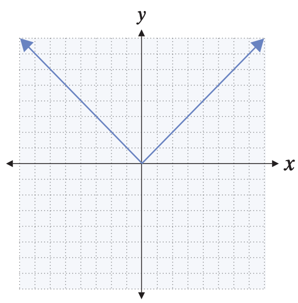
The function $f(x) = \frac{1}{x}$



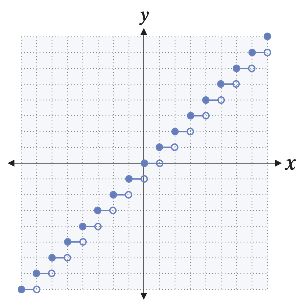
The functions e^x and $\ln(x)$



The function $f(x) = \sqrt{x}$ or $x^{\frac{1}{2}}$

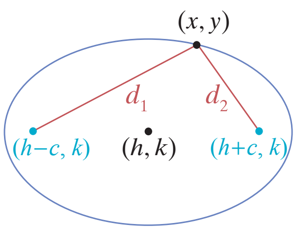


The function $f(x) = |x|$



The function $f(x) = [\![x]\!]$

Conic Sections



ellipse

$a, b > 0$ with $a > b$. The standard form of the equation for the ellipse centered at (h, k) with major axis of length $2a$ and minor axis of length $2b$ is:

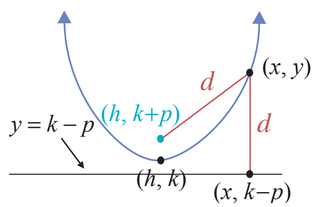
1. $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$
(major axis is horizontal)

2. $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$
(major axis is vertical)

The foci are located c units away from the center of the ellipse where $c^2 = a^2 - b^2$.
The eccentricity is defined as:

$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}$$

Note: an ellipse with $e = 1$ is a circle (see inside back cover).

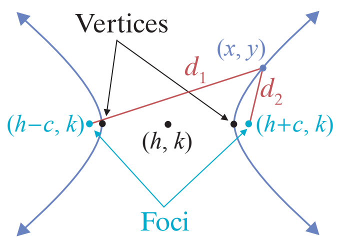


parabola

p is a nonzero real number. The standard form of the equation for the parabola with vertex (h, k) is:

1. $(x-h)^2 = 4p(y-k)$
(vertically-oriented)
The focus is at $(h, k+p)$.
The equation for the directrix is:
$y = k - p$

2. $(y-k)^2 = 4p(x-h)$
(horizontally-oriented)
The focus is at $(h+p, k)$.
The equation for the directrix is:
$x = h - p$



hyperbola

The standard form of the equation for the hyperbola with center at (h, k) is:

1. $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$
(foci are aligned horizontally)

Asymptotes: $y - k = \dfrac{b}{a}(x-h)$

and $y - k = -\dfrac{b}{a}(x-h)$

2. $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$
(foci are aligned vertically)

Asymptotes: $y - k = \dfrac{a}{b}(x-h)$

and $y - k = -\dfrac{a}{b}(x-h)$

The foci are located c units away from the center where $c^2 = a^2 + b^2$.
The vertices are a units away from the center.

Index of Symbols

Symbol	Meaning
$\mathbb{N}$	set of all natural numbers $\{1, 2, 3, 4, 5, ...\}$
$\mathbb{Z}$	set of all integers $\{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$
$\mathbb{Q}$	set of all rational numbers, that is the set of all numbers that can be represented as a ratio
$\mathbb{R}$	set of all real numbers
π	the ratio of the circumference to the diameter of a circle
E	calculator notation for scientific notation (ex. 'aEb' means '$a \times 10^b$')
∞	infinity
i	imaginary unit defined as $\sqrt{-1}$
$\Leftrightarrow$	"is equivalent to"
$\Rightarrow$	"implies"
$\cup$	the union of two sets (i.e. the set of all elements found in either set)
$\cap$	the intersection of two sets (i.e. the set of all elements found in both sets)
$\in$	"is an element of"
$\varnothing$	empty set or the set containing no elements
$\rightarrow$	"approaches"
$\approx$	"approximately equal to"
Δ	"change in"
$\mathbb{R}^2$	the plane of all real x-values by all real y-values (the Cartesian plane)
$\sum_{i=a}^{n}$	summation notation (or sigma notation) used to express the sum of a sequence from a to n
e	base of the natural logarithm
$n!$	"n factorial" stands for the product of all the integers from 1 to n ($0! = 1$ and $1! = 1$)
$\binom{n}{k}$	"n choose k"; combination of n objects taken k at a time

