

Chapter 2. Multiplication and Division of Whole Numbers

In the last chapter you saw that addition and subtraction were inverse mathematical operations. For example, a pay raise of 50 cents an hour is the opposite of a 50 cents an hour pay cut. When you have completed this chapter, you'll understand that multiplication and division are also inverse mathematical operations.

2.1 Multiplication with Whole Numbers

The Multiplication Table

Learning the multiplication table shown below is a basic skill that must be mastered. Do you have to memorize this table? Yes! Can't you just use a calculator? No! You must know this table by heart to be able to multiply numbers, to do division, and to do algebra. To be blunt, until you memorize this entire table, you won't be able to progress further than this page.

MULTIPLICATION TABLE

\times	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Let's get a couple of things out of the way. First, any number times 0 is 0. When we multiply two numbers, we call our answer the *product* of those two numbers. The numbers that are multiplied together are called the *factors* of the product. How much is the product of 0 and 1? $0 \times 1 = 0$.

If you don't already know the product of every combination of the numbers in the table, what you should definitely do is memorize those you don't know.

There are a few ways a multiplication problem can be expressed. Let's consider how to express the multiplication of 23 by 8. We can do it each of these ways:

The product of 23 and 8

$$23 \cdot 8$$

23 times 8

$$23(8)$$

23×8

$$(23)(8)$$

$$\begin{array}{r} 23 \\ \times 8 \\ \hline \end{array}$$

When multiplying two-digit, three-digit and larger numbers, we almost always express multiplication vertically because that's the easiest way to find the product. The numbers we multiply together are called the *factors* and the answer is called the *product*.

Example 1:

$$\begin{array}{r} 23 \\ \times 8 \\ \hline \end{array}$$

SOLUTION: There's a long method and a short method to working out multiplication problems. First we'll do the long method:

$$23 = 20 + 3$$

$$\begin{array}{r} \times 8 \\ 24 \quad (3 \times 8) \\ +160 \quad (20 \times 8) \\ \hline 184 \quad (24 + 160) \end{array}$$

The long method helps you to see how the place value of the digit affects the multiplication process.

The short method is the method we'll use from now on.

$$\begin{array}{r} \overset{2}{2}3 \\ \times 8 \\ \hline 184 \end{array}$$

Here's what we do: $3 \times 8 = 24$. Write down the 4 and carry the 2; $2 \times 8 = 16$. $16 + 2 = 18$. Write down the 18 to get the answer 184.

Example 2:

$$\begin{array}{r} 465 \\ \times 7 \\ \hline \end{array}$$

SOLUTION: Here's how we do this problem using the long method. Pay attention to the place values.

$$\begin{array}{r} 465 = 400 + 60 + 5 \\ \times 7 \\ \hline 35 \quad (7 \times 5) \\ 420 \quad (7 \times 60) \\ \underline{2800} \quad (7 \times 400) \\ 3255 \quad (35 + 420 + 2800) \end{array}$$

Here's the short method:

$$\begin{array}{r} \overset{4}{4}\overset{3}{6}5 \\ \times 7 \\ \hline 3255 \end{array}$$

Here's what we do: $5 \times 7 = 35$; write down the 5 and carry the 3. $6 \times 7 = 42$ plus the 3 we carried is 45. Write down the 5 and carry the 4. $4 \times 7 = 28$ plus the 4 we carried is 32.

Problem Set:

- 1) Find the product of 36 and 5. 2) $\begin{array}{r} 29 \\ \times 9 \\ \hline \end{array}$ 3) $\begin{array}{r} 162 \\ \times 4 \\ \hline \end{array}$
- 4) Find the product of 396 and 3.

Solutions to Problem Set:

- 1) $\begin{array}{r} 36 \\ \times 5 \\ \hline 180 \end{array}$ 2) $\begin{array}{r} 29 \\ \times 9 \\ \hline 261 \end{array}$ 3) $\begin{array}{r} 162 \\ \times 4 \\ \hline 648 \end{array}$ 4) $\begin{array}{r} 396 \\ \times 3 \\ \hline 1188 \end{array}$

Let's try some more complicated problems.

Example 3: $\begin{array}{r} 38 \\ \times 46 \\ \hline \end{array}$

SOLUTION:

$$\begin{array}{r} 38 \\ \times 46 \\ \hline 228 \\ 152 \\ \hline 1748 \end{array}$$

We begin this problem just the way we did the previous problems.

We multiply 38 by 6. Now comes the tricky part. When we start multiplying by 4, we need to move one space to the left, because 4 really represents 40, since $46 = 40 + 6$. When multiplying by 4 start writing your answer directly under 4.

The main thing to remember is to move one space to the left when you put your answer on the second row of numbers.

Example 4: $\begin{array}{r} 275 \\ \times 394 \\ \hline \end{array}$

SOLUTION:

$$\begin{array}{r} 275 \\ \times 394 \\ \hline 1100 \\ 2475 \\ 825 \\ \hline 108350 \end{array}$$

Once again the important thing to remember is to start one place to the left when you begin your second row of numbers, and another space to the left when you begin your third row of numbers. First you multiply 275 by 4 and get 1100. When you multiply by 9, you must place the 5 one space to the left, under the second 0 rather than under the first 0. And when you multiply by 3, place the 5 another one place to the left, directly under the 7.

Problem Set:

- 1) $\begin{array}{r} 71 \\ \times 28 \\ \hline \end{array}$ 2) $\begin{array}{r} 85 \\ \times 49 \\ \hline \end{array}$ 3) $\begin{array}{r} 240 \\ \times 65 \\ \hline \end{array}$ 4) $\begin{array}{r} 8623 \\ \times 539 \\ \hline \end{array}$

Solutions to Problem Set:

1) $\begin{array}{r} 71 \\ \times 28 \\ \hline 568 \\ 142 \\ \hline 1988 \end{array}$	2) $\begin{array}{r} 85 \\ \times 49 \\ \hline 765 \\ 340 \\ \hline 4165 \end{array}$	3) $\begin{array}{r} 240 \\ \times 65 \\ \hline 1200 \\ 1440 \\ \hline 15600 \end{array}$	4) $\begin{array}{r} 8623 \\ \times 539 \\ \hline 77607 \\ 25869 \\ 43115 \\ \hline 4647797 \end{array}$
1,988	4,165	15,600	4,647,797

Multiplying Whole Numbers by Powers of Ten

The following numbers are powers of ten: 10, 100, 1,000, 10,000, and 100,000. These are only *some* examples of powers of ten. What these particular powers of ten all have in common is their first digit, 1, is followed by only zeros. In section 2.2 we'll discuss this at greater length. In this section we'll look at how multiplying a number by a power of ten effects the number.

Quick, what is the product of 145 and 10? The answer is 1,450. How did we do it that fast? We just tacked a 0 on to 145. Next problem: How much is $4,280 \times 10$? The answer is 42,800. Again, we just attached a zero to the end of 4,280. Are you ready for a sweeping generality? To multiply any whole number by ten, just attach a zero to the right of the number. What is the product of 145 and 100? The answer is 14,500. What did we do? We attached two zeros to 145. To multiply any whole number by one hundred, just attach two zeros to the right of the number. Can you guess what we do if we multiply a whole number by 1,000? We just attach three zeros to the right of the number. For example, $145 \times 1,000 = 145,000$. So we can make a general statement: To multiply any whole number by a power of ten, we just attach the same number of zeros to the right of the number, as are in the power of ten. If we multiply by 10 we attach one zero. If we multiply by 100 we attach two zeros. If we multiply by 1,000 we attach three zeros, and so on.

Example 5: Multiply 456 by a) 10 b) 100 c) 1,000 d) 10,000

SOLUTION: a) 4,560 b) 45,600 c) 456,000 d) 4,560,000

Problem Set:

- 1) $23 \times 1,000$ 2) 230×10 3) $500(100)$ 4) $1,237(1,000)$

Solutions to Problem Set:

- 1) 23,000 2) 2,300 3) 50,000 4) 1,237,000

Multiplying by Numbers that End in Zero

The following problem is an example of a shortcut we can use to multiply numbers that end in zeros.

Example 6:

$$\begin{array}{r} 586 \\ \times 500 \\ \hline 293,000 \end{array}$$

Multiply 586×5 , then attach the two zeros to the right of 2930 to get the product 293,000.

SOLUTION:

$$\begin{array}{r} 586 \\ \times 500 \\ \hline 293,000 \end{array}$$

Here's another way to look at this problem.

To multiply this as fast and easily as possible, we'll rewrite this problem.

We'll just drop down the two zeros and multiply 586 by 5.

Example 7:

$$\begin{array}{r} 739 \\ \times 3000 \\ \hline \end{array}$$

SOLUTION:

$$\begin{array}{r} 739 \\ \times 3000 \\ \hline 2,217,000 \end{array}$$

Multiply 739 by 3; then attach the three zeros to the right of 2,217 to get the product 2,217,000.

Problem Set:

1) $\begin{array}{r} 427 \\ \times 400 \\ \hline \end{array}$

2) $\begin{array}{r} 872 \\ \times 600 \\ \hline \end{array}$

3) $\begin{array}{r} 948 \\ \times 800 \\ \hline \end{array}$

4) $\begin{array}{r} 5387 \\ \times 9000 \\ \hline \end{array}$

Solutions to Problem Set:

1) $\begin{array}{r} 427 \\ \times 400 \\ \hline 170,800 \end{array}$

2) $\begin{array}{r} 872 \\ \times 600 \\ \hline 523,200 \end{array}$

3) $\begin{array}{r} 948 \\ \times 800 \\ \hline 758,400 \end{array}$

4) $\begin{array}{r} 5387 \\ \times 9000 \\ \hline 48,483,000 \end{array}$

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2.1 Workbook

Perform the following multiplication problems.

1) 4×25

2) 8×19

3) 19×8

4) 25×12

5) 13×13

6) 91×88

7) Find the product of 25 and 368.

8) Find the product of 679 and 372.