Properties of Absolute Value 1.1

For all real numbers a and b:

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $a \leq |a|$
4. $|ab| = |a||b|$
5. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
6. $|a+b| \leq |a| + |b|$

Properties of Exponents 1.3

1. $a^m \cdot a^n = a^{m+n}$
2. $\frac{a^n}{a^m} = a^{n-m}$
3. $a^{-n} = \frac{1}{a^n}$
4. $(a^n)^m = a^{nm}$
5. $(ab)^n = a^n b^n$
6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Properties of Radicals 1.4

1. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Factoring Special Binomials 1.5

Difference of Two Squares:

$$A^2 - B^2 = (A-B)(A+B)$$

Difference of Two Cubes:

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

Sum of Two Cubes:

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

The Quadratic Formula 2.3

The solutions of the equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagorean Theorem 3.1

Given a right triangle with legs a and b and hypotenuse c:

$$a^2 + b^2 = c^2$$

Distance Formula 3.1

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula 3.1

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Slope of a Line 3.3-3.4

Given two points (x_1, y_1) and (x_2, y_2) on a line, the slope of the line is the ratio:

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Horizontal lines have slope 0.
Vertical lines have undefined slope.
Given a line with slope m:

slope of a parallel line $= m$

slope of a perpendicular line $= -\frac{1}{m}$

Standard Form of a Circle 3.6

The standard form of the equation for a circle of radius r and center (h, k) is

$$(x-h)^2 + (y-k)^2 = r^2.$$

Transformations of Functions 4.4

Horizontal Shifting: The graph of $g(x) = f(x-h)$ is the same as the graph of f, but shifted h units to the right if $h > 0$ and h units to the left if $h < 0$.

Vertical Shifting: The graph of $g(x) = f(x) + k$ is the same as the graph of f, but shifted upward k units if $k > 0$ and downward k units if $k < 0$.

Reflecting with Respect to Axes:

1. The graph of the function $g(x) = -f(x)$ is the reflection of the graph of f with respect to the x-axis.
2. The graph of the function $g(x) = f(-x)$ is the reflection of the graph of f with respect to the y-axis.

Stretching and Compressing:

1. The graph of the function $g(x) = af(x)$ is stretched vertically compared to the graph of f if $a > 1$.
2. The graph of the function $g(x) = af(x)$ is compressed vertically compared to the graph of f if $0 < a < 1$.

Operations with Functions 4.5

Let f and g be functions.

1. $(f+g)(x) = f(x) + g(x)$
2. $(f-g)(x) = f(x) - g(x)$
3. $(fg)(x) = f(x)g(x)$
4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Rational Zero Theorem 5.3

If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ is a polynomial with integer coefficients, then any rational zero of f must be of the form $\frac{p}{q}$, where p is a factor of the constant a_0 and q is a factor of a_n.

Fundamental Theorem of Algebra 5.4

If f is a polynomial of degree n with $n \geq 1$, then f has at least one zero. That is, $f(x) = 0$ has at least one root or solution (note, the solution may be a non-real complex number).

Compound Interest 7.2

An investment of P dollars at an annual interest rate of r, compounded n times per year for t years has an accumulated value of

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}.$$

An investment compounded continuously has an accumulated value of $A(t) = Pe^{rt}$.