9)
$$\begin{array}{r} 2,345 \\ \times 123 \\ \hline \end{array}$$

10)
$$\begin{array}{r} 34,569 \\ \times 1,456 \\ \hline \end{array}$$

11)
$$\begin{array}{r} 28,009 \\ \times 20,111 \\ \hline \end{array}$$

12)
$$\begin{array}{r} 199,909 \\ \times 68,735 \\ \hline \end{array}$$

Multiply each of these numbers by 10:

13) 4

14) 600

15) 70

16) 9,010

17) Multiply each of these numbers by 10, 100, 1000.

a) 145

b) 2,000

c) 876

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Multiply each of these numbers by 100:

18) 72

19) 587

20) 0

21) 12,999

22) 100

23) 100,000

Multiply each of these sets of numbers:

24) $\begin{array}{r} 39 \\ \times 10 \end{array}$

25) $\begin{array}{r} 199 \\ \times 100 \end{array}$

26) $\begin{array}{r} 3490 \\ \times 1000 \end{array}$

27) $\begin{array}{r} 45 \\ \times 40 \end{array}$

28) $\begin{array}{r} 69999 \\ \times 400 \end{array}$

29) $\begin{array}{r} 22 \\ \times 90 \end{array}$

30) $\begin{array}{r} 4876 \\ \times 600 \end{array}$

31) $\begin{array}{r} 1000 \\ \times 1000 \end{array}$

2.2 Exponents

$$2^2 = 2(2) = 4$$

$$2^3 = 2(2)(2)$$

$$2^4 = 2(2)(2)(2)$$

$$2^5 = 2(2)(2)(2)(2)$$

$$4(2)$$

$$4(2)(2)$$

$$4(2)(2)(2)$$

$$8$$

$$8(2)$$

$$8(2)(2)$$

$$16$$

$$16(2)$$

$$32$$

$$3^2 = 3(3) = 9$$

$$3^3 = 3(3)(3)$$

$$6^3 = 6(6)(6)$$

$$1^6 = 1(1)(1)(1)(1)(1) = 1$$

$$9(3)$$

$$36(6)$$

$$27$$

$$216$$

All these problems have one thing in common; they're all multiplying the same number times itself. There's a way to show repeated multiplication of a number times itself. Look at this problem: $4(4)(4)$. This can be written as 4^3 . This is read four to the third power, or we could say four cubed. The 4 is called the **base** and the 3 is called the **power** or the **exponent**. The base is the number you multiply times itself. The exponent, or power, tells you how many of the base you multiply together. Three to the fourth power is written 3^4 , and means $3(3)(3)(3)$, which is equal to 81.

Let's look at a few more examples.

Example 1: 1^4

SOLUTION: $1^4 = 1(1)(1)(1) = 1$

This is read one to the fourth power. 1 is the base; 4 is the exponent. *One to any power is 1.*

Example 2: 5^2

SOLUTION: $5^2 = 5(5) = 25$

This is read five squared. The 5 is the base and the 2 is the exponent.

Example 3: 0^3

SOLUTION: $0^3 = 0(0)(0) = 0$

This is read zero cubed. The 0 is the base, the 3 is the exponent. *Zero to any power (except 0) is 0.*

Example 4: 2^6

SOLUTION: $2^6 = 2(2)(2)(2)(2)(2) = 64$

This is read two to the sixth power. The 2 is the base and the 6 is the power.

Let's look at a special case:

$$2^0 = 1$$

$$3^0 = 1$$
 A basic rule of exponents is: *Anything except 0 to the zero power is 1* (0^0 is not defined).

$$10^0 = 1$$

We can't explain this exception to you until you've taken more math courses.

In the last section we mentioned powers of ten. Let's take a closer look at the number ten raised to a power.

Example 5:

$$10^0 = 1$$

$$10^1 = 10$$

$$10^2 = 10(10) = 100$$

$$10^3 = 10(10)(10) = 1,000$$

$$10^4 = 10(10)(10)(10) = 10,000$$

$$10^5 = 10(10)(10)(10)(10) = 100,000$$

When the number ten is raised to a power its result is called a power of ten. Notice the amount of zeros in the power of ten is the same as the exponent ten was raised to.

Problem Set:

Simplify the following problems and state which number is the base and which is the exponent or power.

1) 8^2

2) 7^1

3) 4^0

4) 2^4

5) 1^6

6) 5^4

7) 10^5

Solutions to Problem Set:

1) $8^2 = 8(8) = 64$

8 is the base.
2 is the exponent.

2) $7^1 = 7$

7 is the base.
1 is the exponent.

3) $4^0 = 1$

4 is the base.
0 is the exponent.

4) $2^4 = 2(2)(2)(2) = 16$

2 is the base.
4 is the exponent.

5) $1^6 = 1(1)(1)(1)(1)(1) = 1$

1 is the base.
6 is the exponent.

6) $5^4 = 5(5)(5)(5)$

$$= 25(5)(5)$$
$$= 125(5)$$
$$= 625$$

5 is the base.
4 is the exponent.

7) $10^5 = 10(10)(10)(10)(10) = 100,000$

10 is the base.
5 is the exponent.

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Simplify the following problems.

1) 5^2

2) 8^2

3) 2^5

4) 2^8

5) 2^3

6) 3^3

7) 5^4

8) 1^{10}

9) 1^{100}

10) 0^5

11) 8^0

12) 6^0

13) 6^2

14) 11^2

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15) 11^3

16) 10^1

17) 10^2

18) 10^3

19) 10^4

20) 10^5

21) 10^0

22) 1^0

2.3 Squares and Roots of Whole Numbers

In the last section we worked with exponents. Some of the problems had exponents of 0, 1, 2, 3, 4, etc. In this section we'll work on problems whose exponent is 2 and how to reverse the process of raising a number to a power by using radicals. The product of a number times itself is called a *perfect square*. The product of 3 and 3 is 9. Nine is a perfect square. Another way to show 3×3 is 3^2 . This can be read, three to the second power or three squared. The word squared is commonly used instead of saying to the second power. In words we would say three squared is nine. In mathematical symbols it would look like this: $3^2 = 9$. The 3 is called the base and the 2 is called the exponent or power. When the power is 2 the base is multiplied times itself.

Example 1: Find the value for 4 squared. Name the base and the power.

SOLUTION: $4^2 = 4(4) = 16$

The four is the base and the two is the power. 16 is called the perfect square.

Example 2: Find the value for 10 squared. Name the base and the exponent.

SOLUTION: $10^2 = 10(10) = 100$

The ten is the base and the two is the exponent. 100 is called the perfect square.

Example 3: See if you can fill in the following table.

PROBLEM	BASE	POWER	PERFECT SQUARE
0^2			
1^2			
2^2			
3^2			
4^2			
5^2			
6^2			
7^2			
8^2			
9^2			
10^2			
11^2			
12^2			

SOLUTION: To example 3:

PROBLEM	BASE	POWER	PERFECT SQUARE
0^2	0	2	0
1^2	1	2	1
2^2	2	2	4
3^2	3	2	9
4^2	4	2	16
5^2	5	2	25
6^2	6	2	36
7^2	7	2	49
8^2	8	2	64
9^2	9	2	81
10^2	10	2	100
11^2	11	2	121
12^2	12	2	144

Before we continue, we strongly advise you to memorize the perfect squares in the previous table.

Example 4: Find the value of 15^2

SOLUTION:

$$\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ 150 \\ \hline 225 \\ 15^2 = 225 \end{array}$$

Problem Set:

Find the perfect square for each of the following numbers.

- 1) 7 2) 11 3) 5 4) 23 5) 100 6) 0

Solutions to Problem Set:

- 1) $7^2 = 7(7) = 49$ 2) $11^2 = 11(11) = 121$ 3) $5^2 = 5(5) = 25$
4) $23^2 = 23(23) = 529$ 5) $100^2 = 100(100) = 10,000$ 6) $0^2 = 0(0) = 0$

Let's take a look at a type of problem that is the reverse of what we just did.

Suppose we asked you what number times itself is 9. We're sure you could tell us the number is 3, because 3 times itself is 9. Instead of writing this type of problem in words, we can write it in symbols in the following way: $\sqrt{9}$. The $\sqrt{\quad}$ symbol is called a *radical*. If we use this symbol, $\sqrt{\quad}$, the little 2 means square root. Square root asks us what number times itself equals the number under the radical. The 2 is used to indicate the base is written twice and multiplied together to get the number under the radical. If no number is written where the 2 is, it's assumed the $\sqrt{\quad}$ means square root. The number under the radical is called the *radicand*. In the term $\sqrt{9} = 3$, the 9 is the radicand and the root is assumed to be 2 for square root. $\sqrt{9}$ is read the square root of 9.

Example 5: Find $\sqrt{64}$.

SOLUTION:

What number times itself is 64? By now you should know $8(8) = 64$, so the answer is 8. $\sqrt{64} = 8$. The square root of 64 is 8.

Suppose you didn't remember that $8(8) = 64$. To figure this out, you could refer to the table you filled in Example 3 or you could try using trial and error. Suppose we guessed the answer is 10: $10^2 = 100$; 100 is too big. Suppose we guessed the answer is 7: $7^2 = 49$; 49 is a little too small. Next we'll guess 8: $8^2 = 64$. It looks like we have our answer.

Example 6: Fill in the following table.

PROBLEM	ROOT	RADICAND	SQUARE ROOT
$\sqrt{0}$			
$\sqrt{1}$			
$\sqrt{4}$			
$\sqrt{9}$			
$\sqrt{16}$			
$\sqrt{25}$			
$\sqrt{36}$			
$\sqrt{49}$			
$\sqrt{64}$			
$\sqrt{81}$			
$\sqrt{100}$			
$\sqrt{121}$			
$\sqrt{144}$			

SOLUTION: To example 6:

PROBLEM	ROOT	RADICAND	SQUARE ROOT
$\sqrt{0}$	2	0	0
$\sqrt{1}$	2	1	1
$\sqrt{4}$	2	4	2
$\sqrt{9}$	2	9	3
$\sqrt{16}$	2	16	4
$\sqrt{25}$	2	25	5
$\sqrt{36}$	2	36	6
$\sqrt{49}$	2	49	7
$\sqrt{64}$	2	64	8
$\sqrt{81}$	2	81	9
$\sqrt{100}$	2	100	10
$\sqrt{121}$	2	121	11
$\sqrt{144}$	2	144	12

Example 7: Find $\sqrt{784}$.

SOLUTION:

What number times itself is 784? This is one most people (including us) don't have memorized, so we'll have to use the trial and error method. This number's pretty big so we'll guess 20. $20^2 = 400$; 400 is too small. Let's try 30. $30^2 = 900$; 30 is too big. 20 is too small and 30 is too big, so let's try 25. $25^2 = 25(25) = 625$, which is a little too small. Let's try 28. $28^2 = 28(28) = 784$. Now we know $\sqrt{784} = 28$. The square root of 784 is 28.

Later in this book we'll show you another method for finding roots. The other method uses prime factorization.

Now that we know how to find square roots, let's take a look at some other roots. Suppose we asked you what number times itself, times itself again is 8? Let "?" represent the number we're trying to find. In other words, $(?)(?)(?) = 8$. Notice that the same number is written three times and multiplied. In mathematical symbols the way we would represent what number times itself, times itself is 8 is $\sqrt[3]{8}$. The answer to the question is $\sqrt[3]{8} = 2$, because $2(2)(2) = 4(2) = 8$. This problem is read the cube root of 8 is 2.

Example 8: Find $\sqrt[3]{64}$.

SOLUTION:

What number times itself, times itself is 64? This is read the cube root of 64. $4(4)(4) = 16(4) = 64$ so $\sqrt[3]{64} = 4$.

Example 9: Find $\sqrt[4]{16}$.

SOLUTION:

$(?)(?)(?)(?) = 16$. This is read the fourth root of 16. To find the correct number we can use the trial and error method. Let's start with 3. $3(3)(3)(3) = 9(3)(3) = 27(3) = 81$. 81 is larger than 64 so we have to try a number smaller than 3. Let's try 2 and see what happens. $2(2)(2)(2) = 4(2)(2) = 8(2) = 16$. Now we know $\sqrt[4]{16} = 2$.

Example 10: Find $\sqrt[5]{1,024}$.

SOLUTION:

$(?)(?)(?)(?)(?) = 1,024$. This is read the fifth root of 1,024. Again we'll use the trial and error method. Let's try 4. $4(4)(4)(4)(4) = 16(4)(4)(4) = 64(4)(4) = 256(4) = 1,024$. Now we know $\sqrt[5]{1,024} = 4$.

Problem Set:

Simplify the following.

- 1) $\sqrt{81}$ 2) $\sqrt{961}$ 3) $\sqrt[3]{1,000}$ 4) $\sqrt[4]{256}$ 5) $\sqrt[5]{243}$

Solutions to Problem Set:

- 1) 9 2) 31 3) 10 4) 4 5) 3

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2.3 Workbook

Simplify the following problems. State the base, power, and perfect square.

1) 6^2

2) 11^2

3) 20^2

4) 25^2

5) 30^2

6) 0^2

7) 27^2

8) 48^2

Simplify the following problems. State the root and radicand.

9) $\sqrt{25}$

10) $\sqrt{144}$

11) $\sqrt{100}$

12) $\sqrt{169}$

13) $\sqrt{81}$

14) $\sqrt{16}$

15) $\sqrt{324}$

16) $\sqrt{961}$

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17) $\sqrt{289}$

18) $\sqrt{361}$

19) $\sqrt[3]{27}$

20) $\sqrt[3]{64}$

21) $\sqrt[3]{125}$

22) $\sqrt[3]{216}$

23) $\sqrt[4]{16}$

24) $\sqrt[4]{81}$

25) $\sqrt[4]{625}$

26) $\sqrt[4]{1,296}$

27) $\sqrt[5]{32}$

28) $\sqrt[5]{243}$

29) $\sqrt{1}$

30) $\sqrt[3]{1}$

31) $\sqrt{0}$

32) $\sqrt[3]{0}$

33) $\sqrt[4]{1}$

34) $\sqrt[10]{1}$

35) $\sqrt[4]{0}$

36) $\sqrt[3]{0}$

2.4 Division with Whole Numbers

Four friends share an eight-slice pizza. How many slices does each person get? The answer is 2. What we did is divide 8 by 4. There are several ways we can express this division:

$$8 \div 4 \quad 4 \overline{)8} \quad 8 \text{ divided by } 4 \quad 4 \text{ into } 8 \quad \frac{8}{4} \quad \frac{8}{4}$$

We would read this as eight divided by four, or we could say the *quotient* of eight and four. We call our answer 2, the *quotient* of 8 divided by 4. The 8 is called the *dividend* and the 4 is called the *divisor*. The dividend always goes inside the division box and the divisor always goes on the outside of the division box. The quotient is always on the top of the box, so it should look like this: The number before the \div is inside the division box.

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array} \quad 4 \overline{)2}8$$

Example 1: What is the quotient of 12 and 4?

SOLUTION: Division is the inverse of multiplication, so this question is really asking us what number times 4 is 12? The quotient of 12 and 4 is 3.

$$4 \overline{)3}12$$

We can always check our division by multiplication. The quotient times the divisor is equal to the dividend.

$$\text{quotient} \times \text{divisor} = \text{dividend} \quad 3(4) = 12$$

Example 2: What is the quotient of 15 and 3?

SOLUTION: $15 \div 3 = 5$ or $\frac{15}{3} = 5$ The quotient of 15 and 3 is 5.

The divisor always goes on the right side of the \div symbol.

Example 3: $576 \div 8$

SOLUTION: Ask yourself, does 8 go into 5? The answer is no, so we do not begin writing our answer over the 5. Does 8 go into 57? Yes, so we start writing our answer over the 7. We know $8(7) = 56$. So we write 7 over the 7 in the 57 and 56 below 57. Next, subtract 56 from 57 and get 1, then bring down the next number, 6. How many times does 8 go into 16? It goes twice, so we write a 2 over the 6 in 576. Now subtract.

$$\begin{array}{r} 7 \\ 8 \overline{)576} \\ - 56 \\ \hline 72 \\ 8 \overline{)576} \\ - 56 \\ \hline 16 \\ - 16 \\ \hline 0 \end{array}$$

Let's check our answer; quotient \times divisor = dividend.

Check:

$$\begin{array}{r} 72 \\ \times 8 \\ \hline 576 \end{array}$$

Three friends share an eight-slice pizza. How many slices does each person get? This time the friends can't have the same number of slices of pizza, because three doesn't divide into eight evenly. If they each have two slices, there will be two slices left over. The two leftover slices are called the *remainder*. Sometimes a number doesn't divide into another number evenly. When this happens, the number we're left with is called the *remainder*. Let's take a look at some division problems that have remainders.

Example 4: $2456 \div 25$

SOLUTION:

$$\begin{array}{r} 9 \\ 25 \overline{)2456} \\ \underline{-225} \\ 20 \end{array}$$

$$\begin{array}{r} 98 \text{ R}6 \\ 25 \overline{)2456} \\ \underline{-225} \\ 206 \\ \underline{-200} \\ 6 \end{array}$$

25 does not go into 24, so we don't begin writing the quotient over the four. 25 does go into 245 nine times. Put the 9 over the 5.

Multiply 9 times 25 and put 225 under the 245. Subtract and bring down the next number.

25 goes into 206 eight times. Multiply 25 times 8. Subtract 200 from 206. This leaves us with a remainder of 6.

To check our answer when there's a remainder, just multiply the quotient times the divisor and then add the remainder. You should get the dividend.

quotient \times divisor + remainder = dividend

Check: $98 \times 25 + 6 = 2,456$.

We have found that our students get confused when a quotient has zeros in it. They're not sure where to put the zeros or how many there should be. Hopefully the next example will make it easier for you to understand this type of problem.

Example 5: $30045 \div 15$

SOLUTION:

$$\begin{array}{r} 2 \\ 15 \overline{)30045} \\ \underline{-30} \\ 0 \end{array}$$

$$\begin{array}{r} 20 \\ 15 \overline{)30045} \\ \underline{-30} \\ 00 \end{array}$$

$$\begin{array}{r} 200 \\ 15 \overline{)30045} \\ \underline{-30} \\ 004 \end{array}$$

$$\begin{array}{r} 2003 \\ 15 \overline{)30045} \\ \underline{-30} \\ 0045 \\ \underline{-45} \\ 0 \end{array}$$

The key to getting the correct answer is being very careful where you write the digits of the quotient. **You also have to write zeros in the quotient when a division can't be done.** 15 goes into 30 twice.

When we multiply and subtract, this leaves us with 0. The next number we bring down is also a 0. 15 doesn't go into 00. We have to show this by writing a 0 in the quotient.

Now we bring down the next number, 4, but 15 doesn't go into 4 either, so we have to put another 0 in the quotient. Bring down the 5. Now divide 15 into 45 and we're finished.

Notice that each digit in the quotient goes directly over the last digit in its corresponding product.

Example 6: $4 \div 4$

SOLUTION: *Any number (except 0) divided by itself is 1.*

$$\begin{array}{r} 1 \\ 4 \overline{)4} \\ \underline{-4} \\ 0 \end{array}$$

Problem Set:

Now it's time for you to try a few. Be careful where you put your zeros. Also, we suggest you check your answers.

1) $46 \overline{)9200}$ 2) $21 \overline{)6405}$ 3) $20 \overline{)11592}$ 4) $51 \overline{)515151}$

Solutions to Problem Set:

$$\begin{array}{l} 1) \begin{array}{r} 200 \\ 46 \overline{)9200} \\ \underline{-92} \\ 00 \\ \underline{-00} \\ 00 \\ \underline{-00} \\ 0 \end{array} \end{array}$$
$$\begin{array}{l} 2) \begin{array}{r} 305 \\ 21 \overline{)6405} \\ \underline{-63} \\ 10 \\ \underline{-00} \\ 105 \\ \underline{-105} \\ 0 \end{array} \end{array}$$
$$\begin{array}{l} 3) \begin{array}{r} 579 \text{ R}12 \\ 20 \overline{)11592} \\ \underline{-100} \\ 159 \\ \underline{-140} \\ 192 \\ \underline{-180} \\ 12 \end{array} \end{array}$$
$$\begin{array}{l} 4) \begin{array}{r} 10101 \\ 51 \overline{)515151} \\ \underline{-51} \\ 05 \\ \underline{-00} \\ 51 \\ \underline{-51} \\ 05 \\ \underline{-00} \\ 51 \\ \underline{-51} \\ 0 \end{array} \end{array}$$

Division with Zeros

Let's take a look at two very important types of division problems:

- $0 \div 4$, which is read 0 divided by 4, or the quotient of 0 and 4, or $0/4$, or $\frac{0}{4}$
- $4 \div 0$, which is read 4 divided by 0, or the quotient of 4 and 0, or $4/0$, or $\frac{4}{0}$

These two problems look very similar, but have very different answers.