Properties of Logarithms 7.3, 7.4

$a > 0$, a is not equal to 1, $x, y > 0$ and r is a real number

$\log_a(x) = y$ and $x = a^y$ are equivalent

$\log_a(a) = 1$

$\log_a(1) = 0$

$\log_a(a^x) = x$

$a^{\log_a(x)} = x$

$\log_a(xy) = \log_a(x) + \log_a(y)$

$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

$\log_a(x^r) = r\log_a(x)$

Change of Base Formula 7.4

$a, b, x > 0$; $a, b \neq 1$;

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Matrices 8.3

The **minor** of the element a_{ij} is the determinant of the $(n-1)$ by $(n-1)$ matrix formed from A by deleting the i^{th} row and the j^{th} column.

The **cofactor** of the element a_{ij} is $(-1)^{i+j}$ times the minor of a_{ij}.

Find the **determinant** of an $n \times n$ matrix by expanding along a fixed row or column.

- To expand along the i^{th} row, each element of that row is multiplied by its cofactor and the n products are then added.
- To expand along the j^{th} column, each element of that column is multiplied by its cofactor and the n products are then added.

Cramer's Rule 8.3

A linear system of n equations $x_1, x_2, \ldots, x_n$ can be written in the form:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases}$$

The solution of the system is given by the n formulas $x_1 = \frac{D_{x_1}}{D}$, $x_2 = \frac{D_{x_2}}{D}$, ..., $x_n = \frac{D_{x_n}}{D}$, where D is the determinant of the coefficient matrix and D_{x_i} is the determinant of the same matrix with the i^{th} column of constants replaced by the column of constants $b_1, b_2, \ldots, b_n$.

Scalar Multiplication 8.4

cA = the matrix such that the element in the i^{th} row and j^{th} column is equal to ca_{ij}.

Matrix Addition 8.4

$A + B$ = the matrix such that the $c_{ij} = a_{ij} + b_{ij}$ (c_{ij} is the element in the i^{th} row and j^{th} column of $A + B$).

Matrix Multiplication 8.4

AB = the matrix such that $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$. (The length of each row in A must be the same as the length of each column on B.)

Properties of Sigma 9.1

1. $\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

2. $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$

3. $\sum_{i=1}^{n} a_i = \sum_{i=1}^{k} a_i + \sum_{i=k+1}^{n} a_i$ (for any $1 \le k \le n-1$)

Summation Formulas 9.1

1. $\sum_{i=1}^{n} 1 = n$

2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Arithmetic Sequences 9.2

General term: (where d is the common difference)

$$a_n = a_1 + (n-1)d$$

Partial sum: $S_n = na_1 + d\left(\frac{(n-1)n}{2}\right) = \left(\frac{n}{2}\right)(a_1 + a_n)$

Geometric Sequences 9.3

General term: $a_n = a_1 r^{n-1}$, where $\left(r = \frac{a_{n+1}}{a_n}\right)$

Partial sum: $S_n = \frac{a_1(1-r^n)}{1-r}$

Infinite sum: $S = \sum_{n=0}^{\infty} a_1 r^n = \frac{a_1}{1-r}$

Permutation Formula 9.5

$${}_nP_k = \frac{n!}{(n-k)!}$$

Combination Formula 9.5

$${}_nC_k = \frac{n!}{k!(n-k)!}$$

(Note that ${}_nC_k$ may also be denoted $\binom{n}{k}$.)

Binomial Coefficient 9.5

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Binomial Theorem 9.5

$$(A+B)^n = \sum_{k=0}^{n}\binom{n}{k}A^k B^{n-k}$$

Multinomial Coefficients 9.5

$$\binom{n}{k_1, k_2, \ldots k_r} = \frac{n!}{k_1!k_2!\ldots k_r!}$$

Multinomial Theorem 9.5

$$(A_1 + A_2 + \ldots + A_r)^n =$$

$$\sum_{k_1+k_2+\ldots+k_r=n}\binom{n}{k_1,\ k_2,\ldots,k_r}A_1^{k_1}A_2^{k_2}\ldots A_r^{k_r}$$