Let's take a closer look at $0 \div 4$. The 0 is the dividend, so it goes inside the division box, and 4 is the divisor, so it goes outside the box.

$$\begin{array}{r} 4 \overline{)0} \end{array} \quad \text{We ask ourselves, what number times 4 would give us 0? 0, of course.}$$

So 0 divided by 4 is 0.

$$\begin{array}{r} 0 \\ 4 \overline{)0} \end{array} \quad \text{Check: } 0 \cdot 4 = 0$$

How about $0 \div 7$? What number times seven would give us 0? 0, of course.

$$\begin{array}{r} 0 \\ 7 \overline{)0} \end{array} \quad \text{Check: } 0 \cdot 7 = 0$$

So, we can say that *zero divided by any number is zero*. The only exception to this rule is $0 \div 0$, which is an indeterminate form. Don't worry about this for now.

$$0 \div 8 = 0$$

$$0 \div 10 = 0$$

$$0 \div 345 = 0$$

$$\frac{0}{10} = 0$$

Now, let's look at the other type of problem, $4 \div 0$. Using a division box, the 0 is on the outside and the 4 is inside the box, like this: $0 \overline{)4}$. Let's ask ourselves, what number times 0 is 4? You can't think of a number times 0 that will give us 4, because there isn't any. Why not? Because 0 times any number is always 0. Whenever we divide by 0, the answer is called *undefined*. So, $4 \div 0$ is undefined. This is a very important concept in mathematics. But don't worry; we'll help you remember this by giving you many examples of this type throughout this book.

So, we can say that *any number divided by zero is undefined*, except for $0 \div 0$.

$7 \div 0$ is undefined. $8 \div 0$ is undefined. $10 \div 0$ is undefined. $\frac{10}{0}$ is undefined.

Problem Set:

- 1) $0 \div 3$ 2) $3 \div 0$ 3) $24 \div 0$ 4) $0 \div 13$

Solutions to Problem Set:

- 1) $0 \div 3 = 0$ 2) $3 \div 0$ is undefined 3) $24 \div 0$ is undefined 4) $0 \div 13 = 0$

Dividing Whole Numbers by Powers of Ten

How much is 9,000 divided by 10? The answer is 900. All we did was remove the last zero from 9000. How much is 9,000 divided by 100? The answer is 90. This time we removed the last two zeros from 9000. What's 9,000 divided by 1000? It's 9. We removed three zeros from 9,000. To divide any whole number by a power of 10, we just remove the same number of zeros from the end of the number as there are zeros in the power of ten.

If the number doesn't end in zero, we don't use this method.

Example 7: Divide 560,000 by a) 10 b) 100 c) 1,000 d) 10,000

SOLUTION: a) $560,000 \div 10 = 56,000$ 10 has one zero, so we removed one zero from the end of 560,000.

b) $560,000 \div 100 = 5,600$ 100 has two zeros, so we removed two zeros from the right of 560,000.

c) $560,000 \div 1,000 = 560$ 1,000 has three zeros, so we removed three zeros from 560,000.

d) $560,000 \div 10,000 = 56$ 10,000 has four zeros, so we removed four zeros from 560,000.

Problem Set:

- 1) Divide 500 by 10 2) $1,200 \div 100$
 3) $90,000 \div 10,000$ 4) Divide 1,000,000 by 100,000

Solutions to Problem Set:

- 1) 50 2) 12 3) 9 4) 10

Averages

Now that you know how to divide, we can work on averages. An average is supposed to be a “representative” score. To find the average of a set of numbers, add them and divide their sum by the amount of numbers you added.

Example 8: If you had exam scores of 80, 70 and 78, what was your average score?

SOLUTION: 80 Add the scores.
 70
 +78
 228

 76 Divide by the amount of scores to find the average score of 76.
 3)228
 -21
 18
 -18
 0

Example 9: Over a five year period you earned \$15,000, \$22,500, \$29,000, \$15,700 and \$15,700. What was your average income over this period?

SOLUTION: 15,000 We'll add up the earnings and divide by five.
 22,500
 29,000
 15,700
 +15,700
 97,900 The sum is \$97,900.

 19,580 Divide the sum by 5.
 5)97900
 -5
 47
 -45
 29
 -25
 40
 -40

Example 10: A class's test grades on exam 1 in a statistics course are 100, 92, 85, 20, 0, 17, 83, 88, 50 and 45. What is the class's average score on the test?

SOLUTION: The sum of the exam grades is 580. There are ten grades so we'll divide 580 by 10. Since we're dividing by a power of ten, we don't need a division box. We'll take a short cut and just drop off one zero from 580 and find their average is 58. Here's a thought question for you. Look at the test scores again. Do you think 58 is a good representation of the test scores? Why or why not?

Problem Set:

Find the average of the following sets of numbers.

- 1) 23, 65, 77
- 2) 100, 400, 500, 700, 300
- 3) The following is a list of IQs of a sample of residents of a particular neighborhood. What's the average IQ? 90, 98, 101, 120, 100, 112, 120, 99, 128 and 132.

Solutions to Problem Set:

- 1) $165 \div 3 = 55$
- 2) $2,000 \div 5 = 400$
- 3) $1,100 \div 10 = 110$

Estimation

Sometimes it's not necessary to find the exact answer to a mathematical problem. When the exact answer is not necessary, estimating an answer is generally faster than actually working out the problem. For example, suppose we wanted to estimate the number of textbooks to order for your classes next semester. You might want to estimate the number of hours you'll work by the end of this year. Estimation can also be used to check the answers to arithmetic problems. For example, if we were asked to find the product of 185 and 291, we could estimate the product to be 200×300 , which is 60,000. If we incorrectly multiplied 185 times 291 and got an answer of 503,835, our estimation of 60,000 would tell us to go back and try the problem again. The exact answer is 53,835. By the time you've finished this section, you'll be able to estimate the value of the answer to a problem. Now that you know how to round a number, add, subtract, multiply, and divide, it's time for you to learn how to estimate your answer.

Example 11: Estimate the product of 324 and 546.

SOLUTION:

Before we can estimate the product we have to decide how accurate an estimation we need. We'll work this problem out in two ways. The first one will not be as accurate an estimate as the second.

First: We'll begin by rounding 324 and 546 to the nearest hundred.

$$300 \times 500 = 150,000$$

Second: We'll begin by rounding 324 and 546 to the nearest ten.

$$320 \times 540 = 172,800$$

The actual product is $324 \times 546 = 176,904$

Example 12: Estimate the quotient of 868 and 31.

SOLUTION:

We'll round 868 and 31 to 900 and 30.

$$900 \div 30 = 30$$

The quotient is approximately 30.

Example 13: Estimate the sum of the following numbers: 39, 41, 62 and 11.

SOLUTION: We'll round the numbers to the nearest ten and then add.

$$40 + 40 + 60 + 10 = 150$$

The sum is approximately 150.

Example 14: If you make \$11 an hour, about how much should your paycheck be for 48 hours of work?

SOLUTION: $10 \times 50 = 500$

Your paycheck should be approximately \$500.

Example 15: Assume your car gets 21 miles per gallon. If you drove 77 miles one day, 82 miles the next day and 41 miles the third day, approximately how many gallons of gas did you use?

SOLUTION: First we'll round the miles to 80, 80, and 40. Now we'll add them.

$$80 + 80 + 40 = 200 \text{ miles.}$$

Next we'll round 21 miles per gallon to 20 miles per gallon and divide the number of miles, 200 by the number of miles per gallon, 20, to find the number of gallons used.

$$200 \div 20 = 10$$

Approximately 10 gallons were used.

Problem Set:

Estimate your answer for each of the following.

1) 28×52

2) $621 \div 22$

3) $62 + 41 + 81 + 99$

4) A bookstore ordered 39 cases of a book. Each case contains 28 copies of the book. Approximately how many books did they order?

5) The employees of a particular company received a \$2 hourly increase in pay. If the company has 212 employees, each of whom works 36 hours a week, approximately how much will the increase cost the company per week?

Solutions to Problem Set:

1) $30 \times 50 = 1,500$

2) $600 \div 20 = 30$

3) $60 + 40 + 80 + 100 = 280$

4) $40 \times 30 = 1,200$

5) $2(200)(40) = \$16,000$

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2.4 Workbook

Perform the following divisions. Check your answers.

1) $396 \div 3$

2) $5 \overline{)1235}$

3) $16 \overline{)368}$

4) $4,844 \div 44$

5) Find the quotient of 234 and 24.

6) Find the quotient of 5,567 and 123.

7) Find the quotient of 0 and 10.

8) Find the quotient of 10 and 0.

9) What number divided by 6 is 7?

10) What number divided by 7 is 6?

11) $37,030 \div 46$

12) $83 \overline{)83,249}$

13) $55 \overline{)110,220}$

14) $360,024 \div 12$

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15) $18 \overline{)10,911}$

16) $505,704 \div 101$

17) $23,457 \div 456$

18) $768 \overline{)88,990}$

19) $0 \div 23$

20) $0 \div 1$

21) $23 \div 0$

22) $1 \div 0$

23) $0 \div 7$

24) $7 \div 0$

Divide each of the following numbers by 10.

25) 130

26) 7,000

27) 1,010

28) 0

29) 80

30) 14,600

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Divide each of the following numbers by 100.

31) 125,000

32) 600

33) 0

34) 9,000

35) 100

Divide each of the following numbers by 1,000.

36) 10,000

37) 0

38) 20,000

39) 85,000

40) 3,400,000

41) Find the average of the following: 7, 12, 9 and 8.

42) If Kim had exam grades of 84, 77, 65, 80 and 94, what was his average?

43) Over a six week period your grocery bills were \$77, \$59, \$81, \$70, \$48 and \$67. What was your average grocery bill?

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- 44) The Fly-by-Night flying school made profits of \$67,200 in the first quarter, \$39,500 in the second quarter, \$52,100 in the third quarter, and \$45,300 in the fourth quarter. How much did their profits average quarterly?
- 45) Kevin worked 261 hours in January, 198 hours in February, 202 hours in March, 244 hours in April, 185 hours in May, and 224 hours in June. What were the average monthly hours during this six month period?

(Estimate the answers 46 through 53)

46) 512×488 (use nearest hundred)

47) $42 \div 22$

48) Find the average of $22 + 32 + 99 + 11$

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- 49) 79 children ate 12 M&M candies each. Approximately how many candies did they eat all together?
- 50) Jerry drove 61 miles on Monday, 77 miles on Tuesday, 99 miles on Wednesday. About how many miles did he average in those three days?
- 51) A clothing factory receives three shipments of fabric. The first shipment contains 1,297 yards, the second shipment contains 2,611 yards, and the third shipment contains 4,275 yards. Estimate (use nearest hundred) the total number of yards of fabric the company received.
- 52) Shown below is the attendance for six Atlanta Falcon football games. Estimate the total attendance (to the nearest ten thousand) for these six games:
- | | |
|--------|--------|
| game 1 | 66,144 |
| game 2 | 69,045 |
| game 3 | 70,197 |
| game 4 | 64,972 |
| game 5 | 58,901 |
| game 6 | 74,675 |
- 53) Nader Ghermezian has \$18,426 in his bank account. He writes checks for \$1,197, \$3,066, and \$793. Estimate (use nearest hundred) how much money is left in his account.

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2.5 Arithmetic Properties

Now we'll make some general statements about the arithmetic operations. These statements are called arithmetic properties. The first property is called the *Addition Property of Zero* (also called the Additive Identity). The Addition Property of Zero says that if you add a 0 to any number, its value stays the same. The way to state this formally follows.

Addition Property of Zero

If a is any number, then: $a + 0 = a$ and $0 + a = a$

Some examples of the Addition Property of Zero are:

$$0 + 3 = 3 \qquad 3 + 0 = 3 \qquad 0 + 4 = 4 \qquad 0 + 16 = 16$$

Notice that, in every case, adding 0 doesn't change the number.

The next property is called the *Commutative Property of Addition*. The Commutative Property of Addition says the order in which you add two numbers doesn't change the sum. For example, $5 + 6$ and $6 + 5$ both equal 11. If you reverse the order in which you add two numbers, the answer doesn't change; it stays the same. The way to state this formally follows.

Commutative Property of Addition

If a and b are any two numbers, then: $a + b = b + a$

Some examples of the Commutative Property of Addition are:

$$\begin{array}{cccc} 2 + 3 = 3 + 2 & 4 + 6 = 6 + 4 & 1 + 2 = 2 + 1 & 10 + 20 = 20 + 10 \\ 5 = 5 & 10 = 10 & 3 = 3 & 30 = 30 \end{array}$$

Notice the order in which we add the numbers doesn't change the answer.

The next property is called the *Commutative Property of Multiplication*. The Commutative Property of Multiplication says the order in which you multiply two numbers doesn't change their product. For example $5(6)$ and $6(5)$ both equal 30. If you reverse the order in which you multiply two numbers, the answer doesn't change. It stays the same. The way to state this formally follows.

Commutative Property of Multiplication

If a and b are any two numbers, then: $a(b) = b(a)$

Some examples of the Commutative Property of Multiplication are:

$$\begin{array}{cccc} 2(3) = 3(2) & 4(6) = 6(4) & 1(2) = 2(1) & 10(20) = 20(10) \\ 6 = 6 & 24 = 24 & 2 = 2 & 200 = 200 \end{array}$$

Notice the order in which we multiply two numbers doesn't change the answer.

Remember, in the Commutative Properties of Addition and Multiplication, the order of the numbers changes, they commute positions.

The next property is the *Associative Property of Addition*. The Associative Property says that if we change which numbers are grouped together, it doesn't change the answer. For example, $2 + (3 + 4) = (2 + 3) + 4$. The order of the numbers hasn't changed, so it's not an example of the Commutative Property of Addition. What has changed is which numbers are grouped together. On the left side of the equal sign the 3 and 4 are grouped together. On the right side of the equal sign the 2 and 3 are grouped together. Now we'll work out the problem to prove to you that we really will get the same answer.

$$\begin{aligned}2 + (3 + 4) &= (2 + 3) + 4 \\2 + 7 &= 5 + 4 \\9 &= 9\end{aligned}$$

Notice the order in which the numbers are grouped doesn't change the answer.

$$2 + (3 + 4) = 9 \text{ and } (2 + 3) + 4 = 9$$

Associative Property of Addition

If a , b and c are real numbers, then: $a + (b + c) = (a + b) + c$.

Some examples of the Associative Property of Addition are:

$$\begin{array}{ll}(1 + 5) + 9 = 1 + (5 + 9) & 6 + (7 + 8) = (6 + 7) + 8 \\6 + 9 = 1 + 14 & 6 + 15 = 13 + 8 \\15 = 15 & 21 = 21\end{array}$$

Notice that the order in which the numbers are grouped doesn't change the answer.

The next property is the *Associative Property of Multiplication*. This is the same idea as the Associative Property of Addition, except now it's multiplication. The order in which we group numbers when we're multiplying wouldn't change the outcome. For example,

$$\begin{array}{ll}2(3 \cdot 4) = (2 \cdot 3)4 & 5(4 \cdot 6) = (5 \cdot 4)6 \\2(12) = 6(4) & 5(24) = 20(6) \\24 = 24 & 120 = 120\end{array}$$

Notice the order in which the numbers are grouped doesn't change the answer.

Associative Property of Multiplication

If a , b and c are real numbers, then: $a(b \cdot c) = (a \cdot b)c$.

Some examples of the Associative Property of Multiplication are:

$$\begin{array}{lll}1(2 \cdot 3) = (1 \cdot 2)3 & 6(7 \cdot 8) = (6 \cdot 7)8 & (5 \cdot 4)3 = 5(4 \cdot 3) \\1(6) = 2(3) & 6(56) = (42)8 & 20(3) = 5(12) \\6 = 6 & 336 = 336 & 60 = 60\end{array}$$

Notice that the order in which the numbers are grouped doesn't change the answer.

Multiplication Property of One

The next property is the *Multiplication Property of One* (also called the Multiplicative Identity). The Multiplication Property of One means any number times 1 is itself. In other words, any number times one is the same number. The formal way to state that follows.

If a is any number, then: $a(1) = a$ and $1(a) = a$.

Some examples of the Multiplication Property of One are:

$$2(1) = 2 \qquad 1(9) = 9 \qquad 12(1) = 12 \qquad 1(10) = 10$$

Multiplication Property of Zero

If a is any number, then: $a(0) = 0$ and $0(a) = 0$.

Some examples of the Multiplication Property of Zero are:

$$2(0) = 0 \qquad 0(2) = 0 \qquad 0(3) = 0 \qquad 3(0) = 0 \qquad 5(0) = 0 \qquad 0(5) = 0$$

We've given you a lot to think about, so let's look at some examples together.

Example 1: $2 + 4 = 4 + 2$

This is an example of the Commutative Property of Addition because the order of the numbers has changed.

Example 2: $3(1) = 3$

This is an example of the Multiplication Property of One.

Example 3: $0 + 5 = 5$

This is an example of the Addition Property of Zero.

Example 4: $4(3 \cdot 2) = (4 \cdot 3)2$

This is an example of the Associative Property of Multiplication, not the Commutative Property because the order of the numbers hasn't changed; it's still 4 then 3 then 2, on both sides of the equal sign. What has changed is which numbers are grouped together inside the parentheses.

Example 5: $(7 + 8) + 5 = 7 + (5 + 8)$

This is an example of both the Commutative and the Associative Properties of Addition, because the order of the numbers has changed. 7, 8, 5 changed to 7, 5, 8, and, there are different numbers grouped together. On the left side of the equation the 7 and 8 are grouped together, but on the right side of the equation the 5 and 8 are grouped together.

Now it's time for you to try some problems.

Problem Set:

State the property or properties illustrated in the following examples.

- | | |
|--------------------------------|----------------------------------|
| 1) $17 + 23 = 23 + 17$ | 2) $5 + (6 + 7) = (5 + 6) + 7$ |
| 3) $9 + 0 = 9$ | 4) $9(0) = 0$ |
| 5) $2 + (8 + 3) = (2 + 8) + 3$ | 6) $(2 \cdot 3)4 = 2(4 \cdot 3)$ |

Solutions to Problem Set:

- 1) This is an example of the Commutative Property of Addition because the order of the numbers changed.
- 2) This is *not* an example of the Commutative Property of Addition because the order of the numbers has not changed. It's an example of the Associative Property of Addition because there are different numbers inside the grouping.
- 3) This is an example of the Addition Property of Zero.
- 4) This is an example of the Multiplication Property of Zero.
- 5) This is an example of the Associative Property of Addition, because there are different numbers inside the grouping. This is *not* an example of the Commutative Property of Addition because the order of the numbers hasn't changed.
- 6) This is an example of *both* the Commutative and Associative Properties of Multiplication, because the order of the numbers has changed *and* there are different numbers inside the groupings.

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State the arithmetic properties each of these problems demonstrates.

1) $2 + 11 = 11 + 2$

2) $3 + 4 = 4 + 3$

3) $2(11) = 11(2)$

4) $3(4) = 4(3)$

5) $2 + (7 + 4) = (2 + 7) + 4$

6) $(2 + 7) + 4 = 2 + (7 + 4)$

7) $(2 + 7) + 4 = (4 + 2) + 7$

8) $3 + (9 + 2) = 9 + (2 + 3)$

9) $9 + 0 = 9$

10) $0 + 5 = 5$

11) $5(6 \cdot 9) = (5 \cdot 6)9$

12) $2(3 \cdot 4) = (2 \cdot 3)4$

13) $16(0) = 0$

14) $3(0) = 0$

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15) $(4 + 5) + 6 = (5 + 4) + 6$

16) $(4 \cdot 11)3 = 3(4 \cdot 11)$

17) $(4 \cdot 11)3 = (3 \cdot 11)4$

18) $(5 \cdot 6)8 = (8 \cdot 6)5$

19) $100 + 500 = 500 + 100$

20) $100(500) = 500(100)$

21) $0(5) = 0$

22) $0(10) = 0$

23) $1(5) = 5$

24) $1(10) = 10$

25) $(2 + 7) + 3 = 3 + (7 + 2)$

26) $(7 + 5) + 11 = 11 + (5 + 7)$

2.6 Order of Operations

The order in which you choose to work out a problem often changes the answer. Let's look at a problem where we'll get two different answers to the same problem because we approached the problem in two different orders.

$$\begin{array}{r} 2 + 4(3) \\ 6(3) \\ 18 \end{array}$$

$$\begin{array}{r} 2 + 4(3) \\ 2 + 12 \\ 14 \end{array}$$

Notice we got two different answers to the same problem. In the problem on the left, we added first $2 + 4 = 6$, then multiplied $6(3) = 18$, and got the *wrong* answer, 18. In the problem on the right we multiplied first $4(3) = 12$, then added $2 + 12 = 14$ to get the correct answer 14. Now you're probably wondering how you're supposed to know to multiply first, and then add. We're glad you asked. Here's a step-by-step outline of the correct order to simplify problems.

Order of Operations

Step 1: **Simplify** everything inside groupings first. Some grouping symbols are $()$, $[]$, $\{ \}$, and $| |$. When there's more than one grouping symbol in a problem, work from the inside of the problem to the outside.

Step 2: **Exponents and Roots**.

Step 3: **Multiplication/Division**—If there's a multiplication and a division in one problem, work from left to right.

Step 4: **Addition/Subtraction**—If there's an addition and a subtraction in one problem, work from left to right.

Now let's simplify that last problem using the order of operations.

$2 + 4(3)$	Look at the problem to see which operations have to be done. Here we
$2 + 12$	have multiplication and addition. Multiplication comes before addition in the
14	order of operations, so we'll multiply first, and then add.

The order of operations is a very important topic, so just to make sure you learn it, we'll give you more order of operations problems in a few other chapters in this book.

Let's work out some more problems together.

Example 1: $6 + 5(4 - 2)$

SOLUTION: According to the order of operations, we should do inside the grouping first: $4 - 2 = 2$. Next multiply $5(2) = 10$. Lastly, add $6 + 10 = 16$.

$$\begin{array}{r} 6 + 5(4 - 2) \\ 6 + 5(2) \\ 6 + 10 \\ 16 \end{array}$$

Example 2: $7 + 2(5 - 2)$

SOLUTION: First do inside the grouping: $5 - 2 = 3$. Then multiply $2(3) = 6$.
Add $7 + 6 = 13$.

$$\begin{aligned} 7 + 2(5 - 2) \\ 7 + 2(3) \\ 7 + 6 \\ 13 \end{aligned}$$

Example 3: $4 + 16 \div 2^3$

SOLUTION: Step 1 is inside groupings, but there aren't any, so we'll go to step 2, exponents: $2^3 = 2(2)(2) = 8$. Now it's time for step 3, division: $16 \div 8 = 2$. Last is step 4, addition: $4 + 2 = 6$.

$$\begin{aligned} 4 + 16 \div 2^3 \\ 4 + 16 \div 8 \\ 4 + 2 \\ 6 \end{aligned}$$

Let's try some more challenging problems. The next couple of problems may look tough, but all you have to do is follow the order of operations and you'll be fine.

Example 4: $6 + 2(5 + 40 \div 5) - 2^4$

SOLUTION: First on the order of operations is inside groupings. Inside the grouping we have a choice of addition or division. We'll divide first: $40 \div 5 = 8$. Next add $5 + 8 = 13$. Now that we've done everything that could be done inside the grouping, we'll look outside the grouping where we have a choice of addition, multiplication, or exponents. Exponents are next on the order of operations: $2^4 = 16$. Now multiply, $2(13) = 26$. Add $6 + 26 = 32$ then subtract, $32 - 16 = 16$.

$$\begin{aligned} 6 + 2(5 + 40 \div 5) - 2^4 \\ 6 + 2(5 + 8) - 2^4 \\ 6 + 2(13) - 2^4 \\ 6 + 2(13) - 16 \\ 6 + 26 - 16 \\ 32 - 16 \\ 16 \end{aligned}$$

Example 5: $(18)(3) \div (2)(3)$

SOLUTION: Be very careful not to make a very common error on this problem. $(18)(3) \div (2)(3)$ is not the same as $(18 \cdot 3) \div (2 \cdot 3)$. In $(18)(3) \div (2)(3)$, parentheses are used to show multiplication—not as a grouping symbol, so we would work left to right. In $(18 \cdot 3) \div (2 \cdot 3)$, parentheses are used to show a grouping, so we would work inside the grouping first. This changes the order in which we should begin this problem. The parentheses in this problem are *not* used as a grouping symbol, but to show multiplication. This example has multiplication and division, so we'll work left to right, $(18)(3) = 54$, $54 \div 2 = 27$, $27(3) = 81$.

$$\begin{aligned} (18)(3) \div (2)(3) \\ 54 \div (2)(3) \\ 27(3) \\ 81 \end{aligned}$$

Problem Set:

Try the following order of operation problems.

1) $6 + 4(5 - 1)$

2) $8 + 2^3$

3) $40 \div 10 - 2$

4) $40 \div (10 - 2)$

5) $(16 - 4^2) \div (10 - 2 \cdot 3)$

Solutions to Problem Set:

1) $6 + 4(5 - 1)$
 $6 + 4(4)$
 $6 + 16$
 22

Inside the grouping first, $5 - 1 = 4$.
Multiply $4(4) = 16$.
Add $6 + 16 = 22$.

2) $8 + 2^3$
 $8 + 8$
 16

Exponents before addition: $2^3 = 2(2)(2) = 8$.
Add $8 + 8 = 16$.

3) $40 \div 10 - 2$
 $4 - 2$
 2

Divide first: $40 \div 10 = 4$.
Subtract $4 - 2 = 2$.

4) $40 \div (10 - 2)$
 $40 \div 8$
 5

Inside the grouping first, $10 - 2 = 8$.
Divide $40 \div 8 = 5$.

5) $(16 - 4^2) \div (10 - 2 \cdot 3)$
 $(16 - 16) \div (10 - 6)$
 $0 \div 4$
 0

Inside groupings first; exponents before subtraction: $4^2 = 16$.
Multiplication before subtraction, $2 \cdot 3 = 6$.
Finish inside the groupings, then divide.
Be careful, $0 \div 4 = 0$, but $4 \div 0$ is undefined.
This is carefully explained in Section 2.3.

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2.6 Workbook

Apply the order of operations to the following problems.

1) $24 + 6 \div 6$

2) $5 - 5 \div 5$

3) $16 \div 4 \div 4$

4) $100 \div 5 \div 5$

5) $16 \div (4 \div 4)$

6) $100 \div (5 \div 5)$

7) $16 \div 4^2$

8) $100 \div 5^2$

9) $5(4)^2$

10) $3(10)^2$

11) $2 + 4(5 + 6 \div 6)$

12) $100 - 50 \div [10 - (2 \cdot 3) + 1]$

13) $(8 \div 8) \div (4 - 2^2)$

14) $(2 \div 2) \div (16 - 2^4)$

15) $(4 - 2^2) \div (8 \div 8)$

16) $(16 - 2^4) \div (2 \div 2)$

2.6 Workbook

17) $8 \div (8 \div 4) - 2^2$

18) $(10 \div 2)^2 - 2^4$

19) $5^3 - 5^2 \div 5$

20) $3 + [4(15 \div 5) \div 6]$

21) $(7 + 5) \div (5 - 3)$

22) $40 + 5(6 + 4 \div 2)$

23) $7 + (5 \div 5) - 3$

24) $4 + 5(6 + 4) \div 2$

Challenge Problems

25) $5(3)^4 + 16 \div 4 - 2^2$

26) $3 + 2[8 + 5(6 - 4)]$

27) $8^2 \div (9 - 1)^2$

28) $7^2[6 + 4(10 + 12)]$

29) $9^2 + 4^2 \div 2^3$

30) $[(6 \div 3)(8 \div 2)] \div 8$

31) $6^2[5 + 2(8 - 2)^2]$

32) $5^4 \div 5^3 \div 5^0$

2.7 Applications

Now we're going to apply a lot of what you've learned in chapters 1 and 2 about adding, subtracting, multiplying, and dividing. The question students always ask about these applications is: how do you know what to do? In other words, how do you know when you're supposed to add, subtract, multiply, or divide? By the time you've completed this section, you'll no longer need to ask that question.

Applications in Addition and Subtraction

So far we've done straightforward numerical problems. Now we'll do some word problems. You'll need to figure out whether each of these problems calls for addition, subtraction, or a combination of addition and subtraction.

Example 1: Josephina earned \$48 on Monday, \$39 on Tuesday, and \$53 on Wednesday. How much did she earn all together?

SOLUTION: Whenever a question asks for "all together," we have to add.
 $\$48 + \$39 + \$53 = \$140.$

Example 2: Max had \$206 with him when he got to the mall. When he left the mall he had \$97. How much did he spend at the mall?

SOLUTION: Max has less than he started with, so this is a subtraction problem. Whenever we have to find the difference between the original amount and the remaining amount we subtract.
 $\$206 - \$97 = \$109.$

Example 3: Paul had \$571 in his checking account at the beginning of the month. A few days later he deposited \$359 and wrote a check for \$175. How much money did he have left in his account at the end of the month if he didn't write any more checks or make any more deposits?

SOLUTION: Paul began the month with \$571 in his account. His \$359 deposit raised his account balance to \$930 ($\$571 + \$359 = \930). Then his balance went down to \$755 after he wrote a check for \$175 ($\$930 - \$175 = \755).

So far it has been pretty easy to tell from the wording of the problems whether to add, subtract, or to add and subtract. Let's try some problems that are a bit harder.

Example 4: Mark's score on the exam was 92. Sarah's score was 26 points lower. What was Sarah's score on the exam?

SOLUTION: To get a lower score you have to subtract. Start with Mark's score of 92 and subtract 26 to get Sarah's score. $92 - 26 = 66.$

Example 5: The difference between two numbers is 149. If the smaller number is 568, how much is the larger number?

SOLUTION: To find the larger number add 149 to the smaller number, 568.
 $568 + 149 = 717$.

Example 6: When you started climbing a mountain you were 5,844 feet above sea level. When you got to the top, you were 11,909 feet above sea level. How far did you climb?

SOLUTION: To find the difference in the feet above sea level, subtract.
 $11,909 \text{ feet} - 5,844 \text{ feet} = 6,065 \text{ feet}$.

Problem Set:

- 1) The Chicago Cubs scored two runs in the third inning, one run in the fourth inning, five runs in the sixth inning, and three runs in the ninth inning. How many runs did they score in the entire nine inning game?
- 2) The boss brought \$450 to work on Friday. She paid one of her workers \$225 and she paid her other worker \$190. How much money did the boss have left?
- 3) Jason left home with 26 marbles. He gave 9 marbles to Melissa and Sue gave him 15. How many marbles did Jason end up with?
- 4) Hans weighs 126 pounds and Kerry weighs 89 pounds. What is the difference in their weights?
- 5) When you were leaving on a trip, your car's odometer read 14,199. When you arrived at your destination, your odometer read 16,056. How far did you travel?

Solutions to Problem Set:

- 1) $2 + 1 + 5 + 3 = 11$ runs
- 2) $450 - 225 - 190 = \$35$
- 3) $26 - 9 + 15 = 32$ marbles
- 4) Difference means subtract. $126 - 89 = 37$ pounds
- 5) $16,056 - 14,199 = 1,857$ miles

Applications in Multiplication and Division

Example 7: Roberto has 48 folders. Each folder holds 39 pages. How many pages are in the folders?

SOLUTION: To find the total number of pages, we need to multiply the number of pages in each folder (39) by the number of folders (48):

$$\begin{array}{r} 39 \\ \times 48 \\ \hline 312 \\ 156 \\ \hline 1872 \text{ pages} \end{array}$$

Example 8: An end-of-the-year bonus of \$967,486 is divided equally among 26 employees. How much is each person's share?

SOLUTION: To find out how much each person's share is, we have to divide the bonus, \$967,486, by the number of employees, 26.

$$\begin{array}{r} 37211 \\ 26 \overline{)967486} \\ \underline{-78} \\ 187 \\ \underline{-182} \\ 54 \\ \underline{-52} \\ 28 \\ \underline{-26} \\ 26 \\ \underline{-26} \\ 0 \end{array}$$

Each person's share is \$37,211.

Problem Set:

- 1) There are 23 girls in a club. Each girl brings 72 cookies to a party. How many cookies did they bring all together?
- 2) If Jim Panepinto hit an average of 49 home runs each year and his career lasted 23 years, how many home runs did he hit during his career?
- 3) Every day Sophie brings 140 pennies to her grandmother's house. How many pennies does she bring in 7 days? How many days will it take Sophie to save up \$28?
- 4) If there are 55 M&Ms in a bag, how many bags can be formed from a vat of 5,555 M&Ms?

Solutions to Problem Set:

- 1) There are 23 girls. Each brought 72 cookies. How many cookies did they all bring? We have to multiply.

$$\begin{array}{r} 72 \\ \times 23 \\ \hline 216 \\ 144 \\ \hline 1656 \text{ cookies} \end{array}$$

2) Multiply

$$\begin{array}{r} 49 \\ \times 23 \\ \hline 147 \\ 98 \\ \hline 1127 \end{array}$$

home runs

3)

$$\begin{array}{r} 140 \\ \times 7 \\ \hline 980 \end{array}$$

pennies

There are 100 pennies in one dollar. To change the \$28 to pennies, we multiply by 100: $28 \times 100 = 2800$ pennies. She saves 140 pennies a day, so we divide the 2800 pennies by 140 pennies and get our answer, 20 days.

$$\begin{array}{r} 20 \\ 140 \overline{)2800} \\ \underline{-280} \\ 00 \end{array}$$

- 4) This is a division problem because we're forming smaller groups taken from a larger group.
101 bags of M&Ms can be formed.

$$\begin{array}{r} 101 \\ 55 \overline{)5555} \\ \underline{-55} \\ 05 \\ \underline{-00} \\ 55 \\ \underline{-55} \\ 0 \end{array}$$

2.7 Workbook

- 1) The Philadelphia Eagles scored seven points in the first quarter, ten points in the second quarter, three points in the third quarter, and fourteen points in the fourth quarter. How many points did they score in the entire game?
- 2) Nancy is 19 years older than Ann. If Nancy is 45, how old is Ann?
- 3) The ABC Corporation paid each of its 492 employees \$673. How much was its payroll?
- 4) 4,935 cookies were divided equally among 235 wedding guests. How many cookies did each guest receive?
- 5) A plane flew 808 miles from New York to Chicago, and 2,166 miles from Chicago to San Francisco. What was the entire distance the plane flew? How much closer is Chicago to New York than to San Francisco?
- 6) A truck left the bakery with 241 loaves of bread. It dropped off 137 loaves at one location and picked up 55 loaves at another location. How many loaves of bread were now on the truck?
- 7) Mrs. Hashimoto had a checking account balance of \$2,137. If she wrote checks for \$275 and \$458, how much money was left in her account?
- 8) Ms. Romano paid \$4,712 for 248 ladies' watches. How much did each watch cost?
- 9) Barbara's score on an exam was 83. Mike's score was 19 points lower. What was Mike's score on the exam?
- 10) A bus left Billings with 42 passengers. At Helena 13 people got off the bus. At Spokane 9 people got on the bus. How many people were still on the bus as it left Spokane?

2.7 Workbook

- 11) A total of 15,246 VCRs were shipped to 7 stores. If each store received the same number of VCRs, how many did each store receive?
- 12) 54 counties each sent the state government \$11,250. How much money did the government receive?
- 13) The difference between two numbers is 382. If the larger of the two numbers is 601, how much is the smaller number?
- 14) Jose left the house with \$317. He spent \$129 on clothes and \$84 on groceries. How much money did he have left?
- 15) At James Madison High School there are 1,816 freshmen, 2,003 sophomores, 1,919 juniors, and 1,793 seniors. How many students go to James Madison High School?
- 16) Bob is 23 years older than Sue. If Sue is 49, how old is Bob?
- 17) Phillip worked 13 hours of overtime in January, 19 hours of overtime in February, 12 hours of overtime in March, and 8 hours of overtime in April. How many hours of overtime did he work during these four months?
- 18) 656 gold coins are shared equally by 8 people. How many coins does each person get?
- 19) Kyra left for work with \$283. She spent \$9 on lunch, \$54 on shoes, and \$8 on a watch. How much money did she have left?
- 20) A train traveled 258 miles from Montreal to Albany, and 143 miles from Albany to New York City. What was the total distance the train traveled?

2.7 Workbook

- 21) At the Chinese Communist Party Congress, the delegates from each of 19 provinces cast 275 votes each for the new Prime Minister. How many votes were cast for the Prime Minister?
- 22) If a shipment containing 36,248 cans of tuna fish arrived at a warehouse, and if the shipment was packed in 788 cartons, how many cans were in each carton?
- 23) At James Monroe High School there are 2,017 freshmen, 1,932 sophomores, 1,819 juniors, and 1,687 seniors. How many students go to James Monroe High School?
- 24) If Hans is 34 years older than Joe, how old is Joe if Hans is 61?
- 25) There are 214 passengers on a plane bound for Detroit. At Detroit, 98 passengers get off the plane and 77 get on. How many people are now on the plane?
- 26) A family has an annual income of \$42,180. How much is its monthly income?
- 27) A shoe store orders 123 pairs of shoes for a total of \$5,658. If the store paid the same amount for each pair of shoes, how much was each pair of shoes?
- 28) A truck with 311 cases of soda dropped off 154 cases at one location and picked up 38 cases at another. How many cases of soda were now on the truck?
- 29) Mr. O'Boyle had a checking account balance of \$5,071. If he made a deposit of \$1,250, and then wrote a check for \$425, how much money would be in his account?
- 30) The difference between two numbers is 457. If the smaller of the two numbers is 366, how much is the larger number?

2.7 Workbook

- 31) How many four ounce cups can be filled from a 256-ounce bottle?
- 32) A store was open 16 hours on Monday, 13 hours on Tuesday, 17 hours on Wednesday, 10 hours on Thursday, and 15 hours on Friday. How many hours was the store open during these five days?
- 33) During the 1990s the population of Los Alamos rose by 4,199. If the population of the city was 16,394 in the year 2000; how much was the population of Los Alamos in 1990?
- 34) Ms. Hirsch earned \$61,086 in the year 2001. In 2002 she earned \$98,175. How much less did she earn in 2001 than in 2002?
- 35) Karen left home with 36 marbles. If she gave 14 to John and received 19 from Andy, how many marbles did Karen have now?
- 36) American Airlines flies 14 flights from Chicago to Los Angeles every day. If an average of 319 passengers are on each plane, estimate (to the nearest ten) the number of passengers American flies from Chicago to Los Angeles every day.
- 37) Nariman Behravesh left his three flocks of sheep to his children. There were 129 sheep in the first flock, 178 sheep in the second flock and 144 sheep in the third flock. If the sheep were divided equally among his 11 children, approximately how many sheep did each child get?

Chapter 2 Review

1)
$$\begin{array}{r} 85 \\ \times 6 \\ \hline \end{array}$$

2)
$$\begin{array}{r} 162 \\ \times 8 \\ \hline \end{array}$$

3)
$$\begin{array}{r} 942 \\ \times 5 \\ \hline \end{array}$$

4)
$$\begin{array}{r} 235 \\ \times 64 \\ \hline \end{array}$$

5)
$$\begin{array}{r} 837 \\ \times 425 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 7496 \\ \times 563 \\ \hline \end{array}$$

7) Multiply each of these numbers by 10, 100, and 1,000.

a) 2

b) 10

c) 360

d) 3,100

8)
$$\begin{array}{r} 317 \\ \times 200 \\ \hline \end{array}$$

9)
$$\begin{array}{r} 928 \\ \times 4000 \\ \hline \end{array}$$

10) Simplify each of the following.

a) 6^2

b) 2^6

c) 4^0

d) 3^3

11)
$$5 \overline{)640}$$

12)
$$7 \overline{)1533}$$

13)
$$25 \overline{)1875}$$

14)
$$49 \overline{)9641}$$

15)
$$40 \overline{)7306}$$

16)
$$85 \overline{)17340}$$

17) Divide each of these numbers by 10, 100, and 1,000.

a) 416,000

b) 1,000,000

c) 82,000

d) 9,000

18) Find the average of 79, 24, 90, 61, and 56.

19) State the arithmetic properti(es) each of these problems demonstrates.

a) $2(8) = 8(2)$

b) $3 + (1 + 6) = (3 + 1) + 6$

c) $(5 \cdot 9) \cdot 2 = (2 \cdot 9) \cdot 5$

d) $4(0) = 0$

- 20) How much is $8 + 3(7 - 2)$?
- 21) How much is $2 + 3^3$?
- 22) How much is $20(6 - 2)$?
- 23) How much is $(16 \div 2^2) - 4$?
- 24) A bus with 29 passengers made two stops. At the first stop 8 people got on and 4 got off. At the second stop 3 got on and 2 got off. How many people were left on the bus?
- 25) The Miami Dolphins scored 12 points in the first quarter, 7 points in the second quarter, 14 points in the third quarter, and 3 points in the fourth quarter. How many points did they score in the entire game?
- 26) Ms. Koshima had a checking account balance of \$5,167. If she wrote checks for \$325, \$578, and \$290, how much money was left in her account?
- 27) The XYZ Corporation paid each of its 519 employees a bonus of \$622. How much money did the company pay out in bonuses?
- 28) A total of 964 cartons of food was divided equally among four food pantries. How many cartons did each food pantry receive?
- 29)
$$\begin{array}{r} 76 \\ \times 6 \\ \hline \end{array}$$
- 30)
$$\begin{array}{r} 465 \\ \times 98 \\ \hline \end{array}$$
- 31)
$$\begin{array}{r} 6243 \\ \times 257 \\ \hline \end{array}$$
- 32) Multiply each of these numbers by 10, 100, and 1,000.
- a) 64 b) 4,000 c) 6 d) 70
- 33)
$$\begin{array}{r} 4128 \\ \times 5000 \\ \hline \end{array}$$
- 34) Simplify each of these.
- a) 4^3 b) 3^4 c) 10^0 d) 10^3

35) $8\overline{)4064}$

36) $61\overline{)14387}$

37) $60\overline{)72120}$

38) Divide each of these numbers by 1,000, 10,000, and 100,000.

a) 3,500,000

b) 500,000

c) 434,000,000

39) A winning lottery prize of \$14,500,000 is split 100 ways. How much is each person's share?

40) If there are 116 cans of tuna fish in a carton, how many cans are there in 2,900 cartons?

41) State the arithmetic property(ies) each of these problems demonstrates.

a) $3 + 6 = 6 + 3$

b) $5 + 0 = 5$

c) $2(4 \cdot 3) = (2 \cdot 4)3$

d) $(2 + 3) + 4 = 2 + (4 + 3)$

42) Simplify the following.

a) $4 + 2(8 - 4)$

b) $(3 \cdot 4^2) - 5^2$

c) $(20 \div 5)^2 - 3^2$

d) $(3^4 - 9^2) \div (60 - 3^3)$

43) A band of explorers set off on a journey of 5,122 miles. After they covered 1,847 miles, how much further did they have to go?

44) One hundred ninety-six stockholders each held four thousand shares. How many shares did they hold all together?

45) If Atul Gupta consumed 1,944 calories on Monday, 2,012 on Tuesday, 1,796 on Wednesday, 1,834 on Thursday, and 2,007 on Friday. About how many calories did he consume all together?

46) If Abdul Hanna earns \$19 an hour, estimate how much he earns for 303 hours of work.

Chapter 2 • Test 1

1)
$$\begin{array}{r} 34 \\ \times 5 \\ \hline \end{array}$$

2)
$$\begin{array}{r} 192 \\ \times 7 \\ \hline \end{array}$$

3)
$$\begin{array}{r} 829 \\ \times 45 \\ \hline \end{array}$$

4)
$$\begin{array}{r} 209 \\ \times 687 \\ \hline \end{array}$$

5)
$$\begin{array}{r} 2,983 \\ \times 3,487 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 10,982 \\ \times 2,981 \\ \hline \end{array}$$

7) Multiply each of these numbers by 10, 100, and 1,000.

a) 5 _____

b) 10 _____

c) 546 _____

d) 4,567 _____

8)
$$\begin{array}{r} 267 \\ \times 300 \\ \hline \end{array}$$

9)
$$\begin{array}{r} 2949 \\ \times 4000 \\ \hline \end{array}$$

10) Simplify each of the following:

a) 3^4

b) 4^3

c) 6^0

d) 4^4

11) $8\overline{)984}$

12) $65\overline{)6825}$

13) $32\overline{)128256}$

14) $136\overline{)3808}$

15) Divide each of the following numbers by 10, 100, and 1,000.

a) 52,000 _____

b) 1,000,000 _____

c) 720,000 _____

d) 1,000 _____

16) Find the average of the following test scores: 94, 68, 82, and 80. _____

17) State the arithmetic properti(es) each of these problems demonstrates.

a) $3(4) = 4(3)$

b) $2(5 \cdot 7) = 5(7 \cdot 2)$

18) How much is $4 + 3(32 - 8 \div 2^3)$?

19) How much is $(2^0 - 1^{20}) \div 5^2$?

20) A bus with 55 passengers made two stops. At the first stop 10 people got on and 11 got off. At the second stop 3 got on and 21 got off. How many people were left on the bus?

21) Miss Iorio had a checking account balance of \$6,243. If she wrote checks for \$235, \$3,456, and \$45, how much was left in her account?

22) A total of 1,410 basketballs was shipped to six stores; if each store received an equal amount of basketballs, how many basketballs did each store receive?

23) How many 8-ounce cups of water can be filled from a one gallon container? (There are 128 ounces in a gallon).

24) If the heights of four brothers are 70 inches, 75 inches, 72 inches and 67 inches, what's the average height of the brothers in inches?

25) A group of fifteen children went shopping, if they spent an average of \$26 each on toys, how much did they spend all together?

26) A clothing factory receives three shipments of fabric. The first shipment contains 1,297 yards, the second shipment contains 2,611 yards, and the third shipment contains 4,275 yards. Estimate (to the nearest thousand) the total number of yards of fabric the company received.

27) The graduating class of the entire University of California was 27,008. If 14,197 were men, about how many were women?

Chapter 2 • Test 2

1)
$$\begin{array}{r} 96 \\ \times 4 \\ \hline \end{array}$$

2)
$$\begin{array}{r} 298 \\ \times 873 \\ \hline \end{array}$$

3)
$$\begin{array}{r} 598 \\ \times 67 \\ \hline \end{array}$$

4)
$$\begin{array}{r} 402 \\ \times 521 \\ \hline \end{array}$$

5)
$$\begin{array}{r} 4,882 \\ \times 6,798 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 11,993 \\ \times 4,588 \\ \hline \end{array}$$

7) Multiply each of these numbers by 10, 100, and 1,000.

a) 6 _____

b) 76 _____

c) 748 _____

d) 5,870 _____

8)
$$\begin{array}{r} 398 \\ \times 200 \\ \hline \end{array}$$

9)
$$\begin{array}{r} 2,938 \\ \times 5,000 \\ \hline \end{array}$$

Simplify each of the following.

10) a) 2^6 b) 7^0 c) 6^2 d) 5^3

11) $7\overline{)1218}$

12) $26\overline{)15262}$

13) $65\overline{)65130}$

14) $364\overline{)322868}$

15) Divide each of the following numbers by 10, 100, and 1,000.

a) 13,000 _____

b) 640,000 _____

c) 1,000 _____

d) 2,000,000 _____

16) Find the average of the following students' ages: 19, 22, 18, 21, 25. _____

17) State the arithmetic properti(es) each of these problems demonstrates.

a) $2(0) = 0$

b) $3 + (4 + 5) = 4 + (3 + 5)$

18) How much is $5 + 2(16 - 4 \div 2^2)$?

19) How much is $3^4 \div (9^2 - 3^4)$?

20) A bus with 30 passengers made two stops. At the first stop 6 people got on and 11 got off. At the second stop 5 got on and 3 got off. How many people were left on the bus?

21) If a bin contains 11,760 M&M candies, how many packets containing 21 candies each can be formed from the bin?

22) An inheritance of \$46,701 is to be equally split among three sisters. If \$2,000 is spent on legal fees, and \$6,889 on taxes, how much will each sister inherit?

23) A student's grades in this course are: 88, 92, 73, 52 and 0. What's the student's average in the course? How did the zero affect the grade?

24) Mrs. Marek had a checking account balance of \$7,910. She wrote checks for \$67, \$567, \$890 and \$2,334. She also made a deposit of \$567. What's her new account balance?

25) The weights in pounds of a group of children are: 55, 63, 29, 44, 77 and 56. Find the average weight.

26) Estimate the sum of the ages of these family members:

grandfather, 78

grandmother, 76

father, 53

mother, 52

daughter, 16

son, 13

27) Chris' Corner restaurant in Bayonne founded by Anselmo Crisonino, uses about 159 pounds of mozzarella a week. Estimate how many pounds of mozzarella they will use in one year, time.

